**Grade-by-Grade Proposed Revisions Pre-Kindergarten to Grade 12**

**Massachusetts Curriculum Framework for Mathematics**

**November 29, 2016**

Contents

[Pre-Kindergarten and Kindergarten 2](#_Toc466894948)

[Grade I 3](#_Toc466894949)

[Grade 2 5](#_Toc466894950)

[Grade 3 7](#_Toc466894951)

[Grade 4 10](#_Toc466894952)

[Grade 5 14](#_Toc466894953)

[Grade 6 17](#_Toc466894954)

[Grade 7 20](#_Toc466894955)

[Grade 8 24](#_Toc466894956)

[High School Model Algebra I: 25](#_Toc466894957)

[High School Model Geometry 32](#_Toc466894958)

[High School Model Algebra II 34](#_Toc466894959)

[High School Model Integrated Math I 37](#_Toc466894960)

[High School Model Integrated Math II 45](#_Toc466894961)

[High School Model Integrated Math III 52](#_Toc466894962)

[Summary of Proposed Changes to Other Sections of the Framework 55](#_Toc466894963)

**Note:** The tables in this document show only standards that have been changed or moved in the current draft Framework. Further, if a specific change was made to multiple standards at the same grade level (e.g., if all instances of a certain phrase were deleted from that grade’s standards), only one instance of the change is listed for that grade. Please refer to the November 29 draft of the Framework, in which all changes to the standards are tracked, for complete details.

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| Pre-Kindergarten and Kindergarten |
| **2011 Standard** | **Proposed 2016 Standard****Revisions are in red text** | **Rationale for Revision** |
| **PK.CC.4** Count many kinds of concrete objects and actions up to ten, using one-to-one correspondence, and accurately count as many asseven things in a scattered configuration. | Count many kinds of concrete objects and actions upto ten, recognizing the “one more”, “one less” patterns, using one-to-one correspondence, and accurately count as many as seven things in a scattered configuration. | Edited to highlight that recognizing patterns in numbers is key to mathematics and fundamental for algebraic thinking.  |
| **K.CC.1** Count to 100 by ones and by tens. | Count to 100 by ones and by tens. Recognize the “onemore” and “ten more” pattern of counting. | Edited to highlight that recognizing patterns in numbers is key to mathematics and fundamental for algebraic thinking.  |
| **K.CC.6** Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. | Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group for groups with up to ten objects, e.g., by using matching and counting strategies. | Edited to incorporate an accompanying footnote into the original standard to clarify student learning expectations for comparing number of objects in kindergarten. |
| **K.OA.5** Fluently add and subtract within 5. | Fluently add and subtract within 5 including zero. | Edited to include zero as a student learning expectations for fluency in kindergarten. |
| **K.MD.3** Classify objects into given categories; count the numbers of objects in each category and sort the categories by count. | Classify objects into given categories; count the numbers of objects in each category and sort the categories by count for category counts up to and including 10. | Edited to incorporate an accompanying footnote into the original standard to clarify student learning expectations for sorting by count in kindergarten. |

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| Grade I |
| **2011 Standard** | **Proposed 2016 Standard****Revisions are in red text** | **Rationale for revision** |
| **1.OA.1** Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem | Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations (number sentences) with a symbol for the unknown number to represent the problem | Edited to include number sentences.  |
| **1.OA.3** Apply properties of operations as strategies to add and subtract. *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known.* *(Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)* | Apply properties of operations as strategies to add ~~and subtract~~. *Examples: When adding numbers order does not matter. If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known.* *(Commutative property of addition). To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12.(Associative property of addition.) When adding zero to a number, the result is the same number. (Identity property of zero for addition.)* | Edited to include an explanation of the properties of operations to clarify expectation of the standard. Students are not required to use the formal terms of these properties in grade 1 but they are required to apply them when adding. The properties of operations do not apply to subtraction of whole numbers. The identity property of zero is added to formalize understanding about adding zero to a number.  |
| **1.OA.MA.9** Write and solve number sentences from problem situations that express relationships involving addition and subtraction within 20. | Standard moved. |  This standard has the same student learning expectations with the revised standard 1.OA.1 (See above).  |
| **1.NBT.5**  Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.  | Identify arithmetic patterns using 10 more or 10 less using concrete models or drawings and strategies based on place value. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.  | Added the expectation to identify number patterns using strategies to enhance the development of number sense in grade 1 students as they look for and make use of mathematical structures. |

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| **1.MD.MA.5** Identify the values of all U.S. coins and know their comparative values (e.g., a dime is of greater value than a nickel). Find equivalent values (e.g., a nickel is equivalent to 5 pennies). Use appropriate notation (e.g., 69¢). Use the values of coins in the solutions of problems. | Identify the values of all U.S. coins and know their comparative values (e.g., a dime is of greater value than a nickel). Find equivalent values (e.g., a nickel is equivalent to 5 pennies). Use appropriate notation (e.g., 69¢). Use the values of coins in the solutions of problems (up to 100¢). | Edited to provide parameters that clarify the student learning expectation of the standard. |

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| Grade 2 |
| **2011 Standard** | **Proposed 2016 Standard****Revisions are in red text** | **Rationale for revision** |
| **2.OA.2** Fluently add and subtract within 20 using mental strategies. By end of grade 2, know from memory all sums of two one-digit numbers.  | Fluently add and subtract within 20 using mental strategies. By end of grade 2, know from memory all sums and related differences of two ~~one~~ single-digit numbers. | Edits made to clarify wording of the standard and to incorporate the learning expectation of deleted standard 2.OA.2.MA.2.a. (See below.) |
| **2.OA.MA.2.a**.By the end of grade 2, know from memory related subtraction facts of sums of two one-digit numbers. | Standard moved. | This standard has the same student learning expectations with the revised standard 2.OA.2 (See above).  |
| **2.OA.3**  Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.  | Identify patterns in odd and even numbers using concrete models or drawings. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.  | Added patterns and the use of strategies to enhance conceptual development of odd and even numbers in grade 2 students as they look for and make use of mathematical structures. |
| **2.NBT.2**  Count within 1000; skip-count by 5s, 10s, and 100s. | Identify patterns in skip counting. Count within 1000; skip-count by 5s, 10s, and 100s. | Edited to include patterns to enhance the conceptual development of numbers in grade 2 students as they look for and make use of mathematical structures. |
| **2.MD.8**  Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?*  | Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies (up to $10.00), using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?*  | Edited to provide parameters that clarify the student learning expectation of the standard. |
| **2.MD.9**  Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units. | Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Organize and record data. Show the measurements by making a line plot (dot plot) where the horizontal scale is marked off in whole-number units. | Edited to include the words dot plot because it is a more commonly used term and to help clarify the standard about the expectation for using this method for representing data and to maintain the mathematical rigor. Dot Plot added to glossary. |

