Exploring **Fractions** in the 2011 MA Curriculum Framework for Mathematics

Exploration Activity: Fractions From First to Sixth Grade

Massachusetts Department of
ELEMENTARY & SECONDARY
EDUCATION

Session Goal

We will explore:

- **how the shifts in the 2011 Mathematics Curriculum Frameworks help to build conceptual knowledge of fractions**
- **how these fraction concepts progress through the grades.**

Session Overview

I. Setting the Context: Fraction Math Problem

II. Key Shifts and the Fractions Progression

III. Fractions Progression:

- A. Grades 1and 2: Partition Shapes
- B. Grade 3-5: Fraction are Critical Areas
- C. Grade 6:
  - ★ Dividing Fractions by Fractions
  - ★ Connecting Fractions to Ratios and Proportional Relationships

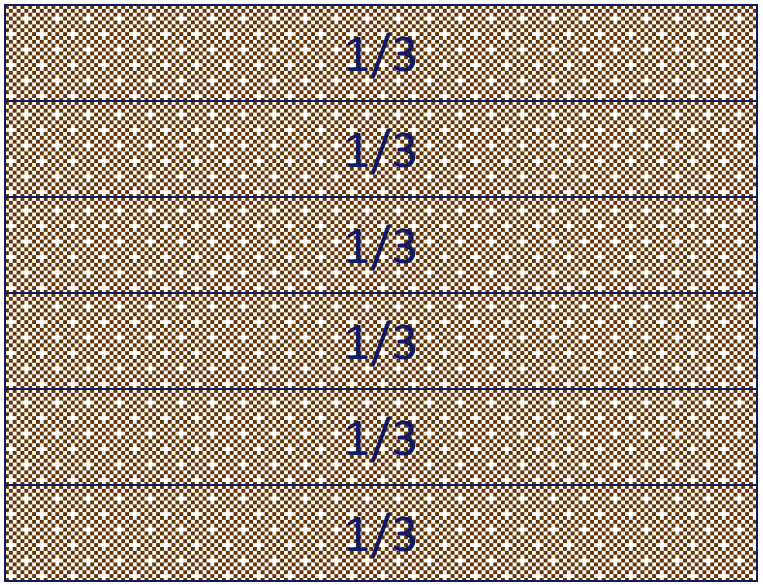
Setting the Context

How many 1/3 cup servings are in 2 cups of raisins?

5.NF.7.c., page 51, Massachusetts Curriculum Framework for Mathematics, March 2011

Setting the Context

How many 1/3 cup servings are in 2 cups of raisins?

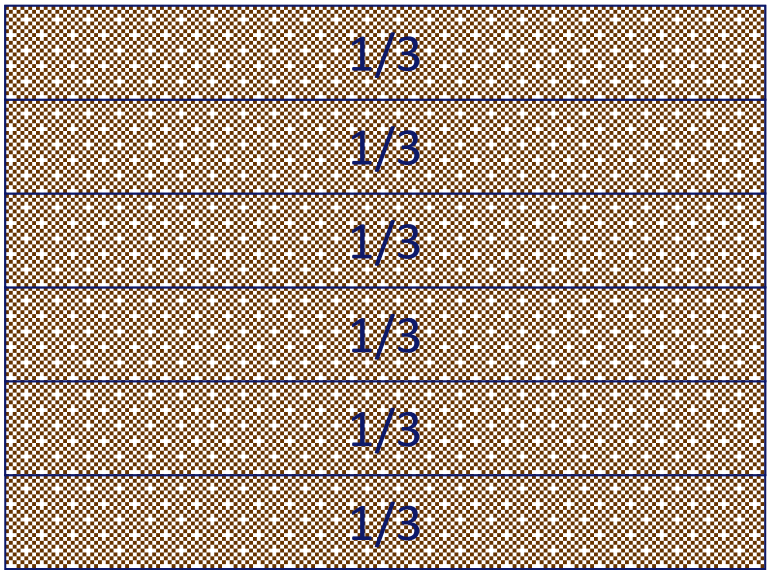


5.NF.7.c., page 51, MP.2, p. 15 Massachusetts Curriculum Framework for Mathematics, March 2011

Setting the Context

How many 1/3 cup servings are in 2 cups of raisins?

2 ÷ 1/3 = 6 servings



5.NF.7.c., page 51, Massachusetts Curriculum Framework for Mathematics, March 2011

II. Key Shifts and the Fractions Progression

The Progression: Fractions to Ratios and Proportional Reasoning

Exploring the key shifts in this progression:

- **Focus**: fractions are identified as Critical Areas in grades 3-5
- **Coherence**: fraction concepts build from grades 1-6
- **Clarity**: precise language, specifying use of a variety of types visual models
- **Rigor**:
  - visual representations to support conceptual understanding
  - ongoing development of the Standards for Mathematical Practice

Domain Progression in the 2011 MA Math Frameworks

PK-8 Domains Progression										
Domains	**PK**	**K**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Counting and Cardinality	MA									
Operations and Algebraic Thinking	MA									
Number and Operations in Base Ten										
Number and Operations - Fractions										
Ratios and Proportional Relationships										
The Number System							MA			
Expressions and Equations										
Functions										
Measurement and Data	MA									
Geometry	MA									
Statistics and Probability										

Coherence Exploration http://www.doe.mass.edu/candi/commoncore/mathexplore/

Fractions Progression and the Standards for Mathematical Practice

Visual representations and modeling help students:

- ★ Understand
- ★ Communicate
- ★ Problem Solve

STANDARDS FOR MATHEMATICAL PRACTICE

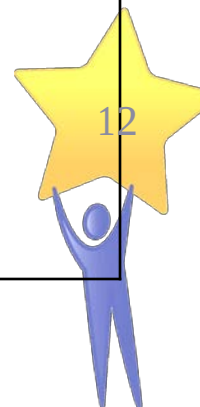
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Visual Models in the New Standards, an Example

4.NF.1 Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using **visual fraction models**, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

Visual Models in the New Standards

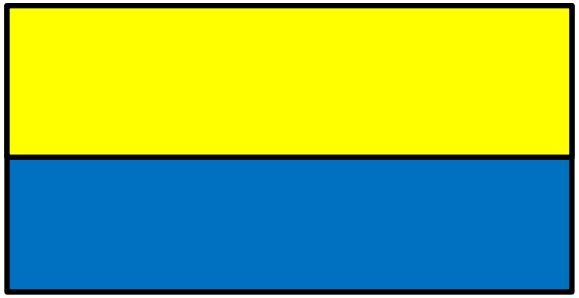
3.NF1.	4.NF 5.
3.NF2 a. number line diagram	4.NF 6. *For example… a number line diagram.*
3.NF2 b. number line diagram	4.NF 7. *e.g., by using a visual model.*
3.NF3. a. number line	Number and Operations—Fractions 5.NF
3.NF3. b. by using a visual fraction model	
3.NF3. c. *Examples: number line diagram.*	5.NF 1.
3.NF3. d. *e.g., by using a visual fraction model.*	5.NF 2. *e.g., by using visual fraction models*
	5.NF 3. *e.g., by using visual fraction models*
4.NF 1. by using visual fraction models	5.NF 4. a. *For example, use a visual fraction model*
4.NF 2. *e.g., by using a visual fraction model.*	5.NF 4. b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares
	5.NF 5. a.
4.NF 3. a.	5.NF 5. b.
4.NF 3. b. *e.g., by using a visual fraction model.*	5.NF 6. *e.g., by using visual fraction models*
4.NF 3. c.	5.NF 7. a. *For example…use a visual fraction model*
4.NF 3. d. *e.g., by using visual fraction models*	5.NF 7. b.*For example… use a visual fraction*
4.NF 4. a. *For example, use a visual fraction model*	5.NF 7. *c.e.g., by using visual fraction models*
4.NF 4. b. *For example, use a visual fraction model*	
4.NF 4. c. *e.g., by using visual fraction models*	



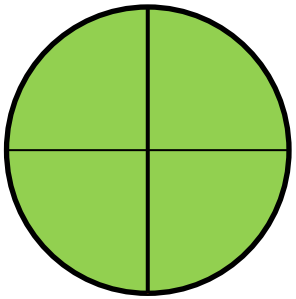
III. Grades One and Two: Partition Shapes

Building Fraction Language in 1st Grade

1.G.3 Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.*



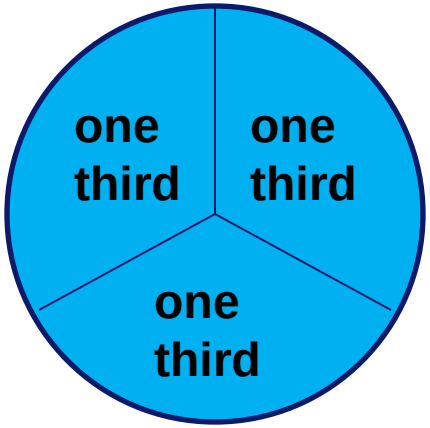
"Half of the rectangle is blue."

