Mathematics Learning Community
Number Sense
Session 13

Title: *Multiplying Fractions*

Common Core State Standards Addressed in the LASW Problem:

5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

Standards for Mathematical Practice Addressed in the MLC Session:

2: Reason abstractly and quantitatively.

4: Model with mathematics.

MLC members make use of mental math in order to quantitatively reason about estimates of fractional products during Math Metacognition. Students are asked to make sense of an area model representation and explain a multiplication problem in which both factors are fractions. By first drawing the model and then second, by interpreting how the model relates to the multiplication performed, students' understanding of fraction multiplication can be assessed.

Standards-Based Teaching and Learning Characteristics in Mathematics Addressed in the MLC Session:

- 5.1 Depth of content knowledge is evident throughout the presentation of the lesson.
- 5.3 Students' prior knowledge is incorporated as new mathematical concepts are introduced.
- 5.4 Student misconceptions are anticipated /identified and addressed.
- 5.5 Classroom strategies incorporate multiple forms of representation.

Session Agenda:

Part I: Mathematical Background
Part II: Math Metacognition
Part III: Looking at Student Work • *Eating Cake* Problem (Grades 6 – 8)
Part IV: Our Learning
Part V: Feedback and Wrap-up

Materials Needed for this Session:

- ✓ Nametags
- ✓ Chart paper and markers
- ✓ Copies of handouts
- ✓ Student Work Samples
- ✓ Index cards
- ✓ Refreshments
- ✓ Highlighters

Possible Ways to Personalize this Session:

- Student work samples are no longer provided for the LASW problem in this session. Instead, MLC members will provide the samples for the given problem. See Page 5 in Part III for more details.
- Two Guiding Questions are provided in the Student Work Analysis Grid – the other two questions can be selected by you or by the group.
- If your group needs more work with multiplying fractions, some additional practice problems are provided on Page 6 in Part III.

Part I: Mathematical Background
Approximate Time: 20 Minutes
Grouping: Whole Group

A. Welcome members of your group to the Math Learning Community. Remind group of **established norms.**

B. Today's Content:

a. The mathematics during this session focuses on multiplying fractions.
b. What do we need to know in order to be able to multiply fractions?
c. Chart ideas to refer to during the Protocol for LASW.
d. It is important to discuss commonly-held **misconceptions** about fractions during this session. One misconception that many students hold is that they **consider a fraction to be two different numbers, instead of one quantity**. One can assess whether or not a student holds this misconception by asking the student to locate a particular fraction on a number line – students will not mark a particular location that the fraction names; instead they may make indications of two distinct locations represented by the fraction. Another way to assess students on this concept is to ask for an equivalent fraction to a particular given fraction. Watch for students who multiply the numerator and denominator by different values. Another commonly-held misconception is that **fractions should behave as whole numbers behave**. For example, when multiplying fractions, a student would expect a resulting product that is greater than either of the factors. This is not always the case with fractions, mixed numbers, and later, with integers. It is very important to discuss how students have developed these whole number concepts that they know and understand that are suddenly undermined with fraction computation. This highlights one of the reasons why fractions are difficult for so many students.
e. Do students understand the properties of arithmetic with whole numbers? Are they then transferring these properties to fractions and mixed numbers? In grade 6, we assume they understand commutative property – are they able to see those generalizations holding true with fractions?
f. The area model can be a powerful tool for the development of fraction multiplication.

C. Relating Content to the Three C's Theme:

a. How do the ways in which students learn to multiply fractions relate to the ways in which they multiply whole numbers?
b. Which attributes or properties of whole numbers hold true for fractions? Which do not?
c. How does context play a role in the development of fraction sense? Composition?

Part II: Math Metacognition
Approximate Time: 25 minutes
Grouping: Whole Group

A. Problem: The problem used for this session is appropriate for Grades 6 – 8.

Estimate the following products.

$$2\frac{1}{3} * \frac{5}{8}$$

$$4\frac{1}{2} * 3\frac{12}{13}$$

B. Solution to Problem:

a. Product 1: Sample estimate: $2 * \frac{1}{2} = 1$

b. Product 1: Actual product: $\frac{7}{3} * \frac{5}{8} = \frac{35}{24} = 1\frac{11}{24}$

c. Product 2: Sample estimate: 4 x 4 = 16

d. Product 2: Actual product: $\frac{9}{2} * \frac{51}{13} = \frac{459}{26} = 17\frac{17}{26}$

C. Problem Intent: (*Note: The problem intent for all Math Metacognition problems is the same).* See Session 2 for more information. In addition, this problem serves as a good start to discussing mixed numbers as quantities and strategies or methods for working with them. This problem also emphasizes the use of estimation, a powerful skill that needs to be formally developed in students and adults alike.