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| Grade 3 |
| **2011 Standard** | **Proposed 2016 Standard****Revisions are in red text** | **Rationale for revision** |
| **3.OA.5**  Apply properties of operations as strategies to multiply and divide. *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15 then 15 × 2 = 30, or by 5 × 2 = 10 then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property).* | Apply properties of operations as strategies to multiply. ~~and divide.~~ *Examples:* When multiplying two numbers order does not matter*. If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication)*. *3 × 5 × 2 can be found by 3 × 5 = 15 then 15 × 2 = 30, or by 5 × 2 = 10 then 3 × 10 = 30. (Associative property of multiplication.)* When multiplying two numbers one number can be decomposed and multiplied*. Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)* When a number is multiplied by 1 the result is the same number. (Identity Property of 1 for multiplication.) | Edited to clarify the properties of operations for multiplication to retain the mathematical rigor of the standard. Students are not required to use the formal terms of these properties in grade 3 but they are required to apply them when multiplying. The identity property for multiplication is added to formalize understanding about multiplying by the number 1.  |
| **3.OA.7**  Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of grade 3, know from memory all products of two one-digit numbers**.** | Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of grade 3, know from memory all products and related divisions of two ~~one~~ single-digit numbers.  | Edits made to clarify wording of the standard and increase student understanding of the relationship between multiplication and division for single digit numbers. |
| **3.OA.8** Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies, including rounding. | Solve two-step word problems using the four operations for problems posed with whole numbers and having whole number answers. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies, including rounding.(Students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order.) | Edited to incorporate accompanying footnotes into the standard itself to clarify the focus in this grade and student expectations. |
| **3.NBT.2** Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. | Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. A range of algorithms may be used. | Edited to incorporate an accompanying footnote into the standard itself to help clarify student expectations. |
| **3.NF Develop understanding of fractions as numbers.** | Develop understanding of fractions as numbers for fractions with denominators 2, 3, 4, 6, and 8. | Edited to incorporate a footnote to clarify the expectation of the standards in this cluster. |
| **3.NF.1**  Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*. | Understand a fraction **1**/***b*** as the quantity formed by 1 part when a whole (a single unit) is partitioned into *b* equal parts; understand a fraction ***a***/***b*** as the quantity formed by *a* parts of size **1**/***b***. | Edit made to clarify the term whole as it relates to fraction expectations in grade 3. |
| **3.NF.2a**  Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line. | a. Represent the unit fraction **1**/***b,*** on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size **1**/***b*** and that unit fraction **1**/***b*** is located **1**/***b*** of a whole unit from 0 ~~the endpoint of the part based at 0 locates the number 1/~~*~~b~~* on the number line. | Edit made to clarify the concept of a unit fraction and its location on a number line.  |
| **3.MD.2** Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. | Measure and estimate liquid volumes and masses of objects using standard metric units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same metric units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. | Edited to clarify the focus of this standard on the metric system.  |
| **3.MD.4** Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters. | Generate and record measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot (dot plot), where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters (for example, measure the length of pencils students are using to the nearest whole, 1/2, and/or ¼; record and display the data). | Edited to clarify student expectations for recording data and for representing data . A context example was provided to maintain the mathematical rigor of the standard. Dot plot added to glossary. |
| **3. MD .6** Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units.) | Measure areas by counting unit squares (square cm, square m, square in, square ft, and non-standard units).  | Edited to clarify the term ‘improvised’ units. |
| **3.MD.7.a**Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths**.** | Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. For example: A 5 meters by 2meter rectangular panel has an area of 10 square meters . | The example was added to focus on the proper use of ‘square unit(s)’ when calculating area. |
| **3.G.1** Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. | Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Compare and classify shapes by their sides and angles (right angle/non-right angle).Recognize rhombuses, rectangles, squares, and trapezoids as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. | Edited to clarify student expectations for comparing and classifying shapes and to clarify a focus on right and non-right angles.  |

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| Grade 4 |
| **2011 Standard** | **Proposed 2016 Standard****Revisions are in red text** | **Rationale for revision** |
| **4.NBT Generalize place value understanding for multi-digit whole numbers.** | Generalize place value understanding for multi-digit whole numbers less than or equal to 1,000,000. | Edited to incorporate a footnote to clarify the expectation of the standards in this cluster. |
| **4.NBT.1** Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.* | Recognize that in a multi-digit whole number, a digit in ~~one~~ any place represents 10 times as much as it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.* | Edited to improve the wording of the standard and understanding of place values. |
| **4.NBT.2** Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. | Read and write multi-digit whole numbers less than or equal to 1,000,000, using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers within 1,000,000 based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. | Edited to provide parameters that clarify the student learning expectation of the standard. |
| **4.NBT.3** Use place value understanding to round multi-digit whole numbers to any place. | Use place value understanding to round multi-digit whole numbers less than or equal to 1,000,000., to any place. | Edited to provide parameters that clarify the student learning expectation of the standard. |
| **4.NBT Use place value understanding and properties of operations to perform multi-digit arithmetic.** | Use place value understanding and properties of operations to perform multi-digit arithmetic of whole numbers less than or equal to 1,000,000. | Edited to incorporate a footnote to clarify the expectation of the standards in this cluster. |
| **4.NF Extend understanding of fraction equivalence and ordering.** | Extend understanding of fraction equivalence and ordering for fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100. | Edited to incorporate a footnote to clarify the expectation of the standards in this cluster. |
| **4.NF.1**  Explain why a fraction *a*/*b* is equivalent to a fraction (*n*×*a*)/(*n*×*b*) by using visual fraction models, with attention to how the numbers and sizes of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. | Explain why a fraction ***a***/***b*** is equivalent to a fraction **(*n* × *a*)**/**(*n* × *b*)** by using visual fraction models, with attention to how the numbers and sizes of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions, including fractions greater than 1. | Edited to clarify the expectation of the standard for Grade 4 includes generating equivalent fractions for fractions greater than 1.  |
| **4.NF Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.** | Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers for fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100. | Edited to incorporate a footnote to clarify the expectation of the standards in this cluster. |
| **4.NF.3.a** Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. | Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. (The whole can be a single unit or a set of objects). | Edited to clarify the meaning of the term ‘whole’ in Grade 4 to address the development of student understanding of the concept of ‘whole’ as it progresses through Grades 3-5.  |
| **4.NF.3.b** Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ;*  *2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.* | Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using drawings or visual fraction models. *Examples:* ***3****/****8****=****1****/****8****+****1****/****8****+****1****/****8*** *;* ***3****/****8****=****1****/****8****+****2****/****8*** *;*  *2* ***1****/****8****= 1 + 1 +****1****/****8****=****8****/****8****+****8****/****8*** *+****1****/****8****.* | Edited to add coherence to 4.NF.3.a. When ‘wholes’ are a set of objects, drawings are an appropriate model. |
| **4.NF.3d**  **Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.**  | Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using drawings or visual fraction models and equations to represent the problem.  | Edited to add drawings as an appropriate model to represent problems involving fractions. |
| **4.NF Understand decimal notation for fractions, and compare decimal fractions.** | Understand decimal notation for fractions, and compare decimal fractions for fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100. | Edited to incorporate a footnote to clarify the expectation of the standards in this cluster. |
| **4.NF.6**  Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as* ***62****/****100*** *; describe a length as 0.62 meters; locate 0.62 on a number line diagram.* | Use decimal notation ~~for~~ to represent fractions with denominators 10 or 100. *For example, rewrite 0.62 as* ***62****/****100*** *; describe a length as 0.62 meters; locate 0.62 on a number line diagram.* | Edited the wording to clarify the expectation of the standard. |

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| **4.MD.4** Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a*  line plot *find and interpret the difference in length between the longest and shortest specimens in an insect collection.*  | Make a  *line plot (dot plot)* representation to display a data set of measurements in fractions of a unit (**1**/**2**, **1**/**4**, **1**/**8**). Solve problems involving addition and subtraction of fractions by using information presented in line plots (dot plots). *For example, from a line plot (dot plot) find and interpret the difference in length between the longest and shortest specimens in an insect collection.*  | Edited to include the words dot plot because it is a more commonly used term and to help clarify the standard about the expectation for sing this method for representing data and to maintain the mathematical rigor. Dot plot added to glossary. |