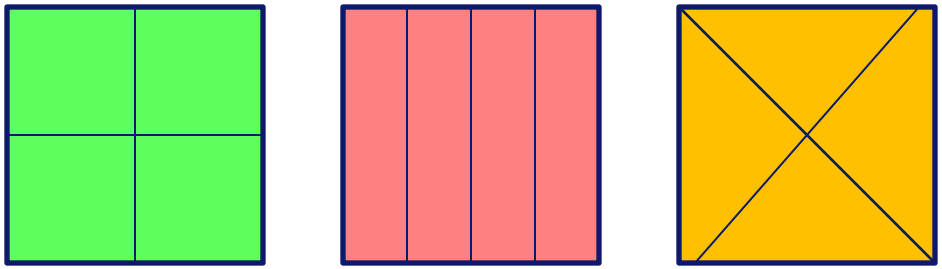


"The circle is composed of 4 fourths or 4 quarters or 4 equal shares."

Building Fraction Language in 2nd Grade

2.G.3 Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.*





"These squares are all composed of 4 fourths, but the fourths look different."

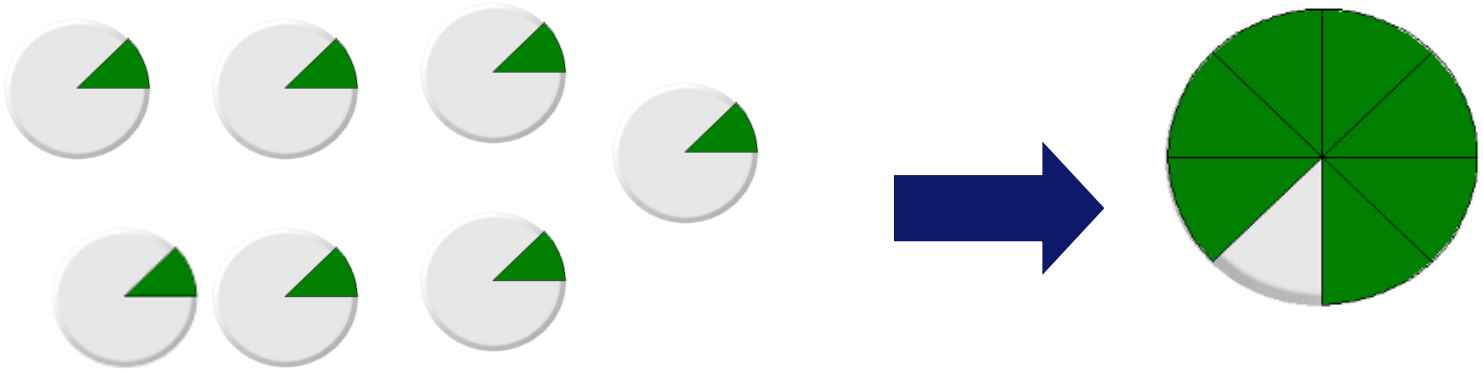
IV. Grades Three through Five: Fractions are Critical Areas in these Grades

Third Grade Fraction Critical Area #2

(2) Students develop an understanding of fractions, beginning with **unit fractions**. Students view fractions in general as being built out of unit fractions, and they use fractions along with **visual fraction models** to represent parts of a whole. Students understand that the size of a fractional part is relative to the **size of the whole**. They solve problems that involve **comparing fractions** by using visual fraction models and strategies based on noticing equal numerators or denominators.

Page 38, Massachusetts Curriculum Framework for Mathematics, March 2011
(bolded for emphasis)

3rd Grade: Fractions are Composed of Unit Fractions

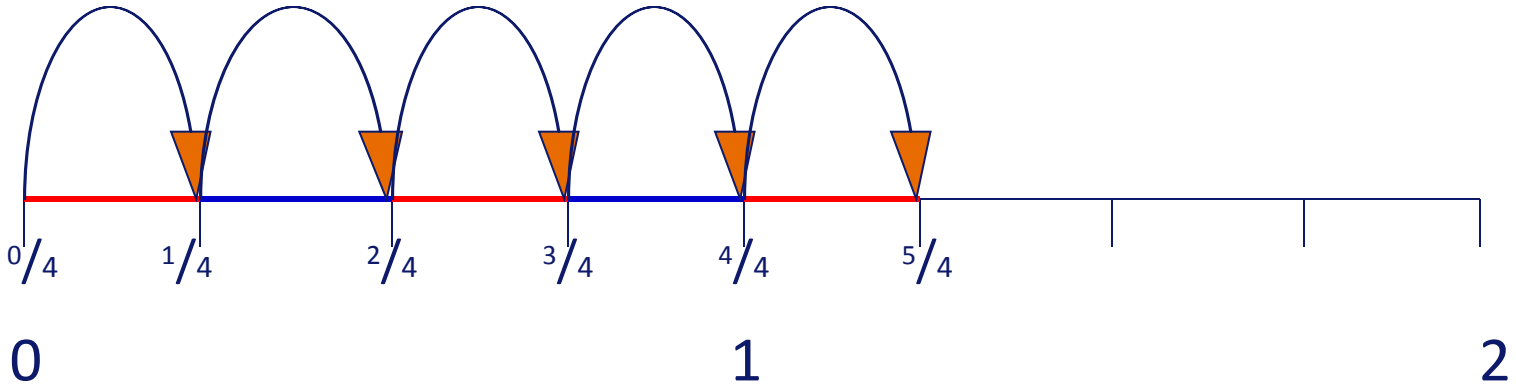


$$\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}=\frac{7}{8}$$

$$\therefore \frac{7}{8}$$

3.NF.1, page 41, MP 3, p. 15 Massachusetts Curriculum Framework for Mathematics, March 2011

3rd Grade: Fractions are Composed of Unit Fractions

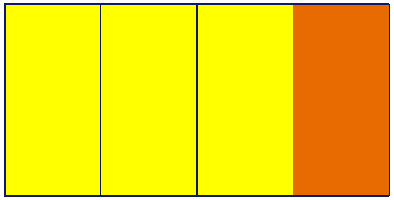


$$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{5}{4}$$

3.NF.2.b., page 41 Massachusetts Curriculum Framework for Mathematics, March 2011

3rd Grade: The Importance of Specifying the Size of the Whole

Example 1: What fraction is represented by the yellow area?

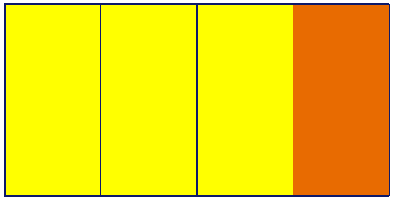


3.NF.3.d., page 41, MP 7, p.16 Massachusetts Curriculum Framework for Mathematics, March 2011

3-5 Number and Operations--Fractions
http://ime.math.arizona.edu/progressions
Project Director: Bill McCallum

3rd Grade: The Importance of Specifying the Size of the Whole

Example 1: What fraction is represented by the yellow area?



Is this ¾ or 1 ½?

3.NF.3.d., page 41, MP 7, p.16 Massachusetts Curriculum Framework for Mathematics, March 2011

3-5 Number and Operations--Fractions
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3rd Grade: The Importance of Specifying the Size of the Whole

Example 2: Which is bigger: ½ or ¼?

