

Phase II Detailed Proposed Revisions PK-12  
Massachusetts Curriculum Framework for Mathematics  
Standards Review October 24, 2016

Pre-Kindergarten and Kindergarten		
2011 Standard	Proposed 2016 Standard	Rationale for Revision
<b>PK.CC.4</b> Count many kinds of concrete objects and Actions up to ten, using one-to-one correspondence, and accurately count as many as seven things in a scattered configuration.	Count many kinds of concrete objects and actions up to ten, <b>recognizing the “one more”, “one less” patterns,</b> using one-to-one correspondence, and accurately count as many as seven things in a scattered configuration.	Edited to include number patterns to enhance the development of number sense in prekindergarten students as they look for and make use of mathematical structures.
<b>K.CC.1</b> Count to 100 by ones and by tens.	Count to 100 by ones and by tens. <b>Recognize the “one more” and “ten more” pattern of counting.</b>	Edited to include number patterns to enhance the development of number sense in prekindergarten students as they look for and make use of mathematical structures.
<b>K.CC.6</b> Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies	Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group <b>for groups with up to ten objects,</b> e.g., by using matching and counting strategies	Edited to incorporate an accompanying footnote into the original standard to clarify student learning expectations for comparing number of objects in kindergarten.
<b>K.OA.5</b> Fluently add and subtract within 5.	Fluently add and subtract within 5 <b>including zero.</b>	Edited to clarify student learning expectations for fluency in kindergarten.
<b>K.MD.3</b> Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.	Classify objects into given categories; count the numbers of objects in each category and sort the categories by count <b>for category counts up to and including 10.</b>	Edited to incorporate an accompanying footnote into the original standard to clarify student learning expectations for sorting by count in kindergarten.

Grade I		
2011 Standard	Proposed 2016 Standard	Rationale for revision
<b>1.OA.2</b> Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.	Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations ( <b>number sentences</b> ) with a symbol for the unknown number to represent the problem.	Edited to remove standard MA.9, which was significantly redundant with the standard but used the term 'number sentences'
<b>1.OA.3</b> Apply properties of operations as strategies to add and subtract. <i>Examples: If <math>8 + 3 = 11</math> is known, then <math>3 + 8 = 11</math> is also known. (Commutative property of addition.) To add <math>2 + 6 + 4</math>, the second two numbers can be added to make a ten, so <math>2 + 6 + 4 = 2 + 10 = 12</math>. (Associative property of addition.)</i>	Apply properties of operations as strategies to add <b>and subtract</b> . <i>Examples: <b>When adding numbers order does not matter.</b> If <math>8 + 3 = 11</math> is known, then <math>3 + 8 = 11</math> is also known. (Commutative property of addition. To add <math>2 + 6 + 4</math>, the second two numbers can be added to make a ten, so <math>2 + 6 + 4 = 2 + 10 = 12</math>. (Associative property of addition.) <b>When adding zero to a number, the result is the same number. (Identity property of zero for addition)</b></i>	Edited to include an explanation of the properties of operations to clarify what the expectation for student learning is and to retain the mathematical rigor. Students are not required to use the formal terms of these properties in grade 1 but they are required to apply them when adding. The properties of operations do not apply to subtraction of whole numbers. The identity property of zero is added to formalize understanding about adding zero to a number.
<b>1.OA.MA.9</b> Write and solve number sentences from problem situations that express relationships involving addition and subtraction within 20.	Standard deleted	Edited to delete standard. This standard has the same student learning expectations with the revised standard 1.OA.2 (See above).
<b>1.NBT.5</b> Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.	<b>Identify arithmetic patterns using 10 more or 10 less using concrete models or drawings and strategies based on place value.</b> Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.	Added number patterns to enhance the development of number sense in grade 1 students as they look for and make use of mathematical structures.
<b>1.MD.MA.5</b> Identify the values of all U.S. coins and know their comparative values (e.g., a dime is of greater value than a nickel). Find equivalent values (e.g., a nickel is equivalent to 5 pennies). Use appropriate notation (e.g., 69¢). Use the values of coins in the solutions of problems.	Identify the values of all U.S. coins and know their comparative values (e.g., a dime is of greater value than a nickel). Find equivalent values (e.g., a nickel is equivalent to 5 pennies). Use appropriate notation (e.g., 69¢). Use the values of coins in the solutions of problems ( <b>up to 100¢</b> ).	Edited to provide parameters that clarify the student learning expectation of the standard.

<p><b>1.G.2</b> Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.</p>	<p>Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, <del>right circular</del> cones, and <del>right circular</del> cylinders) to create a composite shape, and compose new shapes from the composite shape.</p>	<p>The edits were made to improve the coherence in this standard. The wording of the standard is not meant to detract from the intent of composing 2 and 3 dimensional shapes using shapes students are familiar with (cones and cylinders) in the real world.</p>
Grade 2		
2011 Standard	Proposed 2016 Standard	Rationale for revision
<p><b>2.OA.2</b> Fluently add and subtract within 20 using mental strategies. By end of grade 2, know from memory all sums of two one-digit numbers.</p>	<p>Fluently add and subtract within 20 using mental strategies <b>such as counting on; making tens; decomposing a number; using the relationship between addition and subtraction; and creating equivalent but easier or known sums.</b> By end of grade 2, know from memory all sums of two <del>one</del> single-digit numbers <b>and related subtractions.</b></p>	<p>Edits made to remove need for the footnote (a list of strategies). The added wording provides clarity and mathematical rigor and incorporates the learning expectations of MA.2.a.</p>
<p><b>2.OA.2.MA.2.a.</b> By the end of grade 2, know from memory related subtraction facts of sums of two one-digit numbers.</p>	<p>Standard deleted</p>	<p>Edits made to 2.OA.2 (see above) to remove need for this additional standard.</p>
<p><b>2.OA.3</b> Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.</p>	<p><b>Identify patterns in odd and even numbers using concrete models or drawings.</b> Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.</p>	<p>Added patterns to enhance the development off algebraic thinking in grade 2 students as they look for and make use of mathematical structures.</p>
<p><b>2.NBT.2</b> Count within 1000; skip-count by 5s, 10s, and 100s</p>	<p><b>Identify patterns in skip counting.</b> Count within 1000; skip-count by 5s, 10s, and 100s</p>	<p>Added patterns to enhance the development off algebraic thinking in grade 2 students as they look for and make use of mathematical structures.</p>
<p><b>2.MD.8</b> Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies using \$ and ¢ symbols appropriately. <i>Example: If you have 2 dimes and 3 pennies, how many cents do you have?</i></p>	<p>Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies <b>(up to \$10.00)</b>, using \$ and ¢ symbols appropriately. <i>Example: If you have 2 dimes and 3 pennies, how many cents do you have?</i></p>	<p>Edited to provide parameters that clarify the student learning expectation of the standard.</p>