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| Grade 5 |
| **2011 Standard** | **Proposed 2016 Standard****Revisions are in red text** | **Rationale for Revision** |
| **5.OA.1**  Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. | Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. (e.g. (6 x 30) + (6 x 1/2)). | Edited to include an example to clarify the expectation in the standard for grade 5. |
| **5.NBT.1**  Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. | Recognize that in a multi-digit number, including decimals, a digit in any ~~one~~ place represents 10 times as much as it represents in the place to its right and **1**/**10** of what it represents in the place to its left.  | Edited to include the term decimal. |
| **5.NBT.5**  Fluently multiply multi-digit whole numbers using the standard algorithm  | Fluently multiply multi-digit whole numbers (numbers with at least 2 digits x 4 digits, 3 digits x 3 digits) using the standard algorithm.  | Edited to provide parameters that clarify the student learning expectation of the standard. |
| **5.NBT.7** Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. | Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction (and between multiplication and division); relate the strategy to a written method and explain the reasoning used | Edited to retain focus in grade 5 on developing fluency in multi-digit multiplication and division.  |
| **5.NF.2**  Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem.Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example,* *recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2* ***.***  | Solve word problems involving addition and subtraction of fractions referring to the same whole (the whole can be a single unit or a set of objects), including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem.Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example,* *recognize an incorrect result* ***2****/****5*** *+* ***1****/****2*** *=* ***3****/****7****, by observing that* ***3****/****7*** *<* ***1****/****2*** *.*  | Edited to clarify the concept of whole as being a single unit or a set of objects. |
| **5.NF.4**  Interpret the product (***a***/***b***) × *q* as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a* × *q* ÷ *b*. *For example, use a visual fraction model to show (****2****/****3****) × 4 =****8****/****3****, and create a story context for this equation. Do the same with* *(****2****/****3****) × (****4****/****5****) =* ***8****/****15*** *.* (In general, (***a***/***b***) × (***c***/***d***) = ***ac***/***bd***.)  | Interpret the product (***a***/***b***) × *q* as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a* × *q* ÷ *b*. *For example, use a visual fraction model and/or area model to show (****2****/****3****) × 4 =****8****/****3****, and create a story context for this equation. Do the same with* *(****2****/****3****) × (****4****/****5****) =* ***8****/****15*** *.* (In general, (***a***/***b***) × (***c***/***d***) = ***ac***/***bd***.)  | Edited to maintain mathematical rigor by provide options for ways of showing conceptual understanding when multiplying fractions by whole numbers  |
| **5.NF.5.a**  Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. | Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. *For example: 6* ×¾ *is twice as large as 3* × *¾*.  | Edited to provide an example to clarify the standard and to maintain the focus of the standard. |
| **5.NS.MA.1**  Use positive and negative integers to describe quantities such as temperature above/below zero, elevation above/below sea level, or credit/debit. | Standard deleted. | Deleted to maintain focus in Grade 5 on fluency in multi-digit whole numbers and multiplying fractions. Integers are introduced in grade 6. |
| **5.MD.2**  Make a line plot to display a data set of measurements in fractions of a unit. Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.* | Make a  *line plot (dot plot)* to display a data set of measurements in fractions of a unit. Use operations on fractions for this grade to solve problems involving information presented in  *line plot (dot plot) For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.* | Edited to include the words dot plot because it is a more commonly used term and to help clarify the standard about the expectation for sing this method for representing data and to maintain the mathematical rigor. Dot plot added to glossary. |
| **5.MD.4**  Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and non-standard units. | Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and ~~improvised~~ non-standard units. | Edited since ‘non-standard’ is the more frequently used term in measurement. |

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| **5.MD.5.a**  Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. | Find the volume of a right rectangular prism with whole-number ~~side~~ edge lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. | Edited to clarify the focus of this standard to use edge length of a three dimensional prism when finding volume. |
| **5.MD.5.b**  Apply the formula *V* = *l* × *w* × *h* and *V* = *b* × *h* for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems.  | Apply the formula *V* = *l* × *w* × *h* and *V* = ~~b~~ *B* × *h* (where B stands for the area of the base) for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems.  | Edited to clarify the meaning of symbols used in formulas for volumes. |
| **5.G.4**  Classify two-dimensional figures in a hierarchy based on properties. | Classify two-dimensional figures in a hierarchy based on properties. For example, all rectangles are parallelograms, because they are all quadrilaterals with two pairs of opposite, parallel, equal-length sides. | Edited to include an example to clarify the expectation for student learning and to retain the mathematical rigor. |

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| Grade 6 |
| **2011 Standard** | **Proposed 2016 Standard****Revisions are in red text** | **Rationale for Revision** |
| **6.RP Understand ratio concepts and use ratio reasoning to solve problems.** | Understand ratio and rate concepts and use ratio and rate reasoning to solve problems. | Edited to emphasize rate in the ratio and proportional reasoning cluster since it is a critical area at this grade level. |
| **6.RP.1**  Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For e*xample, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”*  | Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities, including the distinction between part : part and part/whole. For e*xample, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.” “The ratio of males to females is 2:3, meaning that 3/5 of the group is female.”* | Edited to add coherence for developing student understanding of ratio relationships and to clarify the expectation for student learning is and to retain the mathematical rigor.  |
| **6.RP.2**  Understand the concept of a unit rate ***a***/***b*** associated with a ratio *a*:*b* with *b* ≠ 0, and use rate language in the context of a ratio relationship. *For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is ¾ cup of flour for each cup of sugar.” “We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger.”*  | Understand the concept of a unit rate ***a***/***b*** associated with a ratio *a*:*b* with *b* ≠ 0, and use rate language in the context of a ratio relationship, *including the use of units*. *For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is ¾ cup of flour for each cup of sugar.” “We paid $75 for 15 hamburgers, which is a rate of ~~$~~5 dollars per hamburger.”*  | Edited to clarify the focus of this standard on developing understanding of unit rates from concepts of ratios. |
| **6.RP.3.d**  Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. | Use ratio reasoning to convert measurement units within and between measurement systems; manipulate and transform units appropriately when multiplying or dividing quantities. Ex. Solve problems that relate the mass of an object to its volume. | Edited to clarify the expectation of conversion between and within measurement system and also to incorporate standard 6.RP.MA.3.e as an example of the type of problems to be solved. |
| **6.RP.MA.3.e**  Solve problems that relate the mass of an object to its volume. |  ~~Solve problems that relate the mass of an object to its volume.~~ | The standard moved and incorporated into 6.RP.3.d.  |
| **6.NS.5** Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge). Use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. | Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge). Use positive and negative integers and other rational numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. | Edited to clarify student expectations for knowing that positive and negative whole numbers including zero comprise a set of numbers called the set of integers and that integers are part of the system of rational numbers.  |
| **6.G.MA.1.a**  Use the relationships among radius, diameter, and center of a circle to find its circumference and area. | Deleted in Grade 6. | Edited the standard and moved to grade 7 in the cluster of standards that develop measurement concepts of circles. |
| **6.G.MA.1.b**  Solve real-world and mathematical problems involving the measurements of circles. | Deleted in Grade 6. | Edited the standard and moved to grade 7 in the cluster of standards that develop measurement concepts of circles. |
| **6.SP.2** Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. | Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center (median, mean, and/or mode), spread (range interquartile range), and overall shape. | Edited to clarify the expectation for student learning about statistical measures in this standard. |
| **6.SP.5.c**  Summarize numerical data sets in relation to their context, such as by: c. Giving quantitative measures of center (median, and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. | Summarize numerical data sets in relation to their context, such as by:c. Giving quantitative measures of center (median, and/or mean), and variability(range and/or interquartile range ~~and/or mean absolute deviation~~), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. | Edited to create a coherent learning progression from grade 6 to grade 7 in relation to summarizing and describing distributions. In grade 6 the focus is on developing understanding of measures of center and spread. In grade 7 students are introduced to the mean absolute deviation as a measure of variability. |

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| Grade 7 |
| **2011 Standard** | **Proposed 2016 Standard****Revisions are in red text** | **Rationale for Revision** |
| **7.RP.3** Use proportional relationships to solve multi-step ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.* | Use proportional relationships to solve multi-step ratio, rate, and percent problems. *Examples: simple interest, tax, markups and ~~markdowns~~ discounts, gratuities and commissions, fees, percent increase and decrease, percent error.* | Edited to maintain coherence and focus in analyzing proportional relationships and ratio/rate learning progression. |
| **7.NS.1.a** Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.* | Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged; if you open a new bank account with a deposit of $30 and then withdraw $30, you are left with a $0 balance.* | Edited to provide an example connecting a financial real world context to the standard and providing for improved conceptual understanding.  |
| **7.NS.1c**  Understand subtraction of rational numbers as adding the additive inverse, *p* – *q* = *p* + (–*q*). Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. | Understand subtraction of rational numbers as adding the additive inverse, *p* – *q* = *p* + (–*q*). Show that the distance between two integers and other rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. | Edited to clarify the different types of rational numbers addressed in this standard. |
| **7.NS.2a**  Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. | Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of integers and other rational numbers by describing real-world contexts. | Edited to clarify the different types of rational numbers addressed in this standard. |
| **7.NS.3** Solve real-world and mathematical problems involving the four operations with rational numbers. | Solve real-world and mathematical problems involving the four operations with integers and other rational numbers. | Edited to clarify the distinction about different sets of numbers in the set of rational numbers that are an expectation in this standard. |
| **7.EE.1**  Apply properties of operations as strategies to add, subtract, factor, and expanding linear expressions with rational coefficients.  | Apply properties of operations as strategies to add, subtract, factor and expand linear expressions with rational coefficients. *For example, 4x + 2 = 2(2x +1) and -3(x – 5/3) = -3x + 5.* | Edited to clarify the expectation for student learning of the focus (applying the properties) and the mathematical rigor. |
| **7.EE.2**  Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that “increase by 5%” is the same as “multiply by 1.05.”* | Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that “increase by 5%” is the same as “multiply by 1.05.” A shirt at a clothing store is on sale for 20% off the regular price, p”. The discount can be expressed as 0.2p. The new price for the shirt can be expressed as p – 0.2p or 0.8p.*  | Edited to give a context example to clarify the expectation of the standard and mathematical rigor of the standard. |
| **7.G.2** Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. | Draw (freehand, with ruler and protractor, and with technology) two-dimensional geometric shapes with given conditions. Focus on ~~constructing~~ drawing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. | Edited to clarify the expectation for student learning and the focus (draw geometric shapes). |
| **7.G.3**  Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.  | Describe the shape of the two-dimensional face of the figures that result from slicing three-dimensional figures such as, ~~in plane sections of~~ right rectangular prisms and right rectangular pyramids. | Edited to improve the clarity of the standard.  |