3.NF.3.d., page 41, MP 7, p.16 Massachusetts Curriculum Framework for Mathematics, March 2011

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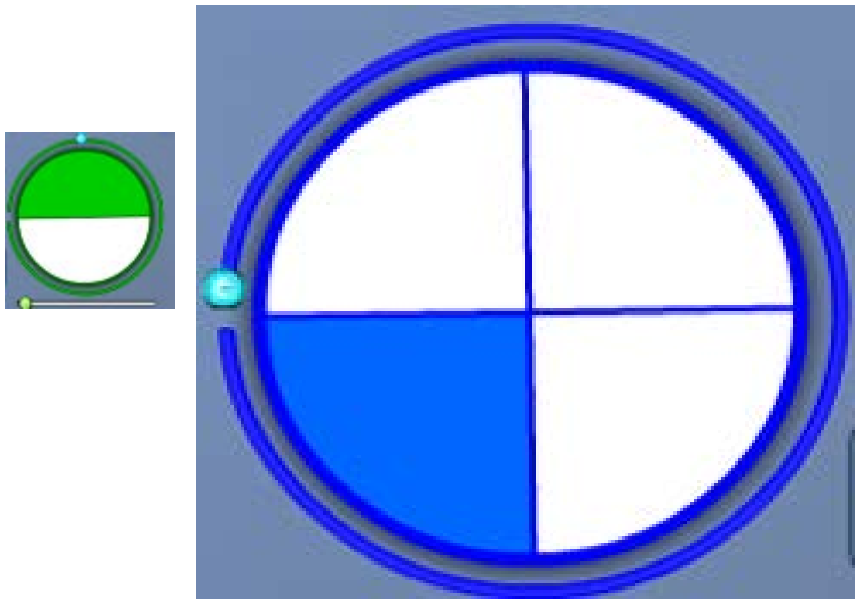
3rd Grade: The Importance of Specifying the Size of the Whole

Example 2: Which is bigger: ½ or ¼?



Illuminations
http://illuminations.nctm.org/ActivityDetail.aspx?id=80

3.NF.3.d., page 41, MP 7, p.16 Massachusetts Curriculum Framework for Mathematics, March 2011

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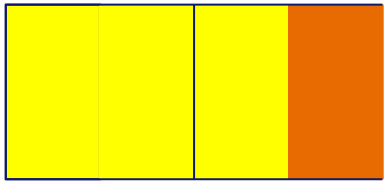

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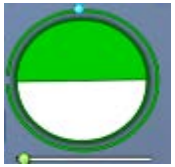

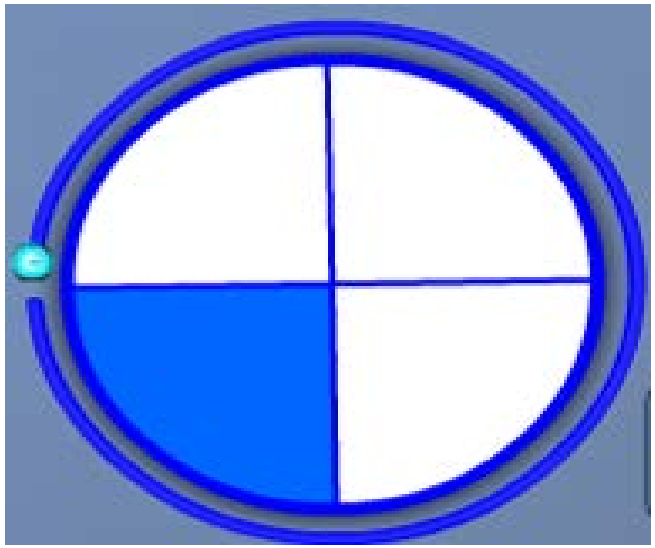

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3.NF.3.d., page 41, MP 7, p.16 Massachusetts Curriculum Framework for Mathematics, March 2011

3-5 Number and Operations--Fractions
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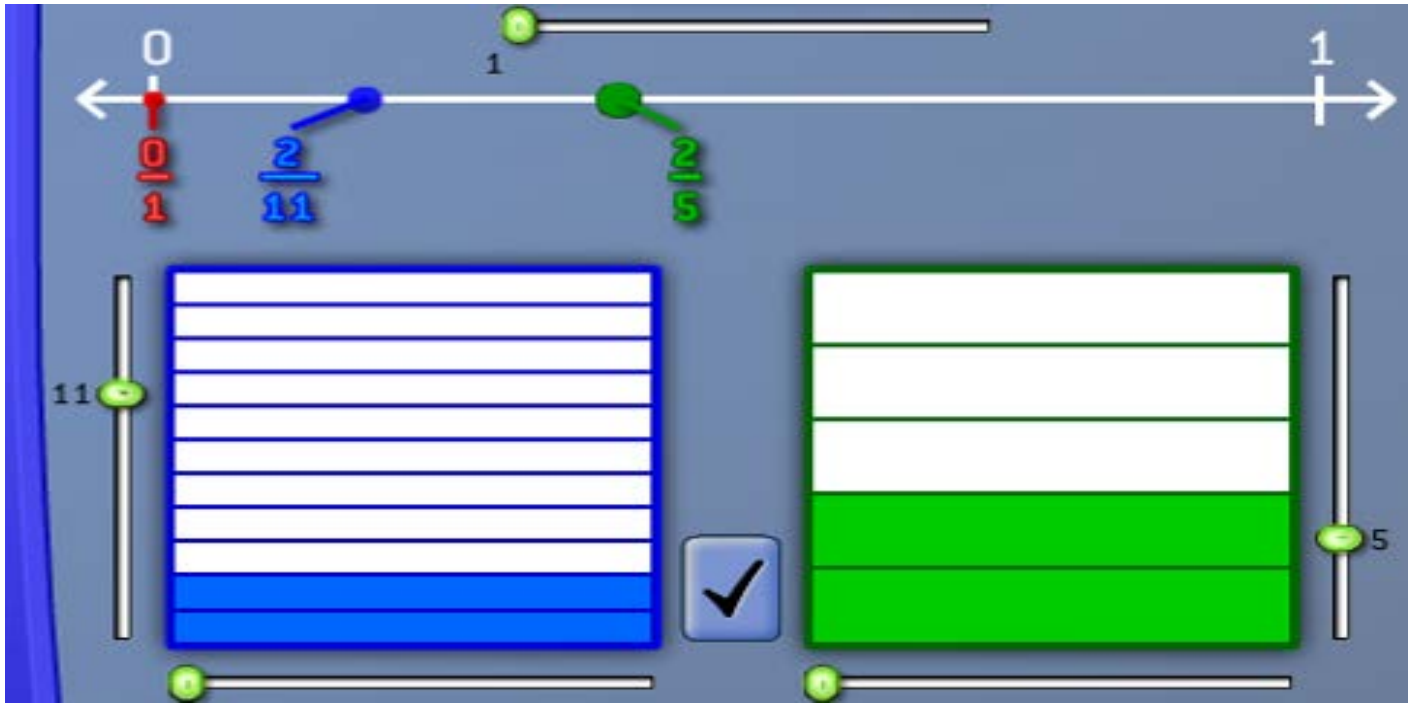
Project Director: Bill McCallum

3rd Grade: Comparing Fractions with Like Numerators



3.NF.3.a, page 41, MP3. p.15 Massachusetts Curriculum Framework for Mathematics, March 2011

Illuminations
http://illuminations.nctm.org/ActivityDetail.aspx?id=80

3rd Grade PARCC Prototype Item

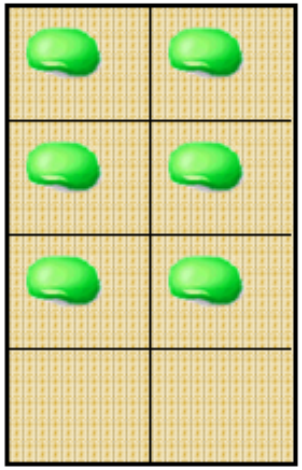
SAMPLE ITEM

Part B

Type a fraction different than 3/4 in the boxes that also represents the fractional part of the farmer's field that is planted with soybeans.

$$\frac{3}{4} = \frac{\square}{\square}$$

Farmer's Fields



Reset

Explain why the two fractions above are equal.

