

# Mathematics Learning Community

## Number Sense

### Session 14

**Title:** *Dividing Fractions*

**Common Core State Standards Addressed in the LASW Problem:**

- 6.NS.1** Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.

**Standards for Mathematical Practice Addressed in the MLC Session:**

- 1:** Make sense of problems and persevere in solving them.      **8:** Look for and express regularity in repeated reasoning.

During both the Math Metacognition and the LASW problem, MLC members and students alike are asked to solve division problems in which either or both the divisor and dividend are fractions. They also search for regular patterns within fraction division in order to give meaning to the “invert and multiply” rule of thumb.

**Standards-Based Teaching and Learning Characteristics in Mathematics Addressed in the MLC Session:**

- 5.1 Depth of content knowledge is evident throughout the presentation of the lesson.
- 5.3 Students’ prior knowledge is incorporated as new mathematical concepts are introduced.
- 5.4 Student misconceptions are anticipated /identified and addressed.
- 5.5 Classroom strategies incorporate multiple forms of representation.

**Session Agenda:**

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|--|
| Part I: Mathematical Background  |
| Part II: Math Metacognition  |
| Part III: Looking at Student Work <ul style="list-style-type: none"><li>• <i>Art Projects</i> Problem (Grades 6 – 8)</li></ul> |
| Part IV: Our Learning  |
| Part V: Feedback and Wrap-up   |

**Materials Needed for this Session:**

- ✓ Nametags      ✓ Chart paper and markers      ✓ Copies of handouts      ✓ Student Work Samples
- ✓ Index cards      ✓ Refreshments      ✓ Highlighters

**Possible Ways to Personalize this Session:**

- Student work samples are no longer provided for the LASW problem in this session. Instead, MLC members will provide the samples for the given problem. See Page 5 in Part III for more details.
- Two Guiding Questions are provided in the Student Work Analysis Grid – the other two questions can be selected by you or by the group.
- If your group needs more work with dividing fractions, sources of additional practice problems are provided on Page 6 in Part III.

## **Part I: Mathematical Background**

*Approximate Time:* 20 Minutes

*Grouping:* Whole Group

- A. Welcome** members of your group to the Math Learning Community. Remind group of **established norms**.
- B. Today's Content:**
- a. The mathematics during this session focuses on dividing fractions.
  - b. What do we need to know in order to be able to divide fractions?
  - c. Chart ideas to refer to during the Protocol for LASW.
  - d. It is important to discuss commonly-held misconceptions about fractions and division during this session.
    - i. Notion of the remainder: how does the portion "left over" after division lead to the concept of fractions itself? In addition, how can students make sense of the situation when you have a fraction of a fractional divisor as a remainder?
    - ii. Reciprocal: how does the concept of the reciprocal play into students' understanding of fractions? What role does the reciprocal play in numerical operations? What misunderstandings may arise from the use of reciprocals within the traditional algorithm?
    - iii. Value of the quotient: Division does not always result in a quotient that is less than the dividend.
  - e. Visual models can be powerful tools used to develop understanding of fraction division, as both a concept and a skill.
- C. Relating Content to the Three C's Theme:**
- a. How do the ways in which students learn to divide fractions relate to the ways in which they divide whole numbers?
  - b. Which attributes or properties of whole numbers hold true for fractions? Which do not?
  - c. How does context play a role in the development of fraction sense?

## Part II: Math Metacognition

Approximate Time: 25 minutes

Grouping: Whole Group

- A. **Problem:** The problem used for this session is appropriate for Grades 6 - 8.

Consider the following problem:

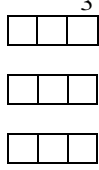
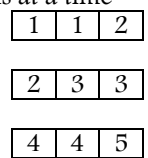
How many  $\frac{2}{3}$ 's are in 3?

Be prepared to explain how you solved the problem.