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| **7.G.4** Know the formulas for the area and circumference of a circle and solve problems; give an informal derivation of the relationship between the circumference and area of a circle. | Circles and measurement:1. Know that a circle is created by connecting all of the points equidistant from a fixed point called the center of the circle.
2. Explore and understand the relationships among the radius, diameter, circumference of a circle.
3. Explore and understand the relationship among the radius, diameter, and area of a circle.
4. Know the formulas for the area and circumference of a circle and solve problems.
5. Give an informal derivation of the relationship between the circumference and area of a circle.
 | Edited to improve the coherence of the learning progression and expectations for students to know, calculate, and solve problems related to circumferences and areas of circles. |
| **7.G.5**  Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and use them to solve simple equations for an unknown angle in a figure.  | Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write simple equations and use them to solve ~~simple equations~~ for an unknown angle in a figure.   | Edited to improve the clarity of the standard. |
| **7.G.MA.7**  Solve real-world and mathematical problems involving the surface area of spheres. | Standard deleted. | Deleted to maintain the focus in grade 7 on finding the surface area of objects that can be decomposed into two-and-three dimensional polygonal shapes.  |
| **7.SP.3**  Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.* | Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team. The variability (mean absolute deviation) for each team is about twice the value of their team’s mean; on a dot plot, the separation between the two distributions of heights ~~is~~ would be noticeable.*  | Edited to improve the coherence of the expectation of the standard and clarity. The wording of the example was edited to reinforce the intent of drawing informal comparative inferences about two populations. |

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| Grade 8 |
| **2011 Standard** | **Proposed 2016 Standard****Revisions are in red text** | **Rationale for Revision** |
| **8.EE.8.b**  Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6.* | Solve systems of two linear equations in two variables algebraically (using substitution and elimination strategies), and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6.* | Edited to clarify student expectations for solving systems of equations in Grade 8 and maintain the mathematical rigor of the standard. |
| **8.G.1.a.b.c**  Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments  to line segments of the same length.b. Angles are taken to angles of the same  measure.c. Parallel lines are taken to parallel lines.  | Verify experimentally the properties of rotations, reflections, and translations: a. Lines are ~~taken~~ transformed to lines, and line  segments to line segments of the same length.b. Angles are ~~taken~~ transformed to angles of the same  measure.c. Parallel lines are ~~taken~~ transformed to parallel lines.  | Edited the wording to clarify the focus and expectation of the standard. |
| **8.G.6**  Explain a proof of the Pythagorean Theorem and its converse.  | a. Explore the relationships among the sides of a right triangle. b. Analyze and justify the Pythagorean Theorem and its converse using pictures, diagrams, narratives or models. | Edited to clarify student expectations for understanding and explaining the Pythagorean Theorem in Grade 8 and to maintain the mathematical rigor of the standard. Formal proofs of theorems are introduced in high school mathematics. In grade 8 students will demonstrate their understanding of the Pythagorean Theorem through analyzing and justifying the relation among the sides of a right triangle.  |

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| **High School Model Course Standards: Several standards are repeated in two or more model courses within a pathway (Traditional and Integrated). Footnotes in the 2011 Framework clarified aspects of the duplicated standard relevant to each course. In the 2017 Framework, most model course footnotes have been removed and incorporated into the standard itself**. |
| High School Model Algebra I: |
| **2011 Standard** | **Proposed 2016 Standard****Revisions are in red text** | **Rationale for Revision** |
| **A.SSE Interpret the structure of expressions.** | Interpret the structure of expressions (expressions include linear, quadratic, and exponential with integer exponents). | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this cluster for Algebra I. |
| **A.SSE.1.b** Interpret complicated expressions by viewing one or more of their parts as a single entity*. For example, interpret P(1 + r)t as the product of P and a factor not depending on P.*  | Interpret complicated expressions by viewing one or more of their parts as a single entity*. For example, interpret P(1 + r)t as the product of P and a factor not depending on P, given that P is the principal amount of money that is growing at a rate, r, over a period of time, t, in years­.* | Edited to provide a context for the expression used in the example. |
| **A.SSE.2** Use the structure of an expression to identify ways to rewrite it. *For example, see x4 – y4 as (x2)2 – (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 – y2)(x2+ y2).* | Use the structure of an expression to identify ways to rewrite it. *For example, see ~~x~~~~4~~ ~~– y~~~~4~~ ~~as (x~~~~2~~~~)~~~~2~~~~– (y~~~~2~~~~)~~~~2~~ ~~thus recognizing it~~* *(x + a)2 – b2 as a difference of squares that can be factored as (x + a + b)(x + a – b). ~~(x~~~~2~~~~– y~~~~2~~~~)(x~~~~2~~~~+ y~~~~2~~~~).~~* | Edited the example in the standard to align with the focus and expectations for Algebra I. |
| **A.SSE Write expressions in equivalent forms to solve problems.** | Write expressions in equivalent forms to solve problems (using linear, quadratic, and exponential expressions). | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this cluster for Algebra I. |

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| **A.APR.1**  Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. | ~~Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials~~ Add, subtract, and multiply polynomial expressions, including factoring and/or expanding polynomial expressions, identifying and combining like terms, and applying the Distributive Property. | Edited to incorporate a footnote that clarifies the focus and expectation of this standard for Algebra I. |
| **A.CED Create equations that describe numbers or relationships.**  | Create equations and inequalities that describe numbers or relationships (using linear, quadratic, and exponential equations with integer exponents). | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this cluster for Algebra I. |
| **A.CED.1**  Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. | Create equations and inequalities in one variable to represent a given context and use them to solve problems. Include equations arising from linear, quadratic, and ~~simple rational~~ exponential functions. | Edited to clarify the focus and to incorporate a footnote that clarifies the expectation of this standard for Algebra I. |
| **A.CED.3**  Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* | Represent constraints by linear equations or inequalities, and by systems of linear equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* | Edited to incorporate a footnote that clarifies the focus and expectation of this standard for Algebra I. |
| **A.CED.4**  Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm’s lawV = IR to highlight resistance R.* | Rearrange formulas to highlight a quantity of interest, using ~~the same reasoning as in solving equations~~ properties of equality. *For example, rearrange Ohm’s lawV = IR ~~to highlight resistance R~~  to solve for the variable R. Manipulate variables in formulas used in financial contexts, such as for simple interest ().* | Edited to add a financial application to the standard and to clarify the expectations of this standard for Algebra I. |