<p><b>2.MD.9</b> Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.</p>	<p>Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. <b>Organize and record data.</b> Show the measurements by making a <b>dot plot</b> (line plot), line plot, where the horizontal scale is marked off in whole-number units.</p>	<p>Added the words dot plot to the standard for clarification about the expectation for representing data and to maintain the mathematical rigor. An example of a dot plot was added to the Glossary.</p>
Grade 3		
2011 Standard	Proposed 2016 Standard	Rationale for revision
<p><b>3.OA.5</b> Apply properties of operations as strategies to multiply and divide. <i>Examples: If <math>6 \times 4 = 24</math> is known, then <math>4 \times 6 = 24</math> is also known. (Commutative property of multiplication.) <math>3 \times 5 \times 2</math> can be found by <math>3 \times 5 = 15</math> then <math>15 \times 2 = 30</math>, or by <math>5 \times 2 = 10</math> then <math>3 \times 10 = 30</math>. (Associative property of multiplication.) Knowing that <math>8 \times 5 = 40</math> and <math>8 \times 2 = 16</math>, one can find <math>8 \times 7</math> as <math>8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56</math>. (Distributive property)</i></p>	<p>Apply properties of operations <del>as strategies</del> <b>and divide.</b> when multiplying <b>and divide.</b> <i>Examples: When multiplying two numbers it does not matter which is considered first. If <math>6 \times 4 = 24</math> is known, then <math>4 \times 6 = 24</math> is also known. (Commutative Property) When multiplying more than two numbers it does not matter which two numbers are multiplied first. <math>3 \times 5 \times 2</math> can be found by <math>3 \times 5 = 15</math> then <math>15 \times 2 = 30</math>, or by <math>5 \times 2 = 10</math> then <math>3 \times 10 = 30</math>. (Associative property of multiplication.) When multiplying two numbers one can be decomposed and multiplied. Knowing that <math>8 \times 5 = 40</math> and <math>8 \times 2 = 16</math>, one can find <math>8 \times 7</math> as <math>8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56</math>. (Distributive property.) When a number is multiplied by 1 the result is the same number. (Identity Property for multiplication).</i></p>	<p>Edited to clarify the properties of operations for multiplication to retain the mathematical rigor of the standard. Students are not required to use the formal terms of these properties in grade 3 but they are required to apply them when multiplying. The identity property for multiplication is added to formalize understanding about multiplying by the number 1.</p>
<p><b>3.OA.7</b> Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that <math>8 \times 5 = 40</math>, one knows <math>40 \div 5 = 8</math>) or properties of operations. By the end of grade 3, know from memory all products of two one-digit numbers.</p>	<p>Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that <math>8 \times 5 = 40</math>, one knows <math>40 \div 5 = 8</math>) or properties of operations. By the end of grade 3, know from memory all products of two <del>one</del> single-digit numbers <b>and related divisions.</b></p>	<p>Edits made to simplify reading the standard and to increase the grade level coherence of the relationship between multiplication and division.</p>
<p><b>3.OA.8</b> Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies, including rounding.</p>	<p>Solve two-step word problems using the four operations <b>for problems posed with whole numbers and having whole number answers.</b> Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies, including rounding. <b>(Students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order).</b></p>	<p>Edited to incorporate accompanying footnotes into the standard itself to clarify the focus in this grade and student expectations.</p>

<p><b>3.NBT.2</b> Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.</p>	<p>Fluently add and subtract within 1000 using <b>a range of</b> strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.</p>	<p>Edited to incorporate an accompanying footnote into the standard itself to help clarify student expectations.</p>
<p><b>3.NF.1</b> Understand a fraction <math>1/b</math> as the quantity formed by 1 part when a whole is partitioned into <math>b</math> equal parts; understand a fraction <math>a/b</math> as the quantity formed by <math>a</math> parts of size <math>1/b</math>.</p>	<p>Understand a fraction <math>1/b</math> as the quantity formed by 1 part when a whole (<b>a single unit</b>) is partitioned into <math>b</math> equal parts; understand a fraction <math>a/b</math> as the quantity formed by <math>a</math> parts of size <math>1/b</math>.</p>	<p>Edit made to clarify the term whole as it relates to fraction expectations in grade 3</p>
<p><b>3.NF.2a</b> Represent a fraction <math>1/b</math> on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into <math>b</math> equal parts. Recognize that each part has size <math>1/b</math> and that the endpoint of the part based at 0 locates the number <math>1/b</math> on the number line.</p>	<p>a. Represent <b>the unit</b> fraction <math>1/b</math>, on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into <math>b</math> equal parts. Recognize that each part has size <math>1/b</math> and that <b>unit fraction <math>1/b</math> is located <math>1/b</math> of a whole unit from 0</b> <del>the endpoint of the part based at 0 locates the number <math>1/b</math></del> on the number line.</p>	<p>Edit made to clarify the concept of a unit fraction and its location on a number line.</p>
<p><b>3.MD.2</b> Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.</p>	<p>Measure and estimate liquid volumes and masses of objects using standard <b>metric</b> units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same <b>metric</b> units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.</p>	<p>Edited to clarify the focus of this standard on the metric system</p>
<p>3.MD.4 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.</p>	<p>Generate <b>and record</b> measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a <b>dot plot</b> (line plot), where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters. (<b>e.g. measure the length of pencils students are using to the nearest whole, <math>1/2</math>, and/or <math>1/4</math></b>).</p>	<p>Edited to clarify student expectations for recording data and for representing data . A context example was provided to maintain the mathematical rigor of the standard.</p>
<p>3. MD .6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).</p>	<p>Measure areas by counting unit squares (square cm, square m, square in, square ft, and <b>non-standard</b> units).</p>	<p>Edited to clarify the term ‘improvised’ units.</p>
	<p><b>MA.7.e. Provide appropriate units for area and calculations of area. For example: a 5 m by 2 m rectangular panel has an area of 10 square meters.</b></p>	<p>This new standard was added to focus on the proper use of units when calculating areas in measures using standard units.</p>