D. Discuss the estimation strategies used. (*Note: Refer to the **Sample Estimation Strategies** on Page 4*). Consider having MLC members try out the second product with a new strategy to see if the same conclusions are reached. Be sure to provide enough time for discussion of this metacognition problem. Additional questions to consider:

- What mathematics did you call on to estimate the products? Did this vary from one product to the next? People might say they used different mathematical ideas to estimate the products for these two different problems (i.e., distributive property for #1, rounding up for #2). This discussion can serve as a way to tie back to the ideas brought up during some of the first conversations of the MLC, where students have different strategies for whole number operations.
- As a branch-off to this discussion, consider this point: one way of doing mathematics does not necessarily serve all of us all of the time. It often depends on the numbers or problem at hand. A good tool in one's problem-solving arsenal is the ability to connect one problem-solving strategy to another. A much higher – level of thinking is required for students to call on and know which strategy will be the most effective for any given problem. This then leads into a broader idea: so what does this say about how we teach mathematics in our classrooms if we only teach one method in one way?
- How does this problem differ from the metacognition problem in Session 12? In particular, estimation in Session 12 did not allow us to be accurate enough because we were comparing quantities that were relatively close in value. Here, we are not making a comparison between two quantities – we are instead looking to determine the estimated value of one quantity.

Sample Estimating Strategies

'Quick & Dirty' Methods

Using Benchmark Fractions	Rounding Using Decimal Equivalents
Strategy: Round the fraction to the nearest benchmark: **0, ¼, ½, ¾, 1** Example: $2\frac{1}{3}*\frac{5}{8}$ $2\frac{1}{3}$ is closest to $2\frac{1}{2}$ $\frac{5}{8}$ is equidistant from $\frac{1}{2}$ and $\frac{3}{4}$. Use $\frac{1}{2}$ since we rounded up on the first benchmark. $\frac{1}{2}$ of $2\frac{1}{2} = 1\frac{1}{4}$.	Strategy: Convert to decimal equivalent. If the tenths place is 5 or more, round up to the nearest one. If the tenths place is 4 or less, round down to the nearest one. Example: $2\frac{1}{3}*\frac{5}{8}$ 2.33 x .625 2 x 1 = 2

More Involved Methods

Fraction – Decimal – Percent	Distributive Property
Strategy: Convert to equivalent forms of the given numbers to estimate. Example: $2\frac{1}{3}*\frac{5}{8}$ 2.33 x .625 .625 ~60% 60% = 50% + 10% 50% of 2.33 = 1.16 10% of 2.33 = .23 60% of 2.33 = 1.39 1.39 is approximately $1\frac{4}{10}$ or $1\frac{2}{5}$.	Strategy: Rewrite the problem as a sum multiplied by either another sum or by a single factor. Distribute the sum or the factor. Here, the sum will be the whole number plus the fraction (what was previously the mixed number). Example: $2\frac{1}{3}*\frac{5}{8}$ $2\frac{1}{3} = (2+\frac{1}{3})$ $\frac{5}{8}*(2+\frac{1}{3}) = \frac{5}{8}*2+\frac{5}{8}*\frac{1}{3}$ $\frac{4}{8}*2=1$ + a bit more $\frac{4}{8}*\frac{1}{3}=\frac{1}{6}+$ a bit more $1+\frac{1}{6}+\frac{1}{3} = 1\frac{1}{2}$

The actual product for Product 1 is $1\frac{11}{24}$, which is approximately 1.4583. The closest estimates were obtained using the more involved methods. Benchmark fractions provided a closer estimate for this particular problem, when using a 'Quick & Dirty' method.

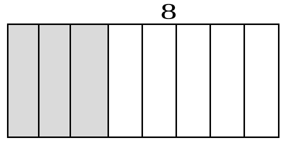
Part III: Looking at Student Work (LASW)
Approximate Time: 50 minutes
Grouping: Refer to protocol

A. Complete the **MLC protocol** with the group.

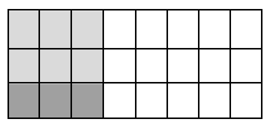
B. **Problem:** The problem used for this session is appropriate for Grades 6 – 8.