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| **A.REI.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. | Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify or refute a solution method. | Edited to be coherent with Standard for Mathematical Practice 3; *Construct viable arguments and critique the reasoning of others.*  |
| **A.REI.4.b**  Solve quadratic equations by inspection (e.g., for *x*2 = 49), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as *a* ± *bi* for real numbers *a* and *b*.  | Solve quadratic equations by inspection (e.g., for *x*2 = 49), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the ~~quadratic formula gives complex solutions and write them as~~ *~~a~~* ~~±~~ *~~bi~~* ~~for real numbers~~ *~~a~~* ~~and~~ *~~b~~*~~.~~  solution of a quadratic equation results in non-real solutions. | Edited to incorporate a footnote that clarifies the focus and expectation of this standard for Algebra I. |
| **A.REI.MA.4.c**  Demonstrate an understanding of the equivalence of factoring, completing the square, or using the quadratic formula to solve quadratic equations. | Standard moved. | This standard has the same student learning expectations with the revised standard A.REI.4.b (See above).  |
| **A.REI.7**  Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line y = –3x and the circle x2 + y2 = 3.* | Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line y = –3x and the ~~circle x~~~~2~~ ~~+ y~~~~2~~ ~~= 3~~ parabola y = x2 + x.* | Edited to incorporate a footnote that clarifies the focus and expectation of this standard for Algebra I. |
| **A.REI.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).  | Understand that the graph of an equation in two variables (equations include linear, absolute value, exponential) is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Show that any point on the graph of an equation in two variables is a solution to the equation. | Edited to incorporate a footnote that clarifies the focus and expectation of this standard for Algebra I and to improve rigor. |

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| **A.REI.11** Explain why the *x*-coordinates of the points where the graphs of the equations *y* = *f*(*x*) and *y* = *g*(*x*) intersect are the solutions of the equation *f*(*x*) = *g*(*x*); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f*(*x*) and/or *g*(*x*) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. | Explain why the *x*-coordinates of the points where the graphs of the equations *y* = *f*(*x*) and *y* = *g*(*x*) intersect are the solutions of the equation *f*(*x*) = *g*(*x*); find the solutions approximately, e.g., using technology to graph the functions and make tables of values. ~~or find successive approximations. Include cases where~~ *~~f~~*~~(~~*~~x~~*~~) and/or~~ *~~g~~*~~(~~*~~x~~*~~) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.~~ | Edited to incorporate a footnote that clarifies the focus and expectation of this standard for Algebra I. |
| **A.REI.12** Graph the solutions to a linear inequality in two variables as a half-plane, and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. | Graph the solutions ~~to~~ of a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set ~~to~~ of a system of linear inequalities in two variables as the intersection of the corresponding half-planes. | Edited to clarify wording of the standard to avoid misconceptions. |
| **F.IF Understand the concept of a function and use function notation.** | Understand the concept of a function ( linear or exponential with integer exponents) and use function notation. | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this cluster for Algebra I. |
| **F.IF Interpret functionsthat arise in applications in terms of the context.** | Interpret functionsthat arise in applications in terms of the context (include linear, quadratic, and exponential functions with integer exponents). | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this cluster for Algebra I. |
| **F.IF.4**  For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior ~~and periodicity~~.* | Edited to be consistent with the focus for Algebra I expectations for functions (linear, quadratic, and exponential functions). Periodic functions are studied in Algebra II. |
| **F.IF Analyze functions using different representations.** | Analyze functions using different representations (include linear, quadratic, and exponential functions with integer exponents). | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this cluster for Algebra I. |

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| **F.IF.7.b.** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.  | Graph ~~square root, cube root, and~~ piecewise-defined functions, including step functions and absolute value functions.  | Edited to incorporate a footnote that clarifies the focus and expectation of this standard for Algebra I. |
| **F.IF.7.e**. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. | Graph exponential ~~and logarithmic~~ functions, showing intercepts and end behavior.~~, and trigonometric functions, showing period, midline, and amplitude~~ | Edited to incorporate a footnote that clarifies the focus and expectation of this standard for Algebra I. |
| **F.IF.8.a**  Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.  | Use the process of factoring and completing the square in a quadratic function to show zeros, maximum/minimum ~~extreme~~ values, and symmetry of the graph, and interpret these in terms of a context.  | Edited to clarify the wording of the standard to be consistent with the key features of linear, exponential and quadratic functions. |
| **F.IF.8.b**  Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, and y = (1.2)t/10, and classify them as representing exponential growth or decay.* | Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, and y = (1.2)t/10, and classify them as representing exponential growth or decay. Apply to financial situations such as Identify appreciation/depreciation rate for the value of a house or car some time after its initial purchase. ().* | Edited to provide an example that connects real world financial application with mathematical expressions. |
| **F.IF.8.MA.c**. Translate among different representations of functions and relations: graphs, equations, point sets, and tables. | Standard Moved. | Moved and incorporate into F.IF.9 that has similar a focus on functions represented in different ways. (See below.) |
| **F.IF.9**  Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* | Translate among different representations of functions: graphs, equations, point sets, and tables. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* | Edited to incorporate standard F.IF.MA.8.c which has a related expectation for representing functions. |

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| **F.IF.MA.10** Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, exponential, or trigonometric. | Standard moved. | Deleted from Algebra I and moved to Algebra II. The student expectations for learning this content; rational, trigonometric, and logarithmic functions, is the focus in Algebra II. The focus in Algebra I is linear, exponential and quadratic functions. |
| **F.BF Build a function that models a relationship** **between two quantities.** | Build a function (linear, quadratic, and exponential functions with integer exponents) that models a relationship between two quantities. | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this Algebra I cluster. |
| **F.BF.2**  Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. | ~~Write~~ Identify arithmetic and geometric sequences written ~~both~~ recursively and ~~with an explicit formula,~~ use them to model situations.~~, and translate between the two forms~~ | Edited to incorporate a footnote that clarifies the focus and expectation of this standard for Algebra I. |
| **F.BF Build new functions from existing functions.**  | Build new functions from existing functions (using linear, quadratic, and exponential functions). | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this cluster for Algebra I. |
| **F.BF.3**  Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *kf*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* | Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *kf*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Utilize technology to experiment with cases and illustrate an explanation of the effects on the graph. *~~Include recognizing even and odd functions from their graphs and algebraic expressions for them.~~* | Edited to clarify the wording and expectation of the standard. |
| **F.BF.4.a**  Find Inverse Functions. Solve an equation of the form *f*(*x*) = *c* for a simple function *f* that has an inverse and write an expression for the inverse. *For example, f(x) =2x3 or f(x) = (x + 1)/(x − 1) for x ≠ 1.* | Find the inverse of a linear function both graphically and algebraically. ~~Solve an equation of the form~~ *~~f~~*~~(~~*~~x~~*~~) =~~ *~~c~~* ~~for a simple function~~ *~~f~~* ~~that has an inverse and write an expression for the inverse.~~ *~~For example, f(x) =2x~~~~3~~ ~~or f(x) = (x + 1)/(x − 1) for x ≠ 1~~.* | Edited to incorporate a footnote that clarifies the focus and expectations of this standard for Algebra I. |

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| **F.LE.3**  Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. | Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically. ~~or (more generally) as a polynomial function~~ | Edited to clarify the expectation of the standard for Algebra I. Polynomial functions are addressed in Algebra II. |
| **F.LE.5**  Interpret the parameters in a linear or exponential function in terms of a context. | Interpret the parameters in a linear or exponential function (of the form f*(x) = bx + k)* in terms of a context. | Edited to incorporate a footnote that clarifies the focus and expectations of this standard for Algebra I. |
| **S.ID.4** Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.  | Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate~~. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.~~  | Edited to incorporate a footnote that clarifies the focus and expectations of this standard for Algebra I. |
| **S.ID.6.** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*  | Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.a. Fit a function to the data; use linear functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. ~~Emphasize linear, quadratic, and exponential models.~~*  | Edited to incorporate a footnote that clarifies the focus and expectations of this standard for Algebra I. |

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| High School Model Geometry |
| **2011 Standard** | **Proposed 2016 Standard****Revisions are in red text** | **Rationale for Revision** |
| **G.CO Prove geometric theorems.** | Prove geometric theorems and, when appropriate, the converse of theorems. | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this Geometry cluster. |
| **G.CO.MA.11.a**  Prove theorems about polygons. *Theorems include: measures of interior and exterior angles, properties of inscribed polygons.* | Derive the formula for the relationship between the number of sides, sums of the interior and sums of the exterior angles of polygons, and apply to the solutions of mathematical and contextual problems. Prove theorems about polygons. *Theorems include: measures of interior and exterior angle. ~~properties of inscribed polygons.~~* | Edited to provide focus and coherence within the standard and preserve the mathematical rigor.  |
| **G.C.MA.3.a**  Derive the formula for the relationship between the number of sides and sums of the interior and sums of the exterior angles of polygons and apply to the solutions of mathematical and contextual problems. | Standard moved. | The content in this standard was incorporated into standard G.CO.MA.11.A where it is more coherently aligned (Proving theorems about polygons).  |
| **G.GPE.4**  Use coordinates to prove simple geometric theorems algebraically*. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, ) lies on the circle centered at the origin and containing the point (0, 2).* | Use coordinates to prove simple geometric theorems algebraically including the distance formula and its relationship to the Pythagorean Theorem*. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, ) lies on the circle centered at the origin and containing the point (0, 2).* | Edited to provide deeper focus of important and specific connections with other standards within the Model Geometry course and preserve mathematical rigor. |

