Report of the Mathematics Review Panel
on the Common Core and Massachusetts Standards
July 2010

The Mathematics Common Core Review Panel is comprised of seven mathematics educators representing the full range of requisite experiences including mathematics instruction at all levels from elementary through university, K-12 mathematics program development and coordination, and university level mathematics program development and coordination. We "met" initially through a one-hour conference call on June 3rd and again in person during a four-and-a-half-hour meeting on June 9th. Our communication since then has consisted of almost daily group e-mails with weekly conference calls. The foci of these discussions have been our individual and collective assessments of the both the ***Common Core State Standards for Mathematics*** (CCM) and the ***Massachusetts Curriculum Framework for Mathematics Working Draft*** (MCFM) dated June 2010. Our assessments were developed through careful deliberation and intense scrutiny of both documents. We entered these discussions with the charge of identifying the general strengths, weaknesses, areas of convergence, and areas of divergence within the two documents. This information is intended to assist the Massachusetts Board of Education with its decision of whether to move forward toward adoption of the Common Core State Standards or to move forward toward completion of, and ultimate adoption of, the revised Massachusetts Curriculum Frameworks.

The Mathematics Common Core Review Panel rates both sets of standards as **excellent** and would be comfortable with state adoption of either the CCM or the MCFM. Both documents would support Massachusetts' quest for rigorous state standards in mathematics and the quest to prepare all students for college and career. In addition, both documents exhibit comprehensive inclusion of the necessary content with generally appropriate rigor, clarity, vertical alignment, and measurability. We applaud the writers of the MCFM for the improvements they have made to the previous version, particularly in grades six through eight. The CCM is complete while the MCFM is still under development and the latter does not currently contain remarks about *mathematical practice* or introductory remarks at each grade level. These two valuable features of the CCM make the document accessible to a wider audience and can easily be adjoined to the MCFM. Given the importance and value we assign to these materials, the panel urges the Massachusetts Department of Elementary and Secondary Education and the Mathematics Frameworks Revision Panel to work toward inclusion of these features in the MCFM should the Board move toward adoption of the MCFM.

The panel highlights the following critical differences between the two documents:

Grades K-5

There is a pattern of mastery of arithmetic computation being generally required one grade level earlier in the MCFM compared to the CCM. For example "Add and subtract up to five-digit numbers accurately and efficiently. Include the conventional algorithm with and without regrouping. (3.N.12)" is listed at Grade 3 in MCFM while Grade 3 in CCM requires "Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. (3.NBT.2)." The equivalent of the Grade 3 Massachusetts standards comes in Grade 4 in the CCM. We believe the MCFM does generally expect mastery of arithmetic computation a bit earlier than the CCM. There are

exceptions, for example in the area of elementary arithmetic and middle school algebra preparation, the division of fractions is introduced and designated for mastery earlier in the CCM than in the MCFM. All but two members of the panel feel the pacing of the CCM is more likely than that of the MCFM to increase achievement for many students. These members argue that the emphasis on taking time for place value understanding and properties of operations helps cement deeper understanding of computation and will ultimately increase Algebra readiness. The members of minority opinion see arithmetic pacing as a weakness in the CCM since it leaves less time for teaching proportional reasoning and algebra readiness in Grades 5 – 7. These members argue that the expectation of mastery of arithmetic concepts at one grade level earlier in the MCFM compared to the CCM is realistic and that students can meet these expectations with the depth of understanding described in the CCM. They cite the fact that the pacing of arithmetic standards in the MCFM, as well as those of some high performing nations such as Singapore, more closely matches that of the *current* Massachusetts Curriculum Frameworks. All panel members agree that Massachusetts students' high level of performance on state, national, and international assessments points to strength in the *current* Massachusetts Curriculum Frameworks and recognize the close relationship between the *current* Massachusetts Curriculum Frameworks and the MCFM. However, the members of majority opinion highlight the fact that student achievement is influenced by many factors beyond mastery of number facts and the standard algorithms.

The CCM uses the word "fluency" where the MCFM uses "automaticity". This discrepancy could cause confusion due to differences in the definitions of these two words. The panel agrees that both *automaticity of number facts*, and *fluency of computation* should be required for Massachusetts students [1]. Psychologists define automaticity as the fast, implicit, and automatic recall of a fact or procedure from long-term memory. Unlike automaticity, fluency involves some processing in working memory. To be fluent, computation must require little effort and be efficient.

The majority of panel members feel that the CCM and the MCFM lead students to significantly similar levels of understanding by the end of grade 5. These members acknowledge the differences at particular grade levels but feel that the *overall* K-5 mathematics experience is substantially similar and that either program of study will position students well for the middle school curriculum. Two panel members feel that the critical differences described above prohibit their agreement with these statements.

Grades 6-8

As was the case with K – 5, there are differences at particular grade levels between the two documents for grades 6 – 8. Examples include the earlier treatment of number line and work with angles in the MCFM and the earlier treatment of systems of linear equations and more extensive standards in statistics and probability in the CCM.

The panel engaged in extensive discussion about Algebra readiness and the preparation for Algebra that students will have depending on which set of standards is adopted. The panel disagrees with the claim of the CCM (p. 84) that students are prepared to take Algebra in 8th grade through mastery of the 7th grade standards as it implies that students can simply skip the 8th

[1] Final Report of the National Mathematics Advisory Panel. See pages 17 and 18.

grade standards defined in the CCM and move directly to a ***full*** Algebra I course. All panel members agree that many students should not be expected to take Algebra I in 8th grade. We also agree that students who do take such a course should complete all standards through 8th grade by the end of 7th grade, whether we follow the standards of the MCFM or those of the CCM.