3.NF.1, page 41, MP 2, MP7 p. 15 Massachusetts Curriculum Framework for Mathematics, March 2011

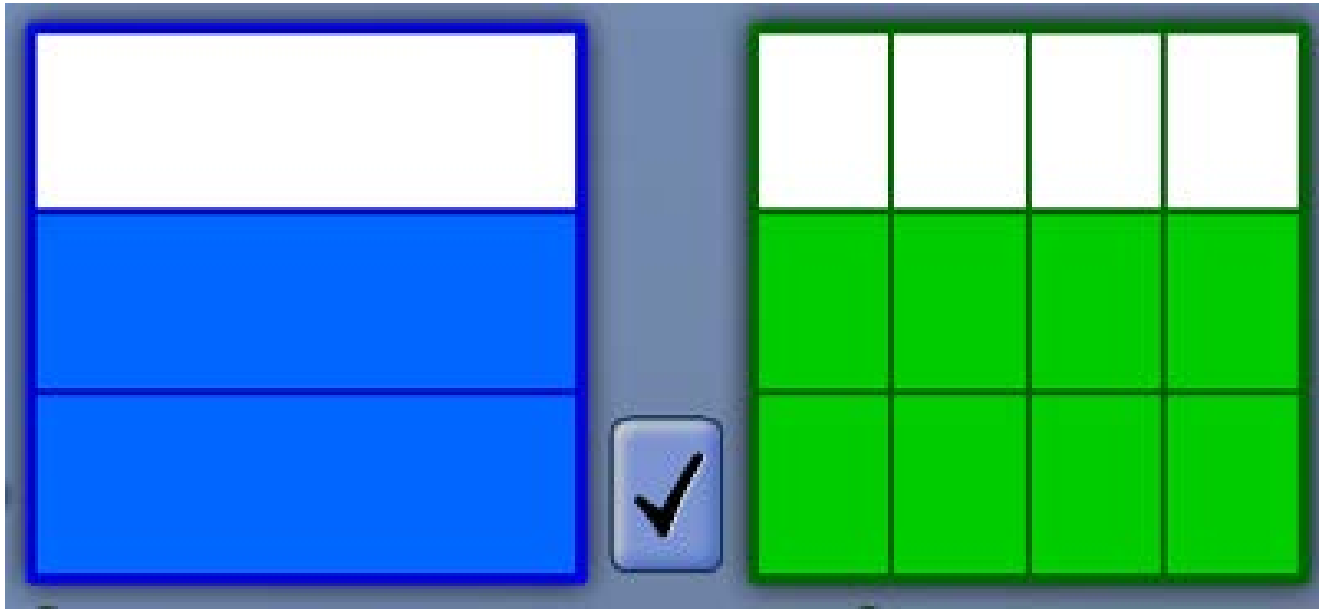
PARCC Prototype Item
http://www.parcconline.org/samples/item-task-prototypes

Fourth Grade Fraction Critical Area #2

(2)Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., 15/9 = 5/3), and they develop methods for generating and **recognizing equivalent fractions**. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the **meaning of fractions** and the meaning of multiplication **to multiply a fraction by a whole number**.

Page 43, Massachusetts Curriculum Framework for Mathematics, March 2011
(bolded for emphasis)

4th Grade: Using the Area Model to Show Equivalence

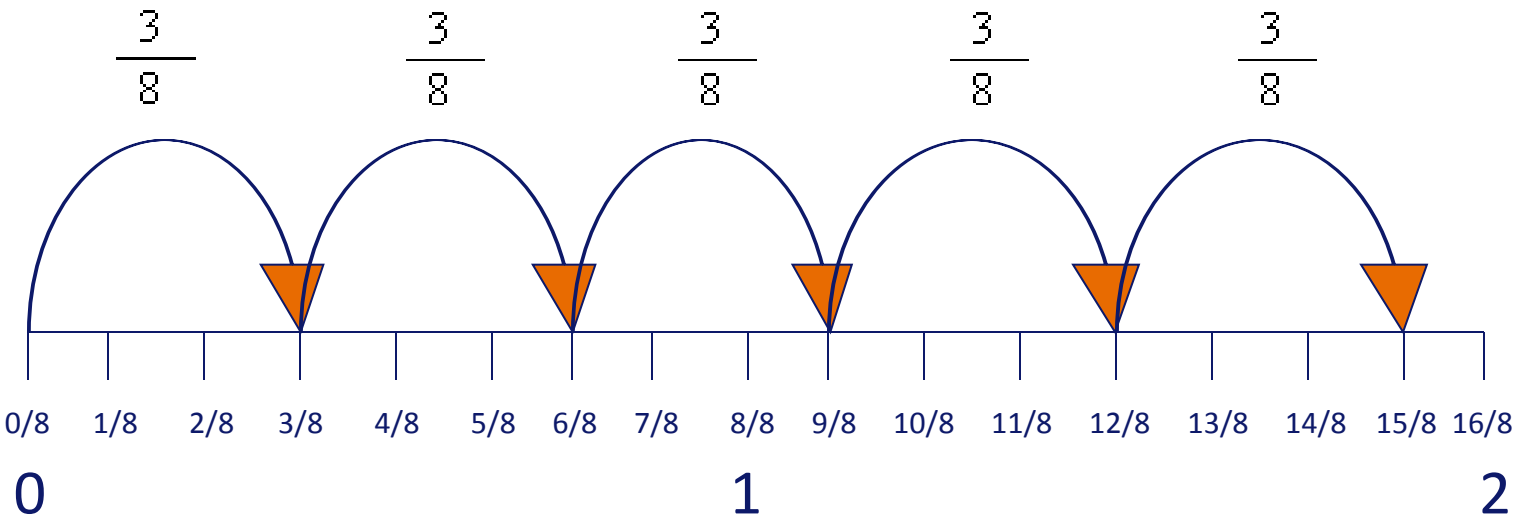


$$\frac{2}{3} = \frac{4 \times 2}{4 \times 3} = \frac{8}{12}$$

4.NF.1, page 46, MP 7, p. 16 Massachusetts Curriculum Framework for Mathematics, March 2011

Illuminations
http://illuminations.nctm.org/ActivityDetail.aspx?id=80

4th Grade: Using a Number Line to Multiply a Unit Fraction by a Whole Number



$$\frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} = \frac{3}{8} \times 5 = \frac{15}{8}$$

4.NF.3.b, page 46, MP. 7, p.16 Massachusetts Curriculum Framework for Mathematics, March 2011

Fifth Grade Fraction Critical Area #1

(1)Students apply their understanding of fractions and fraction models to represent the **addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators**. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and **explain why the procedures for multiplying and dividing fractions make sense**. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

Page 48, Massachusetts Curriculum Framework for Mathematics, March 2011
(bolded for emphasis)

5th Grade Adding Fractions with Unlike Denominators by Replacing them with Equivalent Fractions

$$\frac{2}{3} + \frac{3}{4} = \frac{8}{12} + \frac{9}{12} = \frac{17}{12}$$

5.NF.2, page 50 Massachusetts Curriculum Framework for Mathematics, March 2011

Based on "Fraction Bars" activity found in the National Library of Virtual Manipulatives
http://nlvm.usu.edu/en/nav/topic_t_1.html

5th Grade Adding Fractions with Unlike Denominators by Replacing them with Equivalent Fractions

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Checking work with a tape diagram and number line

5.NF.2, page 50 Massachusetts Curriculum Framework for Mathematics, March 201

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5th Grade Adding Fractions with Unlike Denominators by Replacing them with Equivalent Fractions