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| **S.CP Understand independence and conditional probability and use them to interpret data.** | Understand independence and conditional probability and use them to interpret data from simulations or experiments. | Edited to incorporate a footnote that clarifies the focus and expectations of the standard in this Geometry cluster. |
| **S.MD.7 (+)** Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game) | Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game and replacing with an extra skater). | Edited to incorporate an accompanying footnote that clarifies the example in the standard.  |

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| High School Model Algebra II |
| **2011 Standard** | **Proposed 2016 Standard****Revisions are in red text** | **Rationale for Revision** |
| **A.SS.E.2**  Use the structure of an expression to identify ways to rewrite it. *For example, see x4 – y4 as (x2)2 – (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 – y2)(x2 + y2).* | Use the structure of an expression to identify ways to rewrite it. *For example, see x4 – y4 as (x2)2 – (y2)2, thus recognizing it as a difference of squares that can be factored (x2 – y2)(x2 + y2) and further factored (x - y) x + y)(x - yi)(x + yi).* | Edited the example in the standard to align with the focus and expectations for Algebra II. |
| **A.APR.1**  Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. | Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. Compare the system of polynomials to the system of integers when adding, subtracting, multiplying, and dividing polynomials. | Edited to clarify expectations for student learning and provide coherence between understanding systems and dividing polynomials.  |
| **A.APR.MA.1.a** Divide polynomials. | Standard moved. | This standard has the same student learning expectations with the revised standard A.APR.1 (See above).  |
| **A.CED.4**  Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm’s lawV = IR to highlight resistance R.* | Standard deleted | Deleted the standard in Model Algebra II as the standard is in the Model Algebra I course. |
| F.IF Interpret functions that arise in applications in terms of the context.  | Interpret polynomial, rational, trigonometric, and logarithmic functions that arise in applications in terms of the context.  |   |

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| **F.IF.8.MA.c**. Translate among different representations of functions and relations: graphs, equations, point sets, and tables. | Standard moved. | This standard has the same student learning expectations with the revised standard F.IF.9 (See below). |
| **F.IF.9**  Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* | Translate among different representations of functions ~~and relations~~: graphs, equations, point sets, and tables. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* | Edited to incorporate standard F.IF.MA.8.c which has a related expectation for functions. |
| **New Standard**  **F.IF.MA.10**  | Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, exponential, or trigonometric. | Added to Algebra II (deleted from Algebra I). The expectations for student learning on rational, trigonometric, and logarithmic functions in this standard are a focus of the Algebra II model course. |
| **S.MD.7 (+)** Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game) | Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game and replacing with an extra skater). | Edited to incorporate an accompanying footnote that clarifies the example in the standard. |

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| High School Model Integrated Math I |
| **2011 Standard** | **Proposed 2016 Standard****Revisions are in red text** | **Rationale for Revision** |
| **A.SSE Interpret the structure of expressions.** | Interpret the structure of linear and exponential expressions with integer exponents. | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this Model Math I cluster. |
| **A.SSE.1.b** Interpret complicated expressions by viewing one or more of their parts as a single entity*. For example, interpret P(1 + r)t as the product of P and a factor not depending on P.*  | Interpret complicated expressions by viewing one or more of their parts as a single entity*. For example, interpret P(1 + r)t as the product of P and a factor not depending on P, given that P is the principal amount of money that is growing at a rate, r, over a period of time, t, in years­.* | Edited to provide a context for the expression used in the example. |
| A.CED Create equations that describe numbers or relationships.  | Create equations and inequalities that describe numbers or relationships (using linear and exponential equations with integer exponents). | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this cluster for Model Math I.  |
| **A.CED.1**  Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions | Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and ~~quadratic functions, and simple rational and~~ exponential functions | Edited to clarify the focus and to incorporate a footnote that clarifies the expectation of this standard for Math I. |
| **A.CED.3**  Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* | Represent constraints by linear equations or inequalities, and by systems of linear equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* | Edited to add a financial examples to the standard related to finance and to clarify the expectations of this standard for Math I. |

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| **A.CED.4**  Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm’s lawV = IR to highlight resistance R* | Rearrange formulas to highlight a quantity of interest, using ~~the same reasoning as in solving equations~~ properties of equality. *For example, rearrange Ohm’s lawV = IR ~~to highlight resistance R~~  to solve for the variable R. Manipulate variables in formulas used in financial contexts, such as for simple interest ().* | Edited to add a financial examples to the standard related to finance and to clarify the expectations of this standard for Math I. |
| **A.REI.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. | Explain each step in solving a simple linear equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify or refute a solution method. | Edited to be coherent with Standard for Mathematical Practice 3; *Construct viable arguments and critique the reasoning of others.*  |
| **A.REI.3** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. | Solve linear and exponential (of a form 2x = 1/16) equations and linear inequalities in one variable, including equations with coefficients represented by letters. | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this cluster for Model Math I. |
| **A.REI Solve systems of equations.** | Solve systems of linear equations. | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this Model Math I cluster. |
| **A.REI.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).  | Understand that the graph of an equation (linear and exponential) in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Show that any point on the graph of an equation in two variables is a solution to the equation | Edited to incorporate a footnote that clarified the focus and expectations of this standard for Math I. |

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| **A.REI.11** Explain why the *x*-coordinates of the points where the graphs of the equations *y* = *f*(*x*) and *y* = *g*(*x*) intersect are the solutions of the equation *f*(*x*) = *g*(*x*); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f*(*x*) and/or *g*(*x*) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions | Explain why the *x*-coordinates of the points where the graphs of the equations *y* = *f*(*x*) and *y* = *g*(*x*) intersect are the solutions of the equation *f*(*x*) = *g*(*x*); find the solutions approximately, e.g., using technology to graph the functions and/or make tables of values. ~~or find successive approximations. Include cases where~~ *~~f~~*~~(~~*~~x~~*~~) and/or~~ *~~g~~*~~(~~*~~x~~*~~) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions~~ | Edited to incorporate a footnote that clarifies the focus and expectations in this standard for Model Math I. |
| **A.REI.12** Graph the solutions to a linear inequality in two variables as a half-plane, and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. | Graph the solutions ~~to~~ of a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set ~~to~~ of a system of linear inequalities in two variables as the intersection of the corresponding half-planes. | Edited to clarify wording of the standard to avoid misconceptions. |
| **F.IF Understand the concept of a function and use function notation.** | Understand the concept of a linear or exponential function having integer exponents and use function notation. | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this cluster for Math I. |
| **F.IF Interpret functionsthat arise in applications in terms of the context.** | Interpret linear or exponential functionshaving integer exponents that arise in applications in terms of the context. |  |
| **F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior~~; and periodicity~~.* | Edited to be consistent with the focus for Math I expectations for functions (linear and exponential functions). Periodic functions are studied in Math III. |
| **F.IF Analyze functions using different representations.** | Analyze linear functions and exponential functions with integer exponents using different representations. | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this Model Math I cluster. |
| **F.IF.7.a Graph linear and quadratic functions and show intercepts, maxima, and minima.** | Graph linear ~~and quadratic~~ functions and show intercepts~~,~~ ~~maxima, and minima~~. | Edited to address a footnote that clarifies the focus on linear and exponential functions. For Math I. |