- B. **Solution to Problem:**  $4\frac{1}{2}$
- C. **Problem Intent:** (Note: The problem intent for all Math Metacognition problems is the same). See Session 2 for more information. In addition, this problem serves as a good start to discussing division involving one or more fractions. This problem also sets up what happens when a remainder, or a fraction of a fraction, exists.
- D. **Discuss** how each group member solved the problem. Refer to the sample Problem Solving Strategies on Page 4. Share any strategies that the group itself did not bring forward. Be sure to provide enough time for discussion of this metacognition problem. Ask MLC members:
- What mathematics did you call on to solve this problem?
  - What does the answer to a division problem mean?
  - What does the fractional part in a division problem mean?
- E. A **common question** that arises with a problem such as this is: "Why is my answer expressed in halves rather than thirds?" The visual model of this problem shows  $\frac{3}{3}$  as the whole, with  $\frac{2}{3}$  as the unit. Therefore when you have  $\frac{1}{3}$  of the original whole remaining, you have  $\frac{1}{2}$  of the  $\frac{2}{3}$  unit. Students should be exposed to varied models for division of fractions to learn to accurately express what is the whole and what is the unit.

## Problem Solving Strategies

### Visual Reasoning

|  |   |
|--|---|
| Represent 3 in $\frac{1}{3}$ portions<br> | Dole out 2 of these $\frac{1}{3}$ portions at a time<br> |
|--|---|



### Verbal Reasoning

There are  $3 \frac{1}{3}$ 's in 1. So, there are  $9 \frac{1}{3}$ 's in 3. There are  $2 \frac{1}{3}$ 's in  $\frac{2}{3}$ . So, the problem comes down to determining how many times 2 divides into 9. 9 divided by 2 is 4.5.

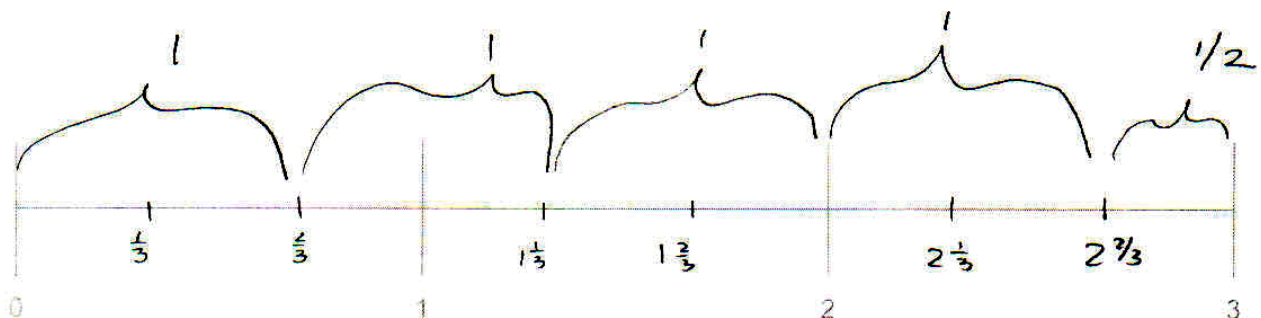
### Numerical Method Using Division Algorithm

$$3 \div \frac{2}{3} = 3 \times \frac{3}{2} = \frac{3}{1} \times \frac{3}{2} = \frac{9}{2} = 4 \frac{1}{2}$$

### Visual Reasoning #2

|  |   |
|--|---|
| Represent 1 whole based on the unit given by the divisor. In this problem, it's thirds.<br> | So, there are $3 \frac{2}{3}$ 's in 1, since $1 \frac{1}{2} = \frac{3}{2}$ .      |
| Consider how many $\frac{2}{3}$ 's are in 1 whole?<br>                                      | To find out how many $\frac{2}{3}$ 's are in 3, multiply $\frac{3}{2} \times 3 =$ |
| In this problem, there is a whole $\frac{2}{3}$ and a half $\frac{2}{3}$ , so there'd be $1 \frac{1}{2} \frac{2}{3}$ 's.   | $\frac{9}{2} = 4 \frac{1}{2}$   |

### Number Line Model



### Part III: Looking at Student Work (LASW)

*Approximate Time:* 50 minutes

*Grouping:* Refer to protocol

- A. Complete the **MLC protocol** with the group.
- B. **Problem:** The problem used for this session is appropriate for Grades 6 - 8.

