**Subject Matter Knowledge Matrix**

**Mathematics Curriculum Framework**

Students in Massachusetts must meet rigorous academic