<p><b>3.G.1</b> Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.</p>	<p>Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). <b>Compare and classify shapes by their sides and angles (right angle/non-right angle).</b> Recognize rhombuses, rectangles, squares, and <b>trapezoids</b> as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.</p>	<p>Edited to clarify student expectations for comparing and classifying shapes and to clarify a focus on right and non-right angles.</p>
<p><b>3.G.2</b> Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. <i>For example, partition a shape into 4 parts with equal areas and describe the area of each part as <math>\frac{1}{4}</math> of the area of the shape.</i></p>	<p>Partition <del>wholes shapes</del> <b>(circle, rectangle, line segment, etc.)</b> into parts with equal areas <b>and/or equal lengths</b>. Express the area of each part as a unit fraction of the whole. <i>For example, partition a shape into 4 parts with equal areas and describe the area of each part as <math>\frac{1}{4}</math> of the area of the shape.</i></p>	<p>Edited to include terms that clarify the focus and expectation of the standard.</p>
<b>Grade 4</b>		
<b>2011 Standard</b>	<b>Proposed 2016 Standard</b>	<b>Rationale for revision</b>
<p><b>4.NBT.1</b> Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. <i>For example, recognize that <math>700 \div 70 = 10</math> by applying concepts of place value and division.</i></p>	<p>Recognize that in a multi-digit whole number, a digit in <del>one</del> <b>any</b> place represents ten times what it represents in the place to its right. <i>For example, recognize that <math>700 \div 70 = 10</math> by applying concepts of place value and division.</i></p>	<p>Edited to avoid confusion about the “one’s” place. In numbers</p>
<p><b>4.NBT.2</b> Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using <math>&gt;</math>, <math>=</math>, and <math>&lt;</math> symbols to record the results of comparisons.</p>	<p>Read and write multi-digit whole numbers <b>to the millions place</b>, using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers <b>within 1,000,000</b> based on meanings of the digits in each place, using <math>&gt;</math>, <math>=</math>, and <math>&lt;</math> symbols to record the results of comparisons.</p>	<p>Edited to provide parameters that clarify the student learning expectation of the standard.</p>
<p><b>4.NBT.3</b> Use place value understanding to round multi-digit whole numbers to any place.</p>	<p>Use place value understanding to round multi-digit whole numbers <b>up to 1000</b>, to any place.</p>	<p>Edited to provide parameters that clarify the student learning expectation of the standard.</p>
<p><b>4.NBT.6</b> Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</p>	<p>Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</p> <p style="text-align: center;"><b>SEE APPENDIX FOR EDITS</b></p>	<p>Edited to include an example to clarify the expectation for student learning and to retain the mathematical rigor.</p>

<p><b>4.NF.1</b> Explain why a fraction <math>a/b</math> is equivalent to a fraction <math>(n \times a)/(n \times b)</math> by using visual fraction models, with attention to how the numbers and sizes of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.</p>	<p>Explain why a fraction <math>a/b</math> is equivalent to a fraction <math>(n \times a)/(n \times b)</math> by using visual fraction models, with attention to how the numbers and sizes of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions, <b>including fractions greater than 1.</b> <i>SEE APPENDIX FOR EDITS</i></p>	<p>Edited to include an example to clarify the expectation for student learning and to retain the mathematical rigor.</p>
<p><b>4.NF.3.a</b> Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.</p>	<p>Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. <b>(The whole can be a single unit or a set of objects)</b></p>	<p>Edited to clarify the meaning of the term ‘whole’ in Grade 4 as the development of student understanding of the concept of ‘whole’ progresses through Grades 3-5.</p>
<p><b>4.NF.3.b</b> Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. <i>Examples: <math>3/8 = 1/8 + 1/8 + 1/8</math>; <math>3/8 = 1/8 + 2/8</math>; <math>2 \frac{1}{8} = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8</math>.</i></p>	<p>Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using <b>drawings</b> or visual fraction models. <i>Examples:</i> <math>3/8 = 1/8 + 1/8 + 1/8</math>; <math>3/8 = 1/8 + 2/8</math>; <math>2 \frac{1}{8} = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8</math>.</p>	<p>Edited to add coherence to 4.NF.3.a. When ‘wholes’ are a sets of objects, visual fraction models are not appropriate to justify decompositions of fractions.</p>
<p><b>4.NF.6</b> Use decimal notation for fractions with denominators 10 or 100. <i>For example, rewrite 0.62 as <b>62/100</b>; describe a length as 0.62 meters; locate 0.62 on a number line diagram</i></p>	<p>Use decimal notation <del>for</del> <b>to represent</b> fractions with denominators 10 or 100. <i>For example, rewrite 0.62 as <b>62/100</b>; describe a length as 0.62 meters; locate 0.62 on a number line diagram</i></p>	<p>Edited the wording to clarify the focus and expectation of the standard.</p>
<p><b>4.MD.4</b> Make a line plot to display a data set of measurements in fractions of a unit (<math>1/2, 1/4, 1/8</math>). Solve problems involving addition and subtraction of fractions by using information presented in line plots. <i>For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.</i></p>	<p>Make a <b>dot plot</b> (line plot) <b>representation</b> to display a data set of measurements in fractions of a unit (<math>1/2, 1/4, 1/8</math>). Solve problems involving addition and subtraction of fractions by using information presented in <b>dot plots</b> (line plots). <i>For example, from a <b>dot plot</b> (line plot) find and interpret the difference in length between the longest and shortest specimens in an insect collection. (See Glossary for examples of dot plots / line plots)</i></p>	<p>Added the words dot plot to the standard for clarification about the expectation for representing data and to maintain the mathematical rigor. Also an example was added to the Glossary.</p>
<p><b>4.G.2</b> Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.</p>	<p>Classify two-dimensional figures based on the presence or absence of parallel or perpendicular <del>lines</del> <b>sides</b>, <b>length of sides</b> <b>and</b>/ or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.</p>	<p>Edited to clarify the focus of this standard on classifying two-dimensional figures by their sides (parallel, perpendicular, equal in length) and angles</p>

Grade 5		
2011 Standard	Proposed 2016 Standard	Rationale for Revision
<b>5.OA.1</b> Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols	Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. (e.g. $(6 \times 30) + (6 \times 1/2)$ )	Edited to include an example to clarify the expectation in the standard for student learning and to retain appropriate mathematical rigor.
<b>5.NBT.1</b> Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.	Recognize that in a multi-digit number, including decimals, a digit in <del>any</del> one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.	Edited to clarify the expectation for student learning and to support coherence in the base ten number system
<b>5.NBT.5</b> Fluently multiply multi-digit whole numbers using the standard algorithm	Fluently multiply multi-digit whole numbers (up to 2 digits $\times$ 4 digits, 3 digits $\times$ 3 digits) using the standard algorithm.	Edited to provide parameters that clarify the student learning expectation of the standard.
<b>5.NF.2</b> Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result $2/5 + 1/2 = 3/7$ , by observing that $3/7 < 1/2$ .	Solve word problems involving addition and subtraction of fractions referring to the same whole (the whole can be a single unit or a set of objects), including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result $2/5 + 1/2 = 3/7$ , by observing that $3/7 < 1/2$ .	Edited to add concepts in a coherent manner across grades for developing understanding of numbers and fractions.
<b>5.NF.4</b> Interpret the product $(a/b) \times q$ as $a$ parts of a partition of $q$ into $b$ equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$ . For example, use a visual fraction model to show $(2/3) \times 4 = 8/3$ , and create a story context for this equation. Do the same with $(2/3) \times (4/5) = 8/15$ . (In general, $(a/b) \times (c/d) = ac/bd$ .)	Interpret the product $(a/b) \times q$ as $a$ parts of a partition of $q$ into $b$ equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$ . For example, use a visual fraction model and/or area model to show $(2/3) \times 4 = 8/3$ , and create a story context for this equation. Do the same with $(2/3) \times (4/5) = 8/15$ . (In general, $(a/b) \times (c/d) = ac/bd$ .)	Edited to maintain mathematical rigor by provide options for ways of showing conceptual understanding when multiplying fractions by whole numbers
<b>5.NF.5.a</b> Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.	Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. For example: $6 \times \%$ is twice as large as $3 \times \%$ .	Edited to clarify student expectations for the standard and to maintain the focus of the standard.