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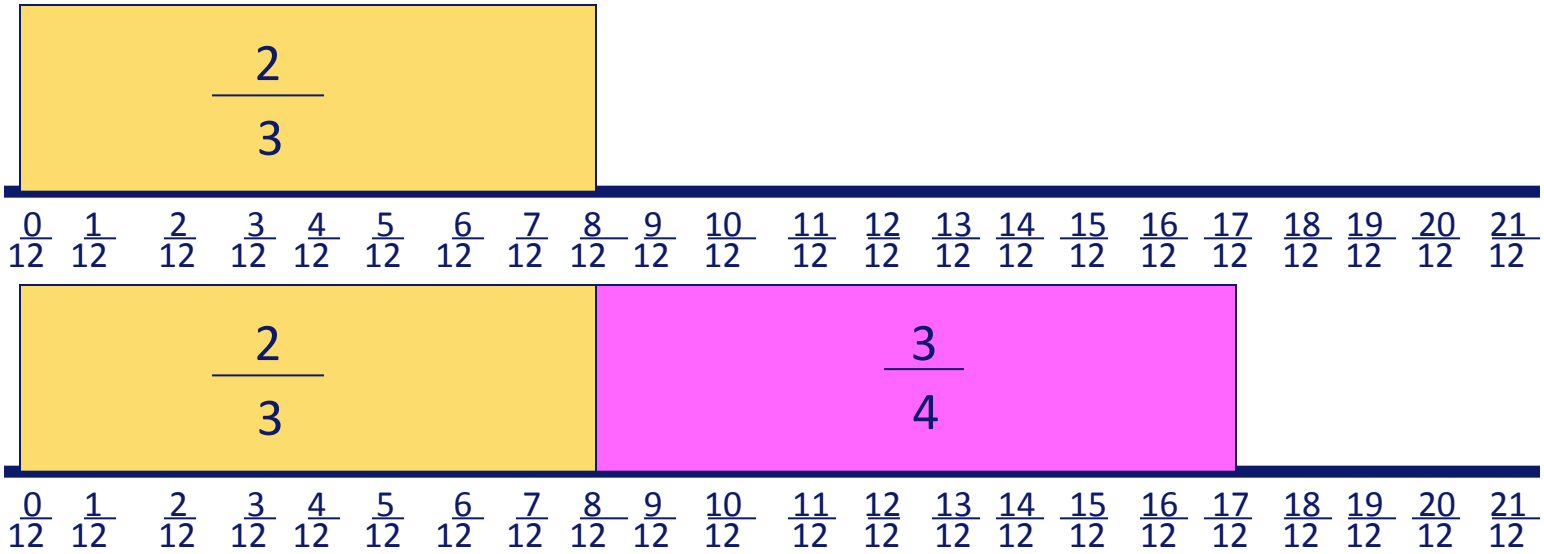
Checking work with a tape diagram and number line



5.NF.2, page 50 Massachusetts Curriculum Framework for Mathematics, March 2011

Based on "Fraction Bars" activity found in the National Library of Virtual Manipulatives
http://nlvm.usu.edu/en/nav/topic_t_1.html

5th Grade: A Context for Multiplication of Fractions

I baked a rectangular sheet cake for a party.

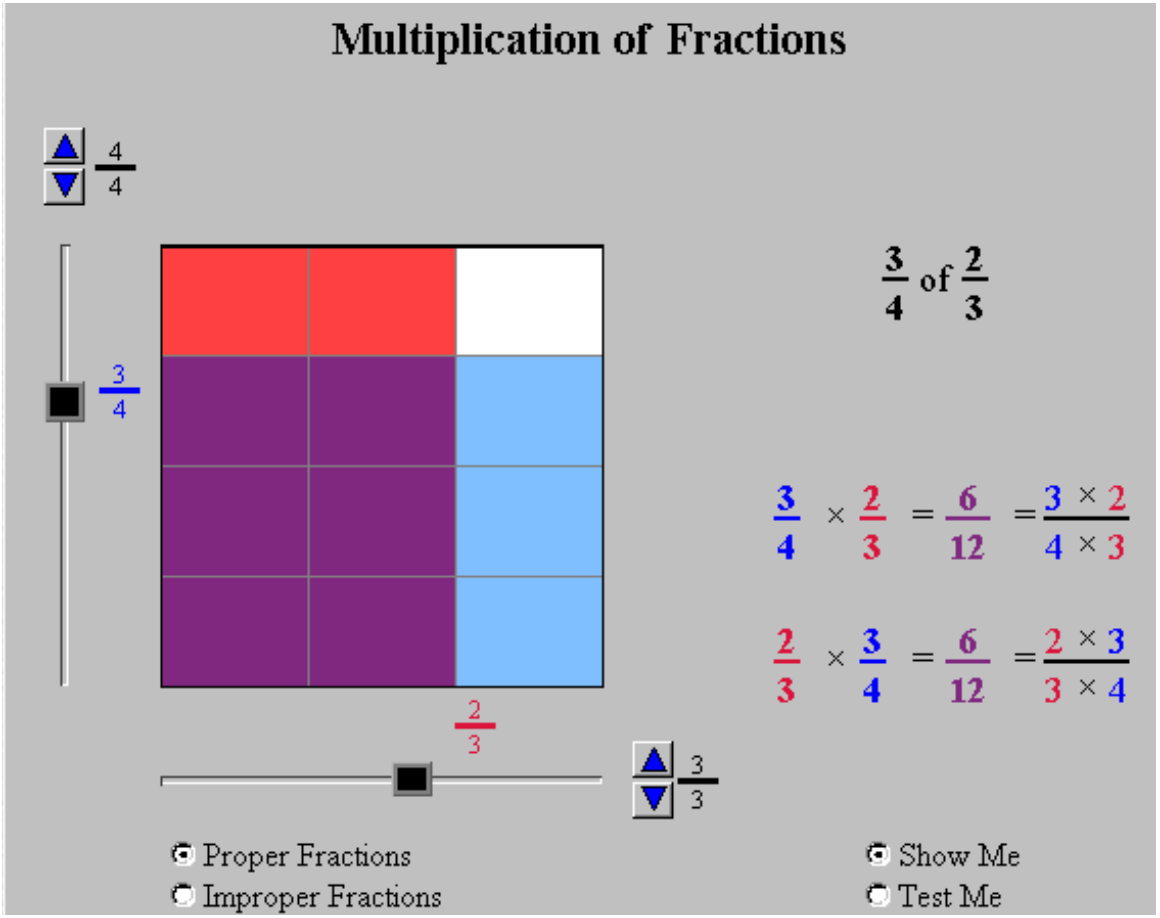
$\frac{3}{8}$ of the cake was left after the party. Draw and label how much of the cake was left.

My son came to visit and ate $\frac{1}{3}$ of the remaining cake. Draw and label how much of the remaining cake he ate.

5.NF.4, page 51, MP 2, 4, p 15 Massachusetts Curriculum Framework for Mathematics, March 2011

Math Learning Community Materials Session 13
http://www.doe.mass.edu/omste/instructional.html

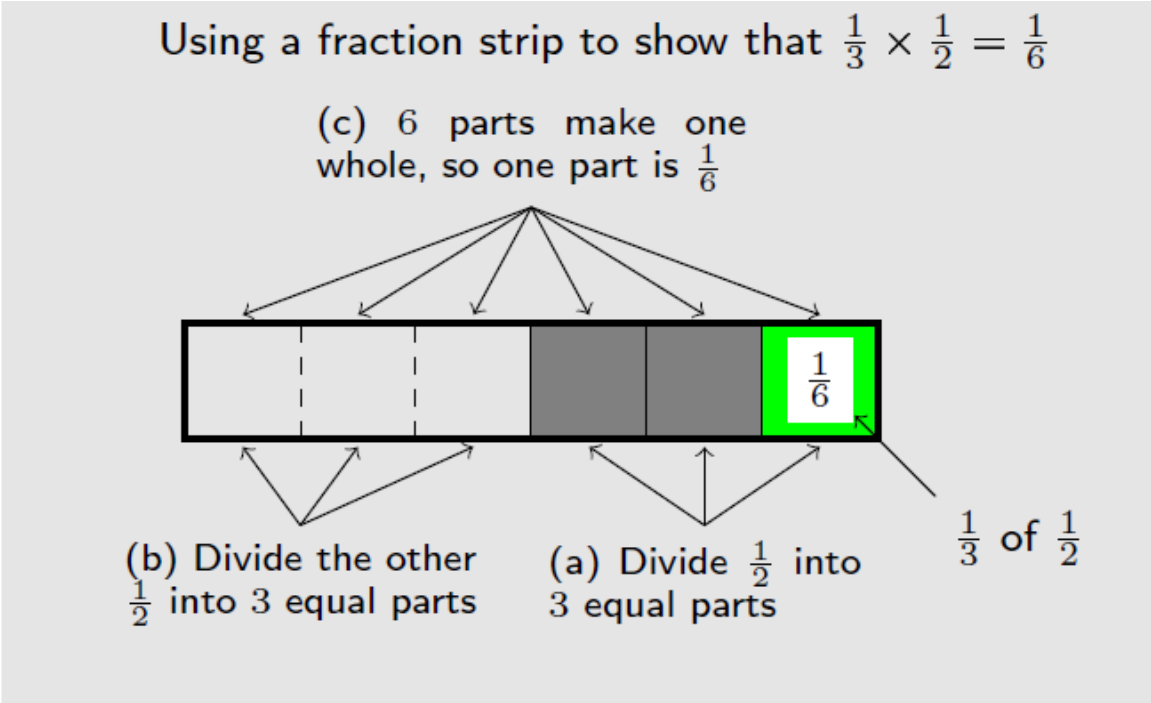
5th Grade: Multiplying Fractions using the Area Model