*Solve each of the following problems. Draw a model to show your thinking.*

- A. Juan has 3 gallons of yellow paint that needs to be put into new containers for an art project. Each container holds  $\frac{2}{7}$  of a gallon of paint. How many containers can Juan fill if he uses up all of the paint?
- B. The art class has  $\frac{5}{6}$  of a gallon of paint that needs to be shared equally among 3 students. How much paint will each student get?
- C. The art class wants to make banners out of construction paper. Each banner takes  $\frac{3}{5}$  yard of paper. How many banners can they make if they have  $2\frac{1}{2}$  yards of paper to use?

- C. **Solutions:** A)  $10\frac{1}{2}$  containers; B)  $\frac{5}{18}$  gallons per person; C)  $4\frac{1}{6}$  banners
- D. **Problem Intent:** This problem provides an opportunity to consider a realistic problem context which involves the three types of fraction division: whole number divided by a fraction; fraction divided by a whole number; and a fraction (or mixed number) divided by a fraction. Visual and verbal reasoning also provide students and adults alike with ways to represent the operation of division by a fraction or of a fraction in a way that makes more sense. In addition, working on these problems using the Visual Reasoning #2 Method found on Page 4 also shows why the use of the traditional algorithm for dividing fractions works.
- E. **Discuss** the following:
- Strategies used to solve this problem.
  - How much experience have group members had with dividing fractions?
  - How can other methods and models be used to represent the process carried out by applying the traditional algorithm?
- F. **Misconceptions/Questions that May Arise:**
- M: Refer to Part I for information on fraction misconceptions.
  - Q: Division of fractions is commonly taught to students using the phrase “invert and multiply,” which highlights the connection between multiplication and division as well as the concept of the reciprocal. However, what understanding is lacking from such instruction? How do students relate the new multiplication problem to the original division problem?
  - Q: How do students interpret remainders when they occur? Problems A and C both have a remainder involved. It is important for teachers to work with these remainders and to determine what they mean and how the remainders relate to their visual models. For example, in A,  $\frac{1}{7}$  of a gallon remains, yet this amount is  $\frac{1}{2}$  of the container size being used.

### Part III: Looking at Student Work (LASW), continued

#### Options for Customization

- G. **Guiding Questions:** Two Guiding Questions have been provided in the Student Work Analysis Grid for this problem. As a way to customize the LASW process, you (or your group) will need to decide on the remaining two questions. You can use the two questions listed below that are specific to this problem or refer to the list of generic questions found on Page 5 in Session 7.
- What evidence is there of proportional reasoning?
  - How is the remainder interpreted?
- H. **Using A Group Member's Student Work:**
- Prior to offering this session to your MLC, you need to collect student work samples for the task: **Art Projects**.*
  - See Page 5 in Session 7 for more details on collecting student work samples. For this task, select 4 samples to discuss during the MLC session. Prior to photocopying samples, mark them as **A, B, C, and D**.
- I. **Additional Practice Problems:** If you think your group needs additional opportunities to model and work with division involving fractions, consider the problems listed below:
- "Bits and Pieces II," *Connected Mathematics 2*, ©Pearson Prentice Hall
    - Whole number divided by a fraction, p. 50: A – F
    - Fraction divided by a whole number, p. 51: A – E
    - Fraction divided by a fraction, p. 52-53: A – E
  - "Number and Operations, Part 2: Making Meaning for Operations," *Developing Mathematical Ideas*, ©Dale Seymour Publications
    - Multiplying and Dividing with Fractions, p. 63: 1 – 8
    - Fractions in Division, p. 69: 1 – 4

## Part IV: Our Learning

*Approximate Time:* 20 Minutes

*Grouping:* Whole Group

- A. **Discussion:** After evidence of student understanding has been discussed as a whole group, you want to facilitate discussion around how the LASW process will impact what teachers do within their classrooms. Some questions to help guide discussion include:
- What do we take away after LASW?
  - What did we learn? About student thinking? About our own knowledge?
    - Refer back to chart made at the beginning of the session
  - How does today's session relate to important mathematical content and pedagogy?
  - How does it impact **your** practice at **your** grade level? (*Note: In order to help teachers connect this session to the mathematics within their own grade level, refer to the information below.*)

### Making Connections Across the Grade Levels

**K – 2:** Work with common fractions begins in this grade band. In addition, some students will begin to explore situations that involve division through real-life contexts, such as when fair sharing. How to interpret the remainder begins to emerge here, as students must decide who gets the last or remaining piece or pieces. (K.CC.5, K.CC.6, K.OA.2, 1.OA.1, 1.G.3, 2.OA.1, 2.OA.3, 2.OA.4, 2.G.2, 2.G.3)