<p><b>5.NS.MA.1</b> Use positive and negative integers to describe quantities such as temperature above/below zero, elevation above/below sea level, or credit/debit.</p>	<p>Standard deleted</p>	<p>Edited to maintain focus in Grade 5 on fluency in multi-digit whole numbers and multiplying fractions. Integers are introduced in grade 6.</p>
<p><b>5.MD.2</b> Make a line plot to display a data set of measurements in fractions of a unit. Use operations on fractions for this grade to solve problems involving information presented in line plots. <i>For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.</i></p>	<p>Make a <b>dot plot</b> (line plot) to display a data set of measurements in fractions of a unit. Use operations on fractions for this grade to solve problems involving information presented in <b>dot plots</b> (line plots). <i>For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.</i></p>	<p>Edited to clarify the term ‘line plot’ and preserve the expectation for representing data.. An example was added to the Glossary.</p>
<p><b>5.MD.4</b> Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and non-standard units.</p>	<p>Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and <b>improvised non-standard</b> units.</p>	<p>Edited the wording to clarify the focus and expectation of the standard.</p>
<p><b>5.MD.5.a</b> Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.</p>	<p>Find the volume of a right rectangular prism with whole-number <b>side edge</b> lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.</p>	<p>Edited to clarify the focus of this standard to use edge length of a three dimensional prism when finding volume.</p>
<p><b>5.MD.5.b</b> Apply the formula <math>V = l \times w \times h</math> and <math>V = b \times h</math> for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems.</p>	<p>Apply the formula <math>V = l \times w \times h</math> and <math>V = <del>b</del> B \times h</math> (<b>where B stands for the area of the base</b>) for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems.</p>	<p>Edited to clarify the meaning of symbols used in formulas for volumes.</p>
<p><b>5.G.4</b> Classify two-dimensional figures in a hierarchy based on properties .</p>	<p>Classify two-dimensional figures in a hierarchy based on properties (e.g. <b>“having parallel sides or “having all sides of equal length”</b>).</p>	<p>Edited to include an example to clarify the expectation for student learning and to retain the mathematical rigor.</p>

Grade 6		
2011 Standard	Proposed 2016 Standard	Rationale for Revision
<b>6.RP.1</b> Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities, <b>including the distinction between part:part and part/whole</b> . For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.” <b>“The ratio of males to females is 2:3, meaning that 3/5 of the group is female.”</b>	Edited to add coherence for developing student understanding of ratio relationships and to clarify the expectation for student learning is and to retain the mathematical rigor.
<b>6.RP.2</b> Understand the concept of a unit rate <b>a/b</b> associated with a ratio $a:b$ with $b \neq 0$ , and use rate language in the context of a ratio relationship. For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is $\frac{3}{4}$ cup of flour for each cup of sugar.” “We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger.”	Understand the concept of a unit rate <b>a/b</b> associated with a ratio $a:b$ with $b \neq 0$ , and use rate language in the context of a ratio relationship, <b>including the use of units</b> . For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is $\frac{3}{4}$ cup of flour for each cup of sugar.” “We paid \$75 for 15 hamburgers, which is a rate of <b>\$5 dollars</b> per hamburger.”	Edited to clarify the focus of this standard on developing understanding of unit rates from concepts of ratios.
<b>6.RP.3.d</b> Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.	Use ratio reasoning to convert measurement units <b>within and between measurement systems</b> ; manipulate and transform units appropriately when multiplying or dividing quantities. <b>Ex. Solve problems that relate the mass of an object to its volume.</b>	Edited to clarify student expectations for the standard and to maintain the focus of the standard on using ratio reasoning to solve problems
<b>6.RP.MA.3.e</b> Solve problems that relate the mass of an object to its volume.	<del>Solve problems that relate the mass of an object to its volume.</del>	The standard was moved to be included with 6.RP.3.d
<b>6.G.MA.1.a</b> Use the relationships among radius, diameter, and center of a circle to find its circumference and area.	<del>Use the relationships among radius, diameter, and center of a circle to find its circumference and area.</del>	Edited by moving the standard to grade 7 to maintain coherence in the cluster of standards that develop measurement concepts of polygons.
<b>6.G.MA.1.b</b> Solve real-world and mathematical problems involving the measurements of circles.	<del>Solve real-world and mathematical problems involving the measurements of circles.</del>	Edited by moving the standard to grade 7 to maintain coherence in the cluster of standards that develop measurement concepts of polygons.
<b>6.SP.2</b> Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.	Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center ( <b>median, mean, and/or mode</b> ), spread ( <b>range interquartile range</b> ), and overall shape.	Edited to clarify the expectation for student learning about statistical measures in this standard

<p><b>6.SP.5</b> Summarize numerical data sets in relation to their context, such as by giving quantitative measures of center (median, and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.</p>	<p>Summarize numerical data sets in relation to their context, such as by giving quantitative measures of center (median, and/or mean), and <del>variability-spread</del> (range and/or interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.</p>	<p>Edited to create a coherent learning progression from grade 6 to grade 7 in relation to summarizing and describing distributions</p>
Grade 7		
2011 Standard	Proposed 2016 Standard	Rationale for Revision
<p><b>7.RP.3</b> Use proportional relationships to solve multi-step ratio and percent problems. <i>Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.</i></p>	<p>Use proportional relationships to solve multi-step ratio, <i>rate</i>, and percent problems. <i>Examples: simple interest, tax, markups and <del>markdowns-discounts</del>, gratuities and commissions, fees, percent increase and decrease, percent error.</i></p>	<p>Edited to maintain coherence and focus in analyzing proportional relationships and ratio/rate learning progression.</p>
<p><b>7.NS.1.a</b> Describe situations in which opposite quantities combine to make 0. <i>For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.</i></p>	<p>Describe situations in which opposite quantities combine to make 0. <i>For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged; if you open a new bank account with a deposit of \$30 and then withdraw \$30, you are left with a \$0 balance.</i></p>	<p>Edited to clarify student expectations for understanding rational numbers (integers). A context example was provided to maintain the focus and mathematical rigor of the standard.</p>
<p><b>7.NS.1.d</b> Apply properties of operations as strategies to add and subtract rational numbers.</p>	<p>Apply properties of operations <del>as strategies to when</del> adding and subtracting rational numbers.</p>	<p>Edited to clarify the expectation for student learning and the focus (applying the properties).</p>
<p><b>7.NS.2.c</b> Apply properties of operations as strategies to multiply and divide rational numbers.</p>	<p>Apply properties of operations <del>as strategies to when</del> multiplying and dividing rational numbers.</p>	<p>Edited to clarify the expectation for student learning and the focus (applying the properties).</p>
<p><b>7.NS.3</b> Solve real-world and mathematical problems involving the four operations with rational numbers</p>	<p><del>Solve real-world and mathematical problems involving the four operations with rational numbers</del></p>	<p>Deleted the standard. The focus and expectation of the standard is in 7.EE.3</p>
<p><b>7.EE.1</b> Apply properties of operations as strategies to add, subtract, factor, and expanding linear expressions with rational coefficients.</p>	<p>Apply properties of operations <del>as strategies to when</del> adding, subtracting, factoring, and expanding linear expressions with rational coefficients. <i>For example, <math>4x + 2 = 2(2x + 1)</math> and <math>-3(x - 5/3) = -3x + 5</math></i></p>	<p>Edited to clarify the expectation for student learning of the focus (applying the properties) and the mathematical rigor.</p>