> I baked a rectangular sheet cake for a party.
> $\frac{3}{8}$ of the cake was left after the party. Draw and label how much of the cake was left.
> My son came to visit and ate $\frac{1}{3}$ of the remaining cake. Draw and label how much of the remaining cake he ate.
> What portion of the whole cake did he eat?

C. **Solution**: Drawing #1 should show 3 ($\frac{1}{8}$ pieces) of the cake remains.



Drawing #2 should show that the son ate $\frac{1}{3}$ of the $\frac{3}{8}$ portion remaining, or $\frac{1}{8}$ of the original cake.



D. **Problem Intent**: This problem provides a visual of the operation of multiplication through the use of the area model. It is important to see multiplication in this way as it gets away from the commonly-held belief that multiplication is simply the process of repeated addition. Also, this problem shows why the use of the traditional algorithm for multiplying fractions works. The area model shows that multiplication of fractions is taking a portion of a portion, thus resulting in a smaller quantity. In this problem, the area model helps students to visualize what the portions of the original whole represent, a cake that was first sliced into eighths. The whole cake can be thought of as $\frac{8}{8}$. Three of those eighths were the remaining portion that was then sliced into thirds. The resulting answer, (product), is a portion of a portion of *the original cake*, which could be expressed at this point as $\frac{24}{24}$. A misconception with the area model is that students often neglect to keep the portion of the portion relative to the original whole. The last question in this problem will highlight this misconception.

E. **Discuss** the following:
   a. Strategies used to solve this problem
   b. How much experience have group members had with using the area model?
   c. How can the area model be used to represent the process carried out by applying the traditional algorithm for multiplication?
   d. If you think your group needs additional opportunities to model and work with fraction multiplication, consider the additional problems on Page 6.

Part III: Looking at Student Work (LASW), continued

F. **Misconceptions/Questions that May Arise:**

a. M: A misconception with the area model is that students often neglect to keep the portion of the portion relative to the original whole.

b. M: Refer to Page 2 in Part I for additional information on fraction misconceptions.

c. Q: In the multiplication of $\frac{1}{3} * \frac{3}{8}$, the traditional algorithm states that one should multiply across the numerators and then multiply across the denominators and this result is the product. What does the 1 x 3 in the numerator represent in the diagram? The 3 x 8?

Options for Customization

G. **Guiding Questions:** Two Guiding Questions have been provided in the Student Work Analysis Grid for this problem. As a way to customize the LASW process, you (or your group) will need to decide on the remaining two questions. You can use the two questions listed below that are specific to this problem or refer to the list of generic questions found on Page 5 in Session 7.

a. What misconceptions or misunderstandings about fractions exist in the work?

b. How is visual reasoning used to solve the problem?

H. **Using A Group Member's Student Work**:

a. *Prior to offering this session to your MLC, you need to collect student work samples for the task:* ***Eating Cake.***

b. See Page 5 in Session 7 for more details on collecting student work samples. For this task, select 4 samples to discuss during the MLC session. Prior to photocopying samples, mark them as **A, B, C, and D**.

I. **Additional Practice Problems**: How do you know if your MLC needs more practice with determining portions of portions and understanding multiplication of fractions? If it takes longer for the group to talk about the metacognition problems in this session and the comparison of properties with whole versus rational numbers. As the facilitator, you need to use your best judgment about discussing these ideas – most teachers have not had time to consider fractions and operations on fractions in this way and therefore need time to better understand the multiplication algorithm.

a. Multiplying fractions using the area model:

- Solve the following: $2\frac{2}{3} * 3\frac{3}{4}$. Use an area model to represent the same problem. Connect the numerical solution to the area model.
- Problems in Investigation 3 in "Bits and Pieces II," *Connected Mathematics 2*, ©Pearson Prentice Hall

b. Multiplicative Inverses:

- Estimate the product of $1\frac{2}{3} * \frac{3}{5}$. Why is the answer 1?
- Is dividing by ½ is the same as multiplying by 2? Given that, is dividing by ¾ same as multiplying by $\frac{4}{3}$? Is this a generalization that always hold true? Can you come up with different problems to represent each of these two situations? How do students and teachers understand and interpret these ideas?

c. **Distributive Property and Mixed Numerals:**

- Page 160 in *Teaching Fractions and Ratios for Understanding* by S. Lamon

Part IV: Our Learning
Approximate Time: 20 Minutes
Grouping: Whole Group

A. **Discussion**: After evidence of student understanding has been discussed as a whole group, you want to facilitate discussion around how the LASW process will impact what teachers do within their classrooms. Some questions to help guide discussion include:
   a. What do we take away after LASW?
   b. What did we learn? About student thinking? About our own knowledge?
      1. Refer back to chart made at the beginning of the session
   c. How does today's session relate to important mathematical content and pedagogy?
   d. How does it impact **your** practice at **your** grade level? *(Note: In order to help teachers connect this session to the mathematics within their own grade level, refer to the information below).*