The panel is agreed that both sets of standards support a strong middle school mathematics program. Most of the differences between the two sets of standards seem to reflect differences in style or differences in the order in which probability/statistics topics are covered. But some differences involve the pacing of preparation for Algebra 1. These differences are inconsequential for students taking Algebra I in Grade 9, but the MCFM's accelerated treatment of proportionality and linearity in Grades 5 - 7 lends itself to the compacting of the curriculum for students who will take Algebra I in Grade 8. While all panel members agree in principle with the previous statement, a majority of panel members feel the CCM's greater emphasis on development of proportional reasoning will result in stronger preparation of all students for studying Algebra I, a point of view that is supported by the Critical Foundations of Algebra section of the final report of the National Mathematics Advisory Panel. The members of minority opinion see the differences as related to pacing but not depth, and believe that the more rapid pacing of the MCFM is well-supported.

Ultimately, all panel members agreed that the CCM and the MCFM both lead students to significantly similar levels of understanding by the end of grade 8. Thus, while differences exist at particular grade levels, the *overall* 6-8 mathematics experience is substantially similar and either program of study will position students well for the high school curriculum beginning with Algebra I.

Grades 9-12

The panel applauds the inclusion of standards for a Pre-Calculus course in the MCFM. To date, the CCM includes model courses for Algebra, Geometry, and Algebra II only. We note that standards beyond those expected in a comprehensive Algebra I through Algebra II sequence are included in the CCM and look forward to assessing additional courses, such as Pre-Calculus, when they become available. Comparing the curricula for Algebra through Algebra II, the MCFM have clearer expectations for linear functions and geometric proof while the CCM includes a good deal more statistics.

Some of the standards listed as "(+)" in the CCM (indicating that "students should learn [them] in order to take advanced mathematics courses" (p.57)) are included in the MCFM standards within the Algebra, Geometry, and Algebra II courses. Examples include logarithmic functions, inverse functions, and some aspects of complex numbers. The review panel is concerned that the (+) standards in the CCM actually generate two tracks. This is troubling in that students may be excluded from advanced mathematics courses/concepts through tracking. If the Board of Education chooses to adopt the CCM, the review panel suggests that the Massachusetts Department of Elementary and Secondary Education consider each (+) standard individually and determine whether or not, for this state, each (+) standard be mandated for inclusion in the Algebra – Algebra II sequence. Such a process will mitigate the concern about the CCM generating two tracks.

Speaking of the CCM and the MCFM in totality, the vertical alignment is evident and appropriate in both documents. However, especially in the CCM, this may not be clear to readers who are not well versed in mathematics and/or education. The verbiage of the standards in the CCM is significantly more mathematical. This, coupled with the inclusion of topics not previously required by the Massachusetts Curriculum Frameworks for Mathematics, absolutely necessitates a plan for teacher professional development and support. If the Board of Education adopts the CCM, the Department of Elementary and Secondary Education would need to create a plan for professional development and support prior to implementation of the standards.

The review panel acknowledges the additional benefits (among them cost) that come with multi-state collaboration through common standards including: common assessments, common teacher professional development, shared production of ancillary materials, and common understandings of what students should know and be able to do, and the continuity of curricula and ease of assimilation afforded to students experiencing inter-state transfer among those states who adopt the Common Core State Standards. This effect will be enhanced (or limited) by the number of states who agree to participate. Further, the Standards for Mathematical Practice included in the CCM highlight the habits of mind that are critical in effective mathematics practice and strengthen the vertical articulation of the CCM. The panel wishes to highlight the need for teacher professional development and classroom materials to specify the effective implementation of the Standards for Mathematical Practice; without which the Standards for Mathematical Practice remain just ideas and will not impact student achievement.

The review panel also acknowledges the additional benefits that would come with adoption of the MCFM. These include a decade of experience implementing the *current* MCFM which is obviously related to the 2010 draft MCFM, state assessments that are already closely aligned to the 2010 draft MCFM, and educator familiarity with the form and structure of the MCFM standards. As indicated above, if the Board of Education chooses to adopt the MCFM, we urge the Department of Elementary and Secondary Education to prioritize the development of features analogous to the CCM's Standards for Mathematical Practice.

In summary, both documents, the Common Core State Standards for Mathematics and the Massachusetts Curriculum Frameworks for Mathematics Working Draft, are well articulated, coherent, sets of standards. Both documents define a comprehensive content that would prepare Massachusetts students for college and career. There are some differences between the sets of standards, and in a few cases the members of this panel disagree on the significance of those differences. However, we all agree that Massachusetts can work well with either document. We can add to whichever document is selected the strengths that are in the document that is not selected.

The most significant features of each of the two documents are as follows:

In the case of the MCFM:

a) Closer alignment with the pacing of arithmetic curriculum standards in high achieving countries, such as Singapore.

b) Its similarity to the current framework, which has served as basis of a decade of successful learning, teaching, and assessment in Massachusetts. This would ease the transition from the current framework to the MCFM.
c) The somewhat greater ease of compacting the grade 5 - 8 curriculum into grades 5 - 7 for those students who are qualified to take a robust Algebra I course in grade 8.

In the case of CCM:
a) Specificity of standards and extended development of approaches to understanding of operations and proportional reasoning through grade 8, which may benefit a majority of students.
b) The efficiencies, savings, and benefits to inter-state transfer students obtainable through multi-state collaboration on curriculum standards and assessment.
c) The arrangement of standards by topic and their wording makes it more evident how the standards connect/relate to one another.

Respectfully submitted by the Mathematics Common Core Review Panel:

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