<p><b>7.EE.2</b> Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. <i>For example, <math>a + 0.05a = 1.05a</math> means that “increase by 5%” is the same as “multiply by 1.05.”</i></p>	<p>Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. <i>For example, <math>a + 0.05a = 1.05a</math> means that “increase by 5%” is the same as “multiply by 1.05.”</i>  <i>A shirt at a clothing store is on sale for 20% off the regular price, <math>p</math>. The discount can be expressed as <math>0.2p</math>. The new price for the shirt can be expressed as <math>p - 0.2p</math> or <math>0.8p</math>.</i></p>	<p>Edited to give a context example to maintain the focus and mathematical rigor of the standard.</p>
<p><b>7.G.2</b> Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle</p>	<p>Draw (freehand, with ruler and protractor, and with technology) <b>two-dimensional</b> geometric shapes with given conditions. Focus on <b>constructing drawing</b> triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.</p>	<p>Edited to clarify the expectation for student learning and the focus (draw geometric shapes).</p>
<p><b>7.G.3</b> Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.</p>	<p>Describe the <b>shape of the two-dimensional face of the</b> figures that result from slicing three-dimensional figures <b>such as,</b> <del>in</del> <b>plane sections of</b> right rectangular prisms and right rectangular pyramids.</p>	<p>Edited to improve the coherence of the standard. The wording of the standard is not meant to detract from the intent of describing geometrical figures and the relationships between them.</p>
<p><b>7.G.4</b> Know the formulas for the area and circumference of a circle and solve problems; give an informal derivation of the relationship between the circumference and area of a circle.</p>	<p>Know the formulas for the area and circumference of a circle and solve problems <b>related to area and circumference;</b> <del>give an informal derivation of the relationship between the circumference and area of a circle.</del></p>	<p>Edited to maintain the focus on solving real world and mathematical problems related to area and circumference of circles.</p>
<p><b>7.G.5</b> Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and use them to solve simple equations for an unknown angle in a figure.</p>	<p>Use facts about supplementary, complementary, vertical, and adjacent angles <del>in a multi-step problem</del> to write <b>simple equations</b> and use them to solve <b>simple equations</b> for an unknown angle in a figure.</p>	
<p><b>7.G.MA.7</b> Solve real-world and mathematical problems involving the surface area of spheres.</p>	<p><del>Solve real-world and mathematical problems involving the surface area of spheres.</del></p>	<p>Deleted to maintain the focus in grade 7 on solving problems involving surface area of objects composed of two-and-three dimensional polygonal shapes.</p>

<p><b>7.SP.3</b> Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. <i>For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.</i></p>	<p>Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. <i>For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team. The variability (mean absolute deviation) <b>for each team</b> is about twice <b>the value of their mean</b>; on a dot plot, the separation between the two distributions of heights <b>is-would be</b> noticeable.</i></p>	<p>Edited to improve the coherence of the expectation of the standard and clarity. The wording of the example is not meant to detract from the intent of drawing informal about two populations.</p>
---	---	--

**Grade 8**

2011 Standard	Proposed 2016 Standard	Rationale for Revision
<p><b>8.NS.1</b> Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.</p>	<p>Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers <b>show</b> recognize that the decimal expansion repeats eventually and <b>convert that</b> a decimal expansion which repeats <b>can be represented by eventually into</b> a rational number.</p>	<p>Edited to improve clarity about the focus of the standard to understand when numbers will have terminating or repeating decimals (rational) and when they will not terminate or repeat (irrational).</p>
<p><b>8.EE.8.b</b> Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. <i>For example, <math>3x + 2y = 5</math> and <math>3x + 2y = 6</math> have no solution because <math>3x + 2y</math> cannot simultaneously be 5 and 6.</i></p>	<p>Solve systems of two linear equations in two variables algebraically (<b>using substitution and elimination strategies</b>), and estimate solutions by graphing the equations. Solve simple cases by inspection. <i>For example, <math>3x + 2y = 5</math> and <math>3x + 2y = 6</math> have no solution because <math>3x + 2y</math> cannot simultaneously be 5 and 6.</i></p>	<p>Edited to clarify student expectations for solving systems of equations in Grade 8 and maintain the mathematical rigor of the standard.</p>
<p><b>8.G.1.a.b.c</b> Verify experimentally the properties of rotations, reflections, and translations:</p> <ol style="list-style-type: none"> <li>Lines are taken to lines, and line segments to line segments of the same length.</li> <li>Angles are taken to angles of the same measure.</li> <li>Parallel lines are taken to parallel lines.</li> </ol>	<p>Verify experimentally the properties of rotations (<b>about the origin</b>), reflections, and translations:</p> <ol style="list-style-type: none"> <li>Lines are <b>taken-transformed</b> to lines, and line segments to line segments of the same length.</li> <li>Angles are <b>taken transformed</b> to angles of the same measure.</li> <li>Parallel lines are <b>taken-transformed</b> to parallel lines.</li> </ol>	<p>Edited the wording to clarify the focus and expectation of the standard.</p>