Making Connections Across the Grade Levels

K – 2: Work with common fractions begins in this grade band. In addition, some students will begin to explore situations that involve multiplication, such as when creating equal groups of objects. Estimation is also a very important skill that begins to be honed at this time (K.CC.5, K.CC.6, 1.OA.1, 1.G.3, 2.OA.3, 2.OA.4, 2.G.2, 2.G.3)

3 – 5: The concept of multiplication with whole numbers is truly developed at this grade band. As seen in these problems, misconceptions will arise later on in school for students who always think multiplication makes a greater quantity. It is important to avoid making blanket statements to suggest such an idea. If students offer this idea up, use a simple common fraction or 0 as a factor to provide them with a counterexample. In addition, work with fractions continues, and the concepts of area and rounding numbers are both developed during these grades. The LASW problem is appropriate for Grades 5 and above to explore and specifically addresses the Cluster of learning standards under 5.NF.4. (3.OA.1, 3.OA.3, 3.OA.5, 3.OA.7, 3.OA.8, 3.NBT.1, 3.NF.1, 3.NF.3 a – d, 3.MD.5 a – b, 3.MD.6, 3.MD.7 a – d, 3.G.2, 4.OA.3, 4.NBT.5, 4.NF.2, 4.NF.4 a – c, 5.NBT.7, 5.NF.2, 5.NF.3, 5.NF.5 a – b, 5.NF.6)

6 – 8: The problems in this session highlight commonly-held misconceptions that should be assessed and addressed during this grade band. It is here that students challenge what they know and understand about whole numbers and how that information translates to fractions. Being able to fluently move amongst equivalent forms of rational numbers (i.e., fractions, decimals, and percents) is also important at this time. Considering the reasonableness of an answer is a skill that can be developed or built upon as well. In addition, the Math Metacognition problems are also appropriate for this grade band. (6.RP.3 a – d, 6.NS.1, 6.NS.6 a – c, 6.NS.7, 6.EE.2 a – c, 6.EE.3, 6.G.1, 7.RP.2 a – d, 7.NS.2 a – d, 7.NS.3, 7.EE.3, 7.G.6)

B. **Writing a Problem or Task**: As a way to synthesize learning from today's session, ask MLC members to come up with a math problem or task that would embody the ideas discussed today. The problem should be appropriate to use at their grade level. Writing these problems will help both you as the facilitator and the other group members to develop a stronger sense of how these mathematical ideas show up in classrooms from grades K – 8. (*Note: See Part IV in Session 1 for more details*).

Part V: Feedback & Wrap-up
Approximate Time: 5 Minutes
Grouping: Individual

A. Closing: Close the session with a message such as: "Hope you leave here with more questions – about student thinking, about your teaching, and ways that we as a group can help support one another." Have group members keep in mind the following: Dialogue, Reflection, and Inquiry are the keys to successful learning.

B. Exit Cards: Pass out exit cards for group members and ask them to provide some feedback to you as the facilitator. Select one or two questions from the list below to help them summarize their thinking about the mathematics from today's session. Collect exit cards so that a summary can be shared during the next session.

Feedback / Exit Card Questions

- How does the mathematics that we explored connect to your own teaching?
- How do I see what we've done today relate to key mathematical ideas or pedagogical content knowledge?
- What idea or discussion topic did you find most interesting from today's session. Why?
- How was this session for you as a learner?
- What ideas were highlighted for you in today's session that you had not previously considered?
- What are you taking away from today's session?

Session References

- "Bits and Pieces II" in *Connected Mathematics 2*, by G. Lappan et al, Prentice Hall Publications, 2006
- *Teaching Fractions and Ratios for Understanding,* by S. Lamon, Lawrence Erlbaum Associates, 1999

Estimate the following products (to the nearest whole number).

$$2\frac{1}{3}*\frac{5}{8}$$

$$4\frac{1}{2}*3\frac{12}{13}$$

LASW Problem

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$\frac{3}{8}$ of the cake was left after the party. Draw and label how much of the cake was left.

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What portion of the whole cake did he eat?

Student Work Analysis for: **Eating Cake**

Student	What is the evidence of understanding of the area model?	Does the student understand what the portions of the cake represent?		
A				
B				
C				
D				