**3 – 5:** The concept of division with whole numbers is truly developed at this grade band. As seen in these problems, misconceptions will arise later on in school for students who always think division makes a lesser quantity. It is important to avoid making blanket statements to suggest such an idea. If students suggest this, use a simple common fraction to provide them with a counterexample. In addition, work with fractions continues. (3.OA.2, 3.OA.3, 3.OA.6, 3.OA.7, 3.OA.8, 3.NF.1, 3.NF.3 a – d, 3.MD.5 a – b, 3.MD.6, 3.MD.7 a – d, 3.G.2, 4.OA.2, 4.OA.3, 4.NBT.6, 4.NF.1, 4.NF.3 a – d, 5.NBT.6, 5.NF.3, 5.NF.7 a – c)

**6 – 8:** The LASW problem is appropriate for Grades 6 to 8 to explore and specifically addresses learning standard 6.NS.1. In addition, the Math Metacognition problems are also appropriate for this grade band. The problems in this session highlight commonly-held misconceptions about fractions, division, remainders, and reciprocals that may not be revealed through the use of the traditional algorithm. It is important that students have varied experiences modeling, interpreting, representing, and analyzing fraction division problems to gain a deeper understanding. (6.RP.3 a – d, 6.NS.2, 6.NS.7, 7.RP.2 a – d, 7.NS.2 a – d, 7.NS.3, 7.EE.3, 7.G.6, 8.NS.1)

- B. **Writing a Problem or Task:** As a way to synthesize learning from today's session, ask MLC members to come up with a math problem or task that would embody the ideas discussed today. The problem should be appropriate to use at their grade level. Writing these problems will help both you as the facilitator and the other group members to develop a stronger sense of how these mathematical ideas show up in classrooms from grades K – 8. (*Note: See Part IV in Session 1 for more details.*)

## Part V: Feedback & Wrap-up

*Approximate Time:* 5 Minutes

*Grouping:* Individual

- A. **Closing:** Close the session with a message such as: “Hope you leave here with more questions – about student thinking, about your teaching, and ways that we as a group can help support one another.” Have group members keep in mind the following: Dialogue, Reflection, and Inquiry are the keys to successful learning.
- B. **Exit Cards:** Pass out exit cards for group members and ask them to provide some feedback to you as the facilitator. Select one or two questions from the list below to help them summarize their thinking about the mathematics from today’s session. Collect exit cards so that a summary can be shared during the next session.

### Feedback / Exit Card Questions

- How does the mathematics that we explored connect to your own teaching?
- How do I see what we’ve done today relate to key mathematical ideas or pedagogical content knowledge?
- What idea or discussion topic did you find most interesting from today’s session. Why?
- How was this session for you as a learner?
- What ideas were highlighted for you in today’s session that you had not previously considered?
- What are you taking away from today’s session?

### Related Student Discourse Video Clip – Math Metacognition Problem

#### *Connecting Mathematical Ideas*

Disc 1: Defending Reasonableness: Division of Fractions

Problem:  $1 \div \frac{2}{3}$

#### Session References

- “Bits and Pieces II” in *Connected Mathematics 2*, by G. Lappan et al, Prentice Hall Publications, 2006
- *Connecting Mathematical Ideas: Middle School Video Cases to Support Teaching and Learning* by J. Boaler and C. Humphreys, Heinemann, 2005
- *Developing Mathematical Ideas: “Number and Operations, Part 2: Making Meaning for Operations,”* by D. Schifter, V. Bastable, and S. Russell, Dale Seymour Publications, 1999.



Consider the following problem:

**How many  $\frac{2}{3}$ 's are in 3?**

Be prepared to explain how you solved the problem.

## LASW Problem

*Solve each of the following problems. Draw a model to show your thinking.*

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Student Work Analysis for: **Art Projects**

| Student | Does the student vary the solution method given the specific problem? | What is the evidence that the student connects visual and numerical reasoning? |  |  |
|---------|---|--|--|--|
| A       |   |  |  |  |
| B       |   |  |  |  |
| C       |   |  |  |  |
| D       |   |  |  |  |