<b>8.G.2</b> Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.	Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations ( <b>about the origin</b> ), reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.	Edited the wording to clarify the focus and expectation of the standard.
<b>8.G.6</b> Explain a proof of the Pythagorean Theorem and its converse.	Explain <del>a proof of</del> the Pythagorean Theorem <del>and its converse</del> . <b>Understand the relationship between the sides of a right triangle in terms of the areas of the squares constructed on each side.</b>	Edited to clarify student expectations for understanding and explaining the Pythagorean Theorem in Grade 8 and to maintain the mathematical rigor of the standard.
<b>High School Model Algebra I</b>		
<b>2011 Standard</b>	<b>Proposed 2016 Standard</b>	<b>Rationale for Revision</b>
<b>NQ.1</b> Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	Use units as a way to <del>understand, identify, and solve</del> <del>and to guide the solution of</del> multi-step real world application problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	Edited to clarify student expectations for using units to solve problems in Algebra I and to maintain the mathematical rigor of the standard.
<b>A.SSE.1.b</b> Interpret complicated expressions by viewing one or more of their parts as a single entity. <i>For example, interpret <math>P(1 + r)^t</math> as the product of <math>P</math> and a factor not depending on <math>P</math>.</i>	Interpret complicated expressions by viewing one or more of their parts as a single entity. <i>For example, interpret <math>P(1 + r)^t</math> as the product of <math>P</math> and a factor not depending on <math>P</math>. An application for this could be to find the principal amount of money that is growing at a rate, <math>r</math>, over a period of time, <math>t</math>, in years</i>	Edited to provide an example that connects real world financial application with mathematical expressions.
<b>A.SSE.2</b> Use the structure of an expression to identify ways to rewrite it. <i>For example, see <math>x^4 - y^4</math> as <math>(x^2)^2 - (y^2)^2</math>, thus recognizing it as a difference of squares that can be factored as <math>(x^2 - y^2)(x^2 + y^2)</math>.</i>	Use the structure of an expression to identify ways to rewrite it. <i>For example, see <del><math>x^4 - y^4</math> as <math>(x^2)^2 - (y^2)^2</math> thus recognizing it</del> <math>(x + a)^2 - b^2</math> as a difference of squares that can be factored as <math>(x + a + b)(x + a - b)</math> <del><math>(x^2 - y^2)(x^2 + y^2)</math>.</del></i>	Edited the example in the standard to clarify the focus and expectations of the standard for Algebra I students.
<b>A.APR.1</b> Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials	<del>Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials</del> Add, subtract, and multiply polynomial expressions, including factoring and/or expanding polynomial expressions, identifying and combining like terms, and applying the Distributive Property.	Edited to clarify the focus and expectation of the standard and maintain the mathematical rigor for Algebra I students.

<p><b>A.CED.1</b> Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions</p>	<p>Create equations and inequalities in one variable <b>to represent a given context</b> and use them to solve problems. Include equations arising from linear and quadratic functions, and <b>simple rational and</b> exponential functions</p>	<p>Edited to clarify the focus and expectation of the standard and maintain the mathematical rigor for Algebra I students.</p>
<p><b>A.CED.3</b> Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. <i>For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.</i></p>	<p>Represent constraints by <b>linear</b> equations or inequalities, and by systems of <b>linear</b> equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. <i>For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.</i></p>	<p>Edited to clarify the focus of the standard is linear equations and inequalities for Algebra I students.</p>
<p><b>A.CED.4</b> Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. <i>For example, rearrange Ohm’s law <math>V = IR</math> to highlight resistance <math>R</math></i></p>	<p>Rearrange formulas to highlight a quantity of interest, using <b>the same reasoning as in solving equations</b> properties of equality. <i>For example, rearrange Ohm’s law <math>V = IR</math> <b>to highlight resistance <math>R</math></b> to solve for the variable <math>R</math>. Manipulate variables in formulas used in financial contexts, such as for simple interest (<math>I = Prt</math>) and simple breakeven (<math>Q = \frac{F}{P - v}</math>).</i></p>	<p>Edited to add examples to the standard related to finance and to clarify the expectations of the standard for Algebra I students.</p>
<p><b>A.REI.1</b> Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.</p>	<p>Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify <b>or refute</b> a solution method.</p>	<p>Edited to be consistent with Standard for Mathematical Practice 3, <i>Construct viable arguments and critique the reasoning of others.</i></p>
<p><b>A.REI.4.b</b> Solve quadratic equations by inspection (e.g., for <math>x^2 = 49</math>), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as <math>a \pm bi</math> for real numbers <math>a</math> and <math>b</math>.</p>	<p>Solve quadratic equations by inspection (e.g., for <math>x^2 = 49</math>), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the <b>quadratic formula gives complex solutions and write them as <math>a \pm bi</math> for real numbers <math>a</math> and <math>b</math></b> solution to a quadratic equation results in non-real solutions.</p>	<p>Edited to address a footnote that clarifies the focus for solution expectations for Algebra I students.</p>
<p><b>A.REI.MA.4.c</b> Demonstrate an understanding of the equivalence of factoring, completing the square, or using the quadratic formula to solve quadratic equations.</p>	<p><b>Standard deleted</b></p>	<p>Deleted the standard due to redundancy in focus and mathematical rigor with standard A.REI.4.b</p>

<p><b>A.REI.7</b> Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. <i>For example, find the points of intersection between the line <math>y = -3x</math> and the circle <math>x^2 + y^2 = 3</math>.</i></p>	<p>Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. <i>For example, find the points of intersection between the line <math>y = -3x</math> and the <del>circle <math>x^2 + y^2 = 3</math></del> parabola <math>y = x^2 + x</math>.</i></p>	<p>Edited to address a footnote that clarifies the focus for linear/quadratic systems and expectations for Algebra I students.</p>
<p><b>A.REI.10</b> Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Show that any point on the graph of an equation in two variables is a solution to the equation</p>	<p>Understand that the graph of an equation in two variables (<del>equations include linear, absolute value, exponential</del>) is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Show that any point on the graph of an equation in two variables is a solution to the equation</p>	<p>Edited to incorporate a footnote that clarified the focus for graphs of equations and expectations for Algebra I students.</p>
<p><b>A/REI.11</b> Explain why the <math>x</math>-coordinates of the points where the graphs of the equations <math>y = f(x)</math> and <math>y = g(x)</math> intersect are the solutions of the equation <math>f(x) = g(x)</math>; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where <math>f(x)</math> and/or <math>g(x)</math> are linear, polynomial, rational, absolute value, exponential, and logarithmic functions</p>	<p>Explain why the <math>x</math>-coordinates of the points where the graphs of the equations <math>y = f(x)</math> and <math>y = g(x)</math> intersect are the solutions of the equation <math>f(x) = g(x)</math>; find the solutions approximately, e.g., using technology to graph the functions, make tables of values. <del>or find successive approximations. Include cases where <math>f(x)</math> and/or <math>g(x)</math> are linear, polynomial, rational, absolute value, exponential, and logarithmic functions</del></p>	<p>Edited to address a footnote that clarifies the focus in Algebra I on linear, quadratic and exponential functions</p>
<p><b>F.IF.4</b> For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. <i>Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.</i></p>	<p>For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. <i>Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; <del>and periodicity.</del></i></p>	<p>Edited to be consistent with the focus for Algebra I expectations for functions (linear, quadratic, and exponential functions). Periodic functions are studied in Algebra II.</p>
<p><b>F.IF.7 b.</b> Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.  <b>e.</b> Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude</p>	<p><b>b.</b> Graph <del>square root, cube root, and</del> piecewise-defined functions, including step functions and absolute value functions.  <b>e.</b> Graph exponential <del>and logarithmic</del> functions, showing intercepts and end behavior. <del>, and trigonometric functions, showing period, midline, and amplitude</del></p>	<p>Edited to address a footnote that clarifies the focus of Algebra I expectations for functions which can include piecewise-defined, step, and absolute value when analyzing key features of their graphs.</p>