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| **F.IF.7 e.** Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude | Graph exponential ~~and logarithmic~~ functions, showing intercepts and end behavior.  ~~and trigonometric functions, showing period, midline, and amplitude~~ | Edited to incorporate a footnote that clarifies the focus and expectations in this standard for Model Math I. |
| **F.IF.9**  Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. | Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one ~~quadratic~~ linear or exponentialfunction and an algebraic expression for another, say which has the larger ~~maximum~~ y-intercept.* | Edited to incorporate a footnote that clarifies the focus and expectations in this standard for Model Math I. |
| **F.IF.MA.10** Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, exponential, or trigonometric. | Standard moved. | Deleted from Math I and moved to Math III. The student expectation for learning rational, trigonometric, and logarithmic functions is the focus in Math III. The focus in Math I is on linear and exponential functions. |
| **F.BF Build a function that models a relationship between two quantities.** | Build a linear or exponential function with integer exponents that models a relationship between two quantities. | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this Model Math I cluster. |
| **F.BF.2**  Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. | Write arithmetic and geometric sequences that are defined ~~both~~ either recursively ~~and~~ or with an explicit formula, use them to model situations, and translate between the two forms. | Edited to clarify the wording of the standard to avoid confusion. |
| **F.BF Build new functions from existing functions.**  | Build new functions (linear and exponential having integer exponents) from existing functions. | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this cluster for Math I. |

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| **F.BF.3**  Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *kf*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* | Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *kf*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Focus on linear and exponential functions and on vertical translations for exponential functions. Utilize technology to experiment with cases and illustrate an explanation of the effects on the graph. ~~Experiment with cases and illustrate an explanation of the effects on the graph using technology~~. *~~Include recognizing even and odd functions from their graphs and algebraic expressions for them.~~* | Edited to incorporate a footnote that clarifies the focus and expectations in this standard for Model Math I. |
| **F.LE Construct and compare linear, quadratic, and exponential models and solve problems.** | Construct and compare linear, ~~quadratic,~~ and exponential models and solve problems. | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this cluster for Math I. |
| **F.LE.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. | Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly. ~~quadratically, or (more generally) as a polynomial function~~ | Edited to clarify the focus and expectation of the standard for Math I. |
| **F.LE.5** Interpret the parameters in a linear or exponential function in terms of a context. | Interpret the parameters in a linear or exponential function (of the form f*(x) = bx + k)* in terms of a context. | Edited to incorporate a footnote that clarifies the focus and expectations in this standard for Model Math I. |
| **G.CO Make geometric constructions.** | Make geometric constructions, formalize with proofs, and explain process. | Edited to incorporate a footnote that clarifies the focus and expectations in this cluster for Math I. |
| **G.GPE.4**  Use coordinates to prove simple geometric theorems algebraically*. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, ) lies on the circle centered at the origin and containing the point (0, 2).* | Use coordinates to prove simple geometric theorems algebraically including the distance formula and its relationship to the Pythagorean Theorem*. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, ) lies on the circle centered at the origin and containing the point (0, 2).* | Edited to include important and specific connections with other standards within the Math I course. |

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| **S.ID.6.**a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*  | a. Fit a function to the data; use linear functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. ~~Emphasize linear, quadratic, and exponential models.~~* | Edited to incorporate a footnote that clarifies the focus and expectations in this standard for Model Math I. |

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| High School Model Integrated Math II |
| **2011 Standard** | **Proposed 2016 Standard****Revisions are in red text** | **Rationale for Revision** |
| **A.SSE Interpret the structure of expressions**. | Interpret the structure of linear, quadratic and exponential expressions. | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this cluster for Math II. |
| **A.SSE.1.b**  Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1 + r)n as the product of P and a factor not depending on P. | Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1 + r)~~n~~t as the product of P and a factor not depending on P.  *An application for this could be the principal amount of money that is growing at a rate, r, over a period of time, t, in years.* | Edited to provide an example that connects real world financial application with mathematical expressions. |
| **A.SSE.2**  Use the structure of an expression to identify ways to rewrite it. *For example, see x4 – y4 as (x2)2 – (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 – y2)(x2+ y2).* | Use the structure of an expression to identify ways to rewrite it. For example, see ~~x~~~~4~~ ~~– y~~~~4~~ ~~as (x~~~~2~~~~)~~~~2~~ ~~– (y~~~~2~~~~)~~~~2~~~~, thus recognizing it as a difference of squares that can be factored as (x~~~~2~~ ~~– y~~~~2~~~~)(x~~~~2~~ ~~+ y~~~~2~~~~)~~ (x + a)2 – b2 as a difference of squares that can be factored as (x + a + b)(x + a – b). | Edited the example in the standard to clarify the focus and expectations of the standard for Math II students. |
| **A.SSE Write expressions in equivalent forms to solve problems.** | Write linear, quadratic, and exponential expressions in equivalent forms to solve problems. | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this cluster for Math II. |
| **A.APR.1**  Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. | ~~Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials~~ Add, subtract, and multiply polynomial expressions, including factoring and/or expanding polynomial expressions, identifying and combining like terms, and applying the Distributive Property. | Edited to incorporate a footnote to clarify the expectation of the standard for Algebra I students. In Algebra I students are introduced to operations of addition, subtraction, and multiplication on polynomials.  |

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| **A.CED.4**  Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm’s lawV = IR to highlight resistance R.* | Rearrange formulas (include formulas with quadratic terms) to highlight a quantity of interest, using ~~the same reasoning as in solving equations~~ properties of equality. *For example, rearrange Ohm’s law~~to highlight resistance R~~  to solve for the variable V. Manipulate variables in formulas used in financial contexts, such as for simple interest ().*  | Edited to add examples to the standard related to finance and to incorporate a footnote to clarify the expectations of the standard for Math II students. |
| **A.REI.4**  Solve quadratic equations in one variable. | Solve quadratic equations in one variable (for equations having real coefficients). | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this Model Math II cluster. |
| **A.REI.7**  Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line y = –3x and the circle x2 + y2 = 3.* | Solve a system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line y = –3x and the parabola, y = x2 + x or the line and the circle x2 + y2 = 3.* | Edited the example in the standard to address a footnote and clarify the focus and expectations of the standard for Math II students. |
| **F.FIF Interpret functions that arise in applications in terms of the context.** | Interpret linear, quadratic, and exponential functions that arise in applications in terms of the context. | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this cluster for Math II. |
| **F.IF.4**  For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior; ~~and periodicity~~.* | Edited to be consistent with the focus for Math II expectations for functions (linear, quadratic, and exponential functions). Periodic functions are studied in Math II. |
| **F.IF.7.b.** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.  | **b.** Graph ~~square root, cube root, and~~ piecewise-defined functions, including step functions and absolute value functions.  | Edited to address a footnote that clarifies the focus of Math II expectations for functions which includes piecewise-defined, step, and absolute value when analyzing key features of their graphs.  |

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| **F.IF.8.a**  Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.  | Use the process of factoring and completing the square in a quadratic function to show zeros, maximum/minimum ~~extreme~~ values, and symmetry of the graph, and interpret these in terms of a context.  | Edited to clarify the wording of the standard to be consistent with the key features of linear, exponential and quadratic functions. |
| **F.IF.8.b**  Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, and y = (1.2)t/10, and classify them as representing exponential growth or decay.* | Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, and y = (1.2)t/10, and classify them as representing exponential growth or decay. Apply to financial situations such as Identify appreciation/depreciation rate for the value of a house or car some time after its initial purchase, .* | Edited to provide an example that connects real world financial application with mathematical expressions. |
| **F.IF.8.MA.c**. Translate among different representations of functions and relations: graphs, equations, point sets, and tables. | Standard moved. | This standard has the same student learning expectations with the revised standard F.IF.9 (See below).  |
| **F.IF.9**  Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* | Translate among different representations of functions and relations: graphs, equations, point sets, and tables. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* | Edited to incorporate standard F.IF.MA.8.c which has a related expectation for functions. |
| **F.IF.MA.10** Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, exponential, or trigonometric. | Standard deleted. | Deleted from Math II and moved to Math III. The student expectation for learning rational, trigonometric, and logarithmic functions is the focus in Math III. The focus in Math II is on linear, quadratic, and exponential functions. |
| **F.BF Build a function that models a relationship between two quantities.** | Build a function (linear, quadratic, and exponential functions) with integer exponents that models a relationship between two quantities. | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this cluster for Math II. |