5.NF.4.b, page 51, MP 7 Massachusetts Curriculum Framework for Mathematics, March 2011

National Library of Virtual Manipulatives
http://nlvm.usu.edu/en/nav/frames_asid_194_g_2_t_1.html?from=topic_t_1.html

5th Grade: Multiplying Fractions using a Tape Diagram

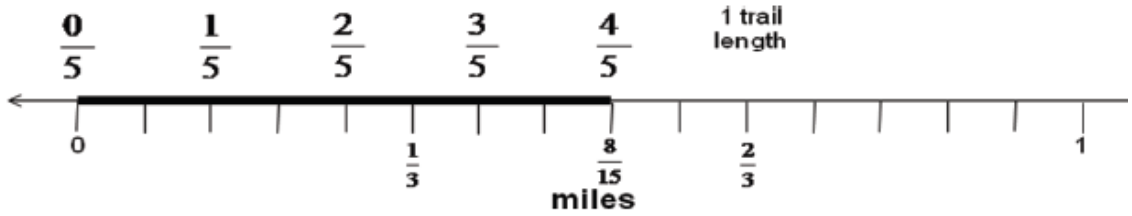


5.NF.4.a, page 51 Massachusetts Curriculum Framework for Mathematics, March 2011

3-5 Number and Operations--Fractions
http://ime.math.arizona.edu/progressions
Project Director: Bill McCallum

5th Grade: Multiplying Fractions using a Double Number Line

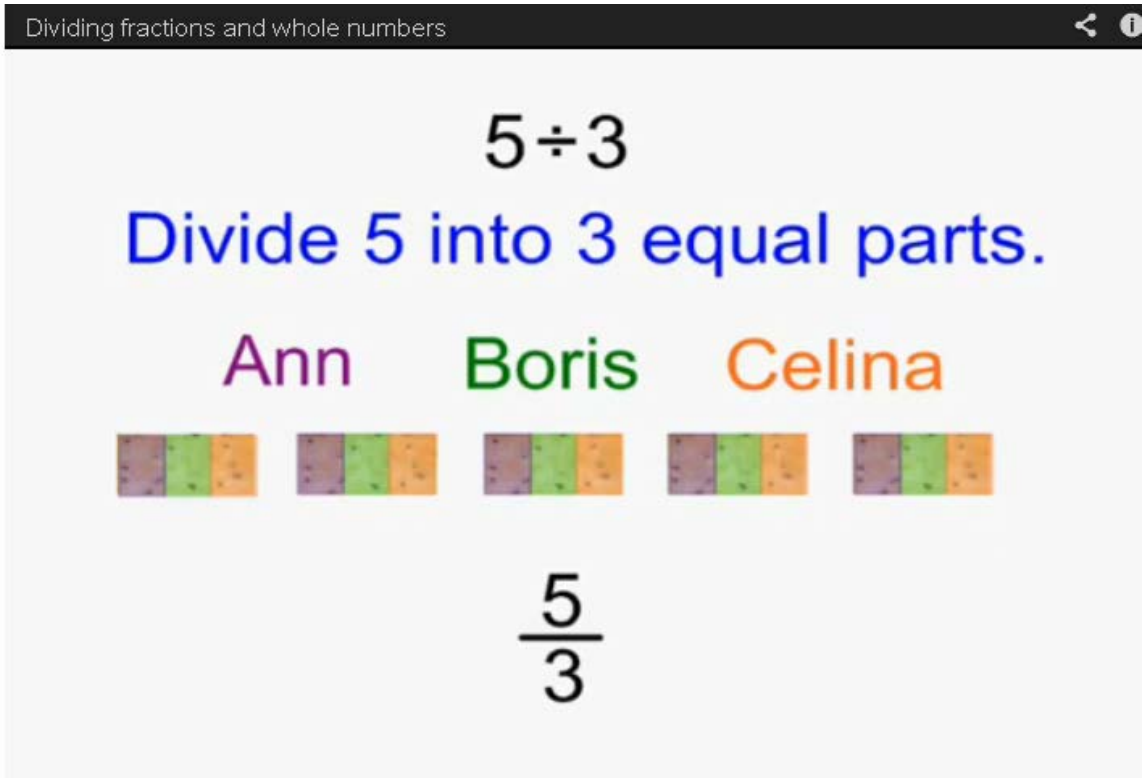
Some students may need to use a double number line to make sense of finding four-fifths of two-thirds. The number line below shows that one-fifth of a trail length is two-fifteenths of a mile, so four-fifths of two-thirds would be eight-fifteenths of a mile.



5.NF.4.b., page 51, MP 2 p.15 Massachusetts Curriculum Framework for Mathematics, March 2011

Rational Numbers Project
http://www.cehd.umn.edu/ci/rationalnumberproject/RNP2/Lesson24.pdf

5th Grade: Relationship Between Division and Fractions



5.NF.3, page 51 Massachusetts Curriculum Framework for Mathematics, March 2011

Illustrative Mathematics
http://www.illustrativemathematics.org/pages/fractions_progression

V. Grade Six: Dividing Fractions by Fractions and connecting Fractions to Ratios and Proportional Relationships

Sixth Grade

6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.

Turn and Talk

Create a context for this problem. Then solve the problem by using a visual model.

$$1\frac{3}{4} \div \frac{1}{2}$$

6.NS.1, page 55, MP 2,3, 4 p.15 Massachusetts Curriculum Framework for Mathematics, March 2011

Based on a discussion from p. 55 Liping Ma Knowing and Teaching Elementary Mathematics