<p><b>F.IF.8 a</b> Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.</p>	<p>Use the process of factoring and completing the square in a quadratic function to show zeros, <b>maximum/minimum extreme-values</b>, and symmetry of the graph, and interpret these in terms of a context.</p>	<p>Edited to clarify the wording of the standard to be consistent with the key features of linear, exponential and quadratic functions.</p>
<p><b>F.IF.8 b</b> Use the properties of exponents to interpret expressions for exponential functions. <i>For example, identify percent rate of change in functions such as <math>y = (1.02)^t</math>, <math>y = (0.97)^t</math>, <math>y = (1.01)^{12t}</math>, and <math>y = (1.2)^{t/10}</math>, and classify them as representing exponential growth or decay.</i></p>	<p>Use the properties of exponents to interpret expressions for exponential functions. <i>For example, identify percent rate of change in functions such as <math>y = (1.02)^t</math>, <math>y = (0.97)^t</math>, <math>y = (1.01)^{12t}</math>, and <math>y = (1.2)^{t/10}</math>, and classify them as representing exponential growth or decay. <b>Apply to financial situations such as Identify appreciation/depreciation rate for the value of a house or car some time after its initial purchase. (<math>V_n = P(1+r)^n</math>)</b></i></p>	<p>Edited to provide an example that connects real world financial application with mathematical expressions.</p>
<p><b>F.IF.8.MA.c.</b> Translate among different representations of functions and relations: graphs, equations, point sets, and tables.</p>	<p><b>Standard deleted</b></p>	<p>Deleted due to redundancy with standard F.IF.9 (Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions).</p>
<p><b>F.IF.MA.10</b> Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, exponential, or trigonometric.</p>	<p><b>Standard Deleted</b></p>	<p>Deleted from Algebra I and moved to Algebra II. The student expectations for learning this content is not coherent with the focus in Algebra I (linear, exponential and quadratic functions)</p>
<p><b>F.BF.2</b> Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms</p>	<p><del>Write</del> Identify arithmetic and geometric sequences <b>written both</b> recursively and <b>with an explicit formula</b>, use them to model situations., <del>and translate between the two forms</del></p>	<p>Edited to address a footnote that clarifies the focus and expectations for Algebra I students and preserves a coherent progression of learning.</p>
<p><b>F.BF.3</b> Identify the effect on the graph of replacing <math>f(x)</math> by <math>f(x) + k</math>, <math>kf(x)</math>, <math>f(kx)</math>, and <math>f(x + k)</math> for specific values of <math>k</math> (both positive and negative); find the value of <math>k</math> given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. <i>Include recognizing even and odd functions from their graphs and algebraic expressions for them.</i></p>	<p>Identify the effect on the graph of replacing <math>f(x)</math> by <math>f(x) + k</math>, <math>kf(x)</math>, <math>f(kx)</math>, and <math>f(x + k)</math> for specific values of <math>k</math> (both positive and negative); find the value of <math>k</math> given the graphs. <b>Utilize technology to experiment with cases and illustrate an explanation of the effects on the graph. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.</b></p>	<p>Edited to clarify the focus and expectation of the standard and maintain the mathematical rigor for Algebra I students.</p>

<p><b>F.BF.4.a</b> Solve an equation of the form <math>f(x) = c</math> for a simple function <math>f</math> that has an inverse and write an expression for the inverse. For example, <math>f(x) = 2x^3</math> or <math>f(x) = (x + 1)/(x - 1)</math> for <math>x \neq 1</math>.</p>	<p>Find the inverse of linear functions function both graphically and algebraically. <del>Solve an equation of the form <math>f(x) = c</math> for a simple function <math>f</math> that has an inverse and write an expression for the inverse. For example, <math>f(x) = 2x^3</math> or <math>f(x) = (x + 1)/(x - 1)</math> for <math>x \neq 1</math>.</del></p>	<p>Edited to clarify the wording of the standard and to be consistent with the focus and expectations for building functions for Algebra I students.</p>
<p><b>F.LE.3</b> Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function</p>	<p>Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically, <del>or (more generally) as a polynomial function</del></p>	<p>Edited to clarify the focus and expectation of the standard for Algebra I students and maintain the mathematical rigor.</p>
<p><b>F.LE.5</b> Interpret the parameters in a linear or exponential function in terms of a context</p>	<p>Interpret the parameters in a linear or exponential function <del>(of the form <math>f(x) = b^x + k</math>)</del> in terms of a context</p>	<p>Edited to incorporate a footnote that clarifies the focus and expectations for Algebra I students.</p>
<p><b>S.ID.4</b> Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.</p>	<p>Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. <del>Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.</del></p>	<p>Edited to address a footnote that clarifies the expectations for Algebra I students while preserving a coherent progression of learning for interpreting data. Introduce in Algebra I; expand in Algebra II.</p>
<p><b>S.ID.6.a.</b> Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.</p>	<p>a. Fit a function to the data; use <b>linear</b> functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. <del>Emphasize linear, quadratic, and exponential models.</del></p>	<p>Edited to address a footnote that clarifies the expectations for Algebra I students while preserving a coherent progression of learning across model course for representing and describing relationships for quantitative data</p>
<p><b>S.ID.6.b</b> Informally assess the fit of a function by plotting and analyzing residuals.</p>	<p><b>Standard deleted</b></p>	<p>Deleted to address a footnote for preserving a coherent progression of learning across model course for representing and describing relationships for quantitative data.</p>

Model Geometry		
2011 Standard	Proposed 2016 Standard	Rationale for Revision
<p><b>G.CO.MA.11.a</b> Prove theorems about polygons. <i>Theorems include: measures of interior and exterior angles, properties of inscribed polygons.</i></p>	<p>Derive the formula for the relationship between the number of sides, sums of the interior and sums of the exterior angles of polygons, and apply to the solutions of mathematical and contextual problems. Prove theorems about polygons. <i>Theorems include: measures of interior and exterior angles, <del>properties of inscribed polygons.</del></i></p>	<p>Edited to provide focus and coherence within the Model Geometry course and preserve the mathematical rigor. The expectation for <i>proving properties of inscribed polygons</i> was incorporated into standard G.C.MA.3 .</p>
<p><b>G.C.MA.3.a</b> Derive the formula for the relationship between the number of sides and sums of the interior and sums of the exterior angles of polygons and apply to the solutions of mathematical and contextual problems.</p>	<p><del>Derive the formula for the relationship between the number of sides and sums of the interior and sums of the exterior angles of polygons and apply to the solutions of mathematical and contextual problems.</del> Prove properties of inscribed polygons.</p>	<p>Edited to provide focus and coherence within the Model Geometry course and preserve the mathematical rigor. The content in the original standard was incorporated into G.CO.MA.11.A where it is more coherently aligned (Proving theorems about polygons). The added content in this standard (prove properties of inscribed polygons) is more coherently aligned with the expectations of other standards in this cluster (<i>proving properties of angles of inscribed polygons</i>) .</p>
<p><b>G.GPE.4</b> Use coordinates to prove simple geometric theorems algebraically. <i>For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point <math>(1, \sqrt{3})</math> lies on the circle centered at the origin and containing the point <math>(0, 2)</math>.</i></p>	<p>Use coordinates to prove simple geometric theorems algebraically <b>including the distance formula and its relationship to the Pythagorean Theorem.</b> <i>For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point <math>(1, \sqrt{3})</math> lies on the circle centered at the origin and containing the point <math>(0, 2)</math>.</i></p>	<p>Edited to provide deeper focus of important and specific connections with other standards within the Model Geometry course and preserve mathematical rigor.</p>
<p><b>S.MD.7 (+)</b> Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game)</p>	<p>Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game <b>and replacing with an extra skater</b>).</p>	<p>Edited to incorporate an accompanying footnote into the original standard to clarify student learning expectations for using probability to make decisions</p>