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| **F.BF Build new functions from existing functions.** | Build new functions from existing functions including quadratic and absolute value functions. | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this cluster for Math II. |
| **F.BF.3**  Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *kf*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* | Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *kf*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Include linear, quadratic and absolute value functions. Utilize technology to experiment with cases and illustrate an explanation of the effects on the graph. ~~Experiment with cases and illustrate an explanation of the effects on the graph using technology~~. *~~Include recognizing even and odd functions from their graphs and algebraic expressions for them.~~* | Edited to clarify the focus and expectation of the standard and maintain the mathematical rigor for Math II students. |
| **F.BF.**4 Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) =2x3 or f(x) = (x + 1)/(x  1) for x ≠ 1. | Standard deleted. | Deleted standard in Math II. Inverse functions are an expectation of Math III. |
| **F.LE.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. | Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically. ~~or (more generally) as a polynomial function~~ | Edited to clarify the focus and expectation of the standard for Math II students and maintain the mathematical rigor. |
| **G.CO Prove geometric theorems** | Prove geometric theorems using a variety of ways of writing proofs, showing validity of underlying reasoning. | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this cluster for Math II. |
| **G.CO.9** Prove theorems about lines and angles. *Theorems include: vertical angles are* *congruent; when a transversal crosses parallel* *lines, alternate interior angles are congruent* *and corresponding angles are congruent;* *points on a perpendicular bisector of a line* *segment are exactly those equidistant from the* *segment’s endpoints.* | Prove theorems about lines and angles. *Theorems* *include: vertical angles are congruent; when a transversal* *crosses parallel lines, alternate interior angles are* *congruent and corresponding angles are congruent, and* *conversely prove lines are parallel; points on a* *perpendicular bisector of a line segment are exactly those equidistant from the segment’s endpoints.* | Edited to clarify the focus and expectation of the standard for Math IIstudents and maintain the mathematical rigor. |

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| **G.CO.10** Prove theorems about triangles. *Theorems include: measures of interior angles* *of a triangle sum to 180°; base angles of* *isosceles triangles are congruent; the* *segment joining midpoints of two sides of a* *triangle is parallel to the third side and half the* *length; the medians of a triangle meet at a* *point.*  | Prove theorems about triangles. *Theorems include:* *measures of interior angles of a triangle sum to 180°;* *base angles of isosceles triangles are congruent, and* *conversely prove a triangle is isosceles; the segment* *joining midpoints of two sides of a triangle is parallel to* *the third side and half the length; the medians of a* *triangle meet at a point.*  | Edited to clarify the focus and expectation of the standard for Math II students and maintain the mathematical rigor. |
| **G.CO.MA.11.a**  Prove theorems about polygons. *Theorems include: measures of interior and exterior angles, properties of inscribed polygons.* | Derive the formula for the relationship between the number of sides, sums of the interior and sums of the exterior angles of polygons, and apply to the solutions of mathematical and contextual problems. Prove theorems about polygons. *Theorems include: measures of interior and exterior angles, ~~properties of inscribed polygons.~~* | Edited to provide focus and coherence within the Model Math II course for proofs about polygons and preserve the mathematical rigor.  |
| **G.SRT Prove theorems involving similarity.** | Prove theorems involving similarity using a variety of ways of writing proofs, showing validity of underlyingreasoning. | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this cluster for Math II. |
| **G.C.MA.3.a**. Derive the formula for the relationship between the number of sides, and sums of the interior and sums of the exterior angles of polygons, and apply to the solutions of mathematical and contextual problems | Standard moved | This standard has the same student learning expectations with the revised standard G.CO.MA.11.a (See above). |
| **New standard** | G.GMD Visualize relationships between two-dimensional and three-dimensional objects.4. Identify the shapes of two-dimensionalcross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. | This standard was added to the Model Math II course as it connects well with other standards in this domain about concepts of volumes and cross sections of two and three dimensional shapes. |

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| High School Model Integrated Math III |
| **2011 Standard** | **Proposed 2016 Standard****Revisions are in red text** | **Rationale for Revision** |
| **A.SSE Interpret the structure of expressions** | Interpret the structure of polynomial and logarithmic expressions, including those involving complex numbers. | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this cluster for Math III. |
| **A.SSE.2** Use the structure of an expression to identify ways to rewrite it. *For example, see x4 – y4 as (x2)2 – (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 – y2)(x2 + y2).* | Use the structure of an expression to identify ways to rewrite it. *For example, see x4 – y4 as (x2)2 – (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 – y2)(x2 + y2) and as (x-y)(x+y)(x-y****i****)(x+y****i****).* | Edited the example in the standard to align with the focus and expectations for Algebra II. |
| **A.APR.1**  Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. | Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. Compare the system of polynomials to the system of integers when adding, subtracting, multiplying, and dividing polynomials. | Edited to clarify expectations for student learning and provide coherence between understanding systems and dividing polynomials.  |
| **A.APR.** **MA.1.a** Divide polynomials.  | Standard moved | This standard has the same student learning expectations with the revised standard A.APR.1 (See above)..  |
| **A.APR Rewrite rational expressions.** | Rewrite rational expressions, focusing on linear and quadratic denominators. | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this cluster for Math III. |
| **A.CED**  **Create equations that describe numbers or relationships**.  | Create equations that describe numbers or relationships, including simple root functions. | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this cluster for Math III. |

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| **A.CED.4**  Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm’s lawV = IR to highlight resistance R.* | Standard deleted. | Deleted the standard in Model Math III as the expectations for rearranging formulas are a focus in Model Math I and Math II model courses.  |
| **F.IF** **Interpret polynomial, rational, trigonometric, and logarithmic functions that arise in applications in terms of the context.** | Interpret polynomial, rational, trigonometric, and logarithmic functions that arise in applications in terms of the context. | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this cluster for Math III. |
| **F.IF.7b** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. | Graph square root, cube root, and piecewise-defined functions, ~~including step functions and absolute value functions~~. | Edited to incorporate a footnote that clarifies the focus and expectations of this standard for Math III. |
| **F.IF.8.a** Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. | Use the process of factoring and completing the square in a quadratic function to show zeros, ~~extreme~~ maximum/minimum values, and symmetry of the graph, and interpret these in terms of a context. | Edited to clarify wording of the standard that relates to quadratic functions. |
| **F.IF.8.MA.c**. Translate among different representations of functions and relations: graphs, equations, point sets, and tables. | Standard moved. | This standard has the same student learning expectations with the revised standard F.IF.9 (See below). |
| **F.IF.9**  Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* | Translate among different representations of functions and relations: graphs, equations, point sets, and tables. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* | Edited to incorporate standard F.IF.MA.8.c which has a related expectation for functions. |
| **New standard** | F.IF.MA.10 Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, exponential, or trigonometric. | Added to Math III (deleted from Math I). The expectations for student learning of rational, trigonometric, and logarithmic functions in this standard is a focus of the Math III model course. |

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| **F.BF Build new functions from existing functions.** | Build new functions from existing functions, including simple radical, rational, and exponential functions | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this cluster for Math III. |
| **S.MD.7** (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game | (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game and replacing the goalie with an extra skater) | Edited to incorporate a footnote to clarify the example in the standard. |

# Summary of Proposed Changes to Other Sections of the Framework

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| **Section** | **Change and Rationale** |
| Introduction | Updated to include Massachusetts standards development and review processes 1993–2017Strengthened and expanded the connection between student learning expectations and college and career readiness and civic preparation |
| Guiding Principles | Literacy in the math content area expanded; new principle added to highlight social and emotional learning in the math classroom |
| Pre-K to 8 Content Standards Introduction | Updated example to reflect current framework  |
| Pre-Kindergarten | Edited coding of standards to reflect grade and domain |
| Grade level introductions grades 5, 6, 7 | Updated grade level introductions |
| High School; conceptual categories | Updated introduction and included charts to provide a visual display showing which conceptual category standards are addressed in each model course |
| High School Model Course introduction | Updated to reflect revisions made to model course content standards |
| Model Course Introductions | Each Model course introduction was updated to reflect the revisions in the model courses |
| High School: Making Decision document | Added a section to provide options and in depth explanation of course taking pathways that accelerate learning in middle school (grade 8) and high school including Algebra I in grade 8 .  |
| Appendices: Standards for Math Practice | To provide Grade span examples of connections between content standards and practice standards closer to their grade level. |
| Appendices: Glossary | Included additional mathematical definitions and terms referenced in the standards |