Some possible solutions

$1\frac{3}{4} + \frac{1}{2}$

Some possible solutions

½ foot of ribbon

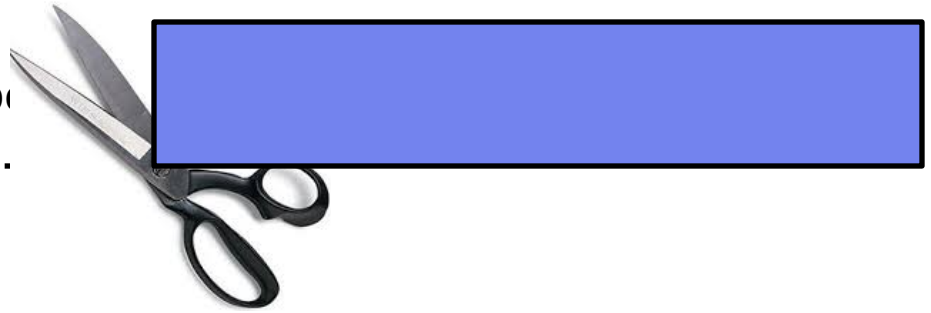
1 half-foot strip

$1\frac{3}{4} \div \frac{1}{2}$

Some possible solutions

½ foot of ribbon

½ foot of ribb



1 half-foot strip

1 half-foot strip

$1\frac{3}{4} \div \frac{1}{2}$

Some possible solutions

½ foot of ribbon	½ foot of ribbon	½ foot of ribbon	

1 half-foot strip

1 half-foot strip

1 half-foot strip

$1\frac{3}{4} \div \frac{1}{2}$

Some possible solutions

½ foot of ribbon	½ foot of ribbon	½ foot of ribbon	¼ foot ribbon

1 half-foot strip

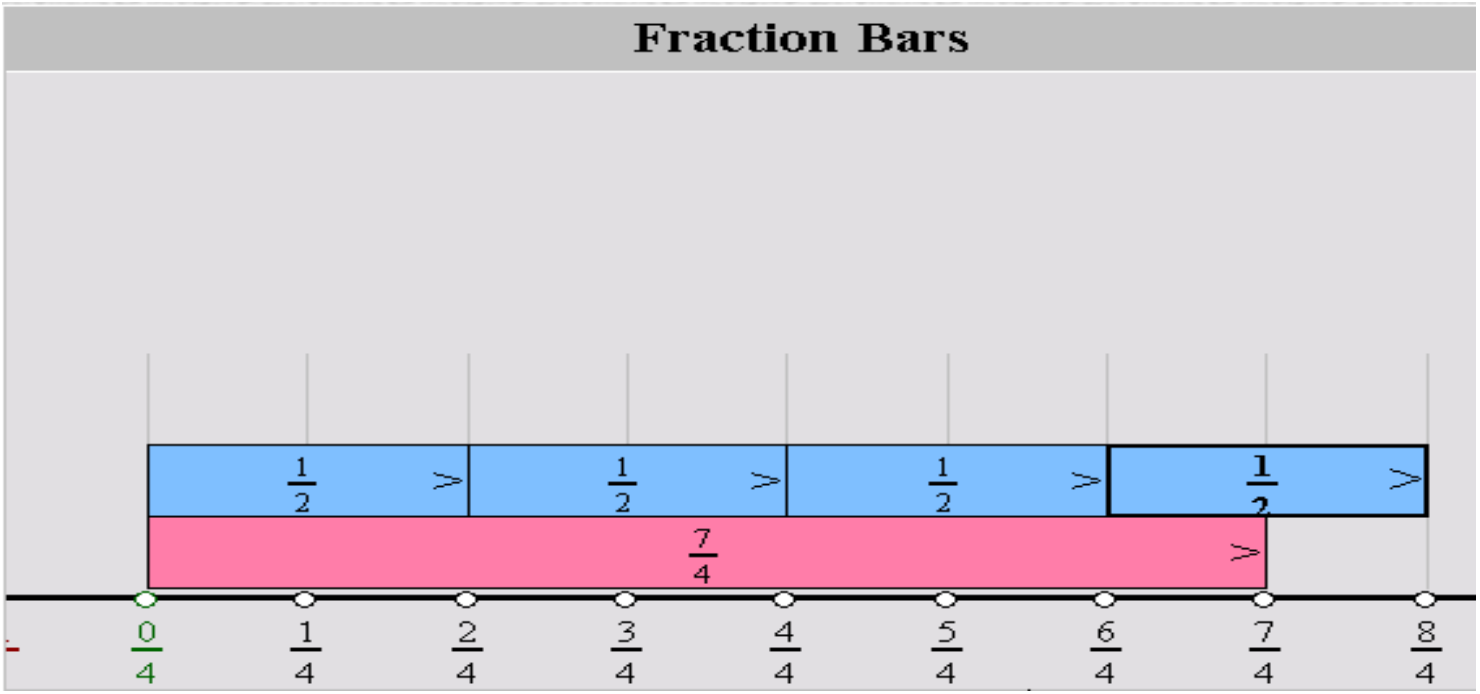
1 half-foot strip

1 half-foot strip

½ half-foot strip

$1\frac{3}{4} \div \frac{1}{2} = 3\frac{1}{2}$

Some possible solutions



National Library of Virtual Manipulatives
http://nlvm.usu.edu/en/nav/frames_asid_265_g_3_t_1.html?open=activities&from=category_g_3_t_1.html

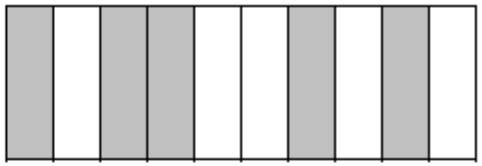
Sixth Grade Ratio and Proportional Reasoning Critical Area # 1

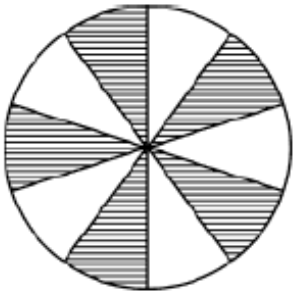
(1) Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they **connect ratios and fractions**. Students solve a wide variety of problems involving ratios and rates.

Page 53, Massachusetts Curriculum Framework for Mathematics, March 2011
(bolded for emphasis)

6th Grade Part-to-Whole to Part-to-Part

Shapes divided into equal parts with some part shaded.: Write the ratios for shaded to non-shaded or shaded to total.





6.RP.1, page 55, MP 7, p 16 Massachusetts Curriculum Framework for Mathematics, March 2011

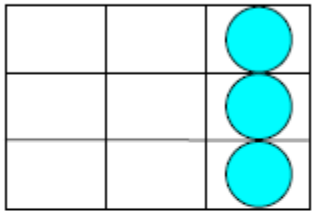
Ratios and Rates Grade 6 Mathematics Model Curriculum Unit
http://www.doe.mass.edu/candi/model/sample.html

6th Grade Part-to-Whole to Part-to-Part

Candies

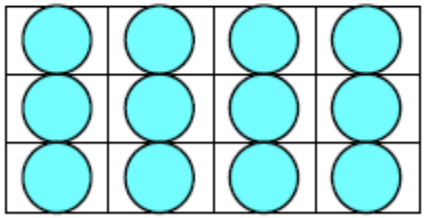
This problem gives you the chance to:
- work with fractions and ratios

1. This is Amy's box of candies.
   She has already eaten 6 of them.



What fraction of the candies has Amy eaten? _____

2. Valerie shares some of the 12 candies from this box.
   She gives Cindy 1 candy for every 3 candies she eats herself.

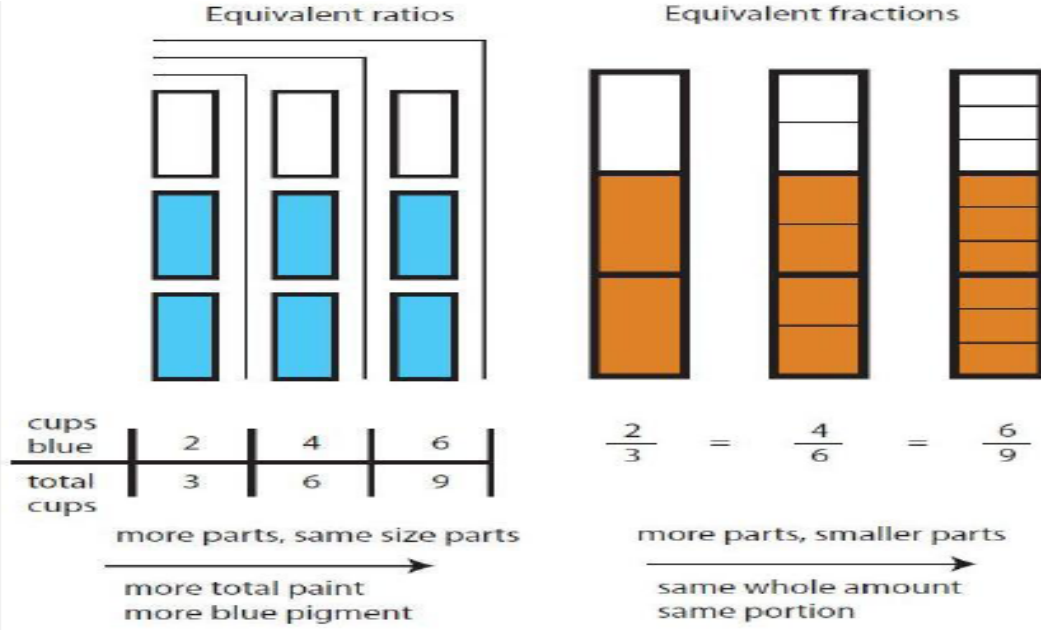


How many candies does she give to Cindy? _____
Show how you figured this out.