Model Algebra II		
2011 Standard	Proposed 2016 Standard	Rationale for Revision
<b>A.SS.E.2</b> Use the structure of an expression to identify ways to rewrite it. <i>For example, see <math>x^4 - y^4</math> as <math>(x^2)^2 - (y^2)^2</math>, thus recognizing it as a difference of squares that can be factored as <math>(x^2 - y^2)(x^2 + y^2)</math>.</i>	Use the structure of an expression to identify ways to rewrite it. <i>For example, see <math>x^4 - y^4</math> as <math>(x^2)^2 - (y^2)^2</math>, thus recognizing it as a difference of squares that can be factored <math>(x^2 - y^2)(x^2 + y^2)</math> and further factored <math>(x - y)(x + y)(x - yi)(x + yi)</math>.</i>	Edited the example in the standard to increase coherence with other standards (N.CN.8) within the Model Algebra II course.
<b>A.APR.1</b> Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. <b>Compare the system of polynomials to the system of integers when adding subtracting, multiplying, and dividing</b>	Edited to clarify expectations for student learning and provide coherence between understanding systems and dividing polynomials.
<b>A.APR.MA.1.a</b> Divide polynomials.	Standard deleted	Deleted this standard in Model Algebra II as the expectation of the standard was included in standard A.APR.1 above.
<b>A.CED.4</b> Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. <i>For example, rearrange Ohm's law <math>V = IR</math> to highlight resistance <math>R</math>.</i>	Standard deleted	Deleted the standard in Model Algebra II as the expectations for solving equations are a focus in Model Algebra I. The standard is in the Model Algebra I course.
<b>F.IF.9</b> Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). <i>For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.</i>	Standard deleted and replaced with F.IF.MA.10	Deleted the standard in the Model Algebra II course. . The expectations for student learning in this standard are a focus of the Algebra II model course. The standard is in the Model Algebra I course
	<b>F.IF.MA.10</b> Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, exponential, or trigonometric.	Added to Algebra II (deleted from Algebra I). The expectations for student learning in this standard are a focus of the Algebra II model course.

<p><b>F.BF.3</b> Identify the effect on the graph of replacing <math>f(x)</math> by <math>f(x) + k</math>, <math>kf(x)</math>, <math>f(kx)</math>, and <math>f(x + k)</math> for specific values of <math>k</math> (both positive and negative); find the value of <math>k</math> given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. <i>Include recognizing even and odd functions from their graphs and algebraic expressions for them.</i></p>	<p>Identify the effect on the graph of replacing <math>f(x)</math> by <math>f(x) + k</math>, <math>kf(x)</math>, <math>f(kx)</math>, and <math>f(x + k)</math> for specific values of <math>k</math> (both positive and negative); find the value of <math>k</math> given the graphs. <b>Utilize technology to experiment with cases and illustrate an explanation of the effects on the graph. Experiment with cases and illustrate an explanation of the effects on the graph using technology.</b> <i>Include recognizing even and odd functions from their graphs and algebraic expressions for them.</i></p>	<p>Edited to clarify the focus and expectation of the standard and maintain the mathematical rigor for Algebra II students.</p>
<p><b>S.MD.7 (+)</b> Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game)</p>	<p>Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game <b>and replacing with an extra skater</b>).</p>	<p>Edited to incorporate an accompanying footnote into the original standard to clarify student learning expectations for using probability to make decisions.</p>

## EDITS adding examples

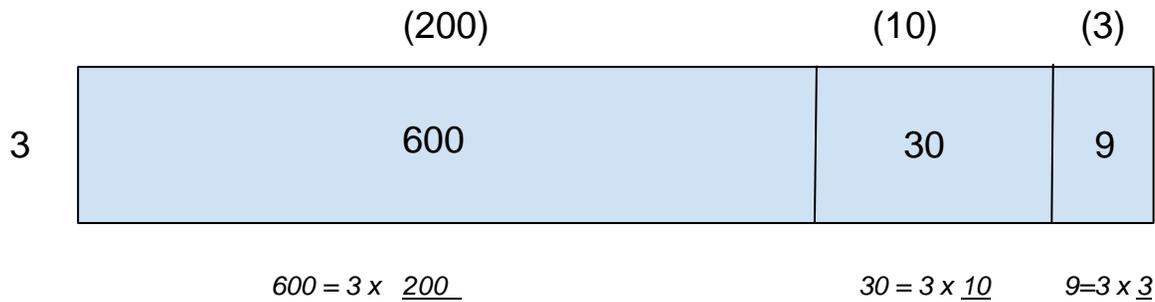
### 4.NBT.6 Example

**Example:**  $639 \div 3$  can be solved with the partial quotients strategy:

$600 \div 3 = 200$ ;  $30 \div 3 = 10$ ;  $9 \div 3 = 3$ ; So,  $639 \div 3 = 200 + 10 + 3 = 213$ .

An illustration below explains the calculations:

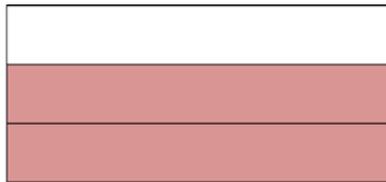
Area Model of using the relationship between multiplication and division with partial quotients strategy



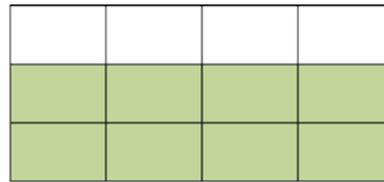
### 4.NF.1 Example

$$\frac{2}{3} = \frac{(4 \times 2)}{(4 \times 3)} = \frac{8}{12}$$

VISUAL FRACTION MODEL



$\frac{2}{3}$  of the whole region is shaded



$\frac{8}{12}$  of the whole region is shaded  
 $\frac{(4 \times 2)}{(4 \times 3)} = \frac{8}{12}$