6.RP.1, page 55 Massachusetts Curriculum Framework for Mathematics, March 2011

Noyce Foundation and Inside Mathematics: http://www.insidemathematics.org/index.php/classroom-video-visits/public-lessons-proportions-a-ratios

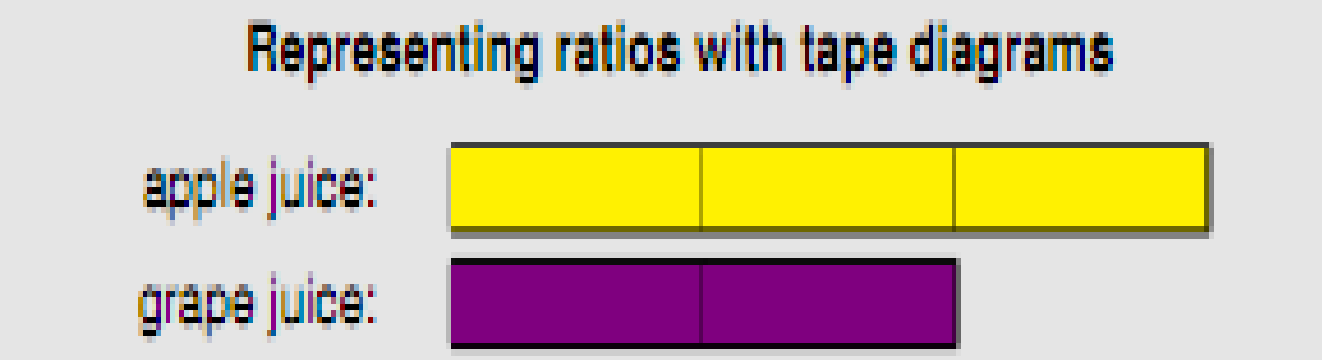
6th Grade: Equivalent Ratios vs. Fractions



6.RP.1, page 55 Massachusetts Curriculum Framework for Mathematics, March 2011

6-7, Ratios and Proportional Relationships http://ime.math.arizona.edu/progressions
Project Director: Bill McCallum

6th Grade: Tape Diagrams
Apple-Grape Juice



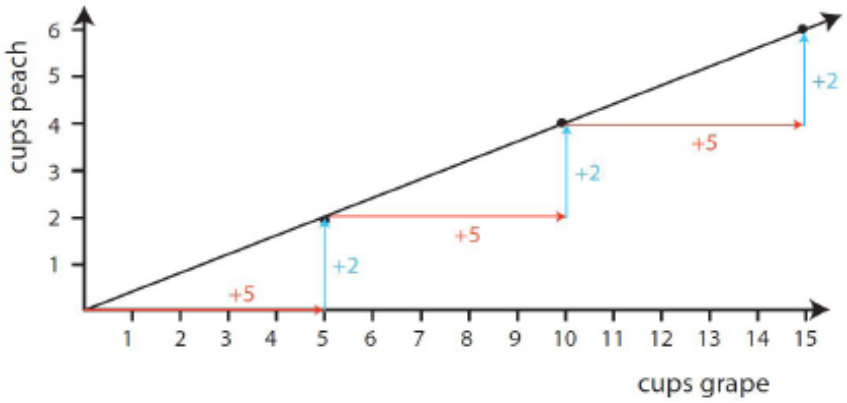
6.RP.1, page 55 Massachusetts Curriculum Framework for Mathematics, March 2011

6-7, Ratios and Proportional Relationships http://ime.math.arizona.edu/progressions
Project Director: Bill McCallum

6th Grade: Visual Representation of Proportional Relationship

cups grape	cups peach
5	2
10	4
15	6
20	8
25	10

+5 +5 +5 +5 (grape) +2 +2 +2 +2 (peach)



6.EE.9, page 57, MP 4 p. 16 Massachusetts Curriculum Framework for Mathematics, March 2011

6-7, Ratios and Proportional Relationships http://ime.math.arizona.edu/progressions

Project Director: Bill McCallum

6th Grade and Beyond: Multiple Representations

Water World Party Plan

LINK SHEET

Verbal Description

Ms. Simpson's students are planning a class party. They do not know how many students are planning on attending. Sophie suggested a party at Water World.

A Water World party includes: swimming, hot dog, chips, and drink.
Cost: $100 to reserve the pool and $5 per person.

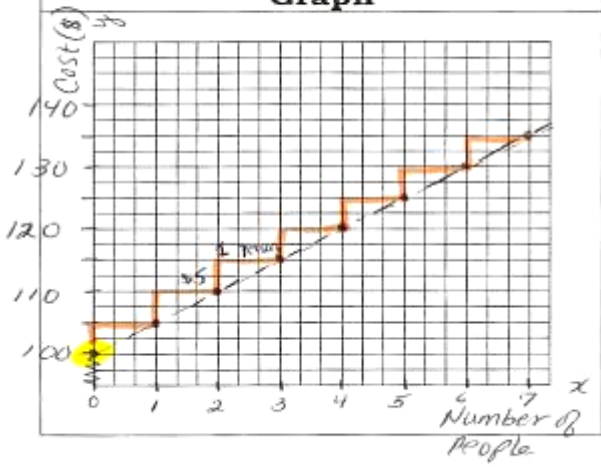
Table of Values

x — y

Number of People	Cost
0	100 + 5(0) = 100
1	100 + 5(1) = 105
2	100 + 5(2) = 110
3	100 + 5(3) = 115
4	100 + 5(4) = 120
10	100 + 5(10) = 150
⋮	
100	100 + 5(100) = 600
x	100 + 5(x)

+1 → +5

Graph



Equation

x = number of people

y = cost

y = 100 + 5(x)

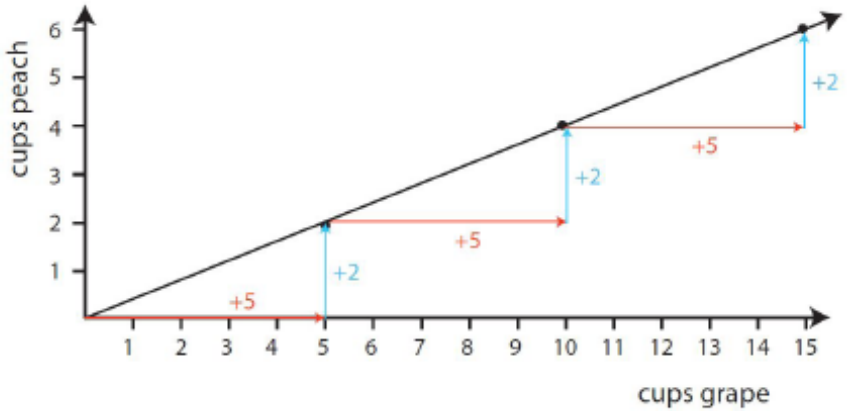
Problem adapted from "Representation as a Vehicle for Solving and Communicating" by K. Preston and A. Garner, AFMS September 2003

PD Offering: Increasing Accessibility to Algebra and Geometry for ALL Students
http://www.doe.mass.edu/apa/sss/support/development.html?section=MATH

6th Grade PARCC Prototype Item

cups grape	cups peach
5	2
10	4
15	6
20	8
25	10

+5 +5 +5 +5 (grape) +2 +2 +2 +2 (peach)



6.RP.3.a page 55, MP 4, p. 16 Massachusetts Curriculum Framework for Mathematics, March 2011

PARCC Prototype Item
http://www.parcconline.org/samples/item-task-prototypes

Turn and Talk

Reflect on the visual models you have looked at today. Which ones do you plan to use at the grade level(s) you teach? Why?

Websites

- ESE: MLC http://www.doe.mass.edu/omste/instructional.html
- ESE: Model Curriculum Units http://www.doe.mass.edu/candi/model/sample.html
- ESE : Professional Development Institutes: http://www.doe.mass.edu/candi/institutes/
- Illuminations http://illuminations.nctm.org/
- Illustrative Mathematics http://www.illustrativemathematics.org/
- Inside Mathematics http://www.insidemathematics.org/
- Library of Virtual Manipulatives http://nlvm.usu.edu/en/nav/grade_g_2.html
- PARCC http://www.parcconline.org/
- Progressions Documents for the CC Standards http://ime.math.arizona.edu/progressions/
- Rational Numbers Project http://www.cehd.umn.edu/ci/rationalnumberproject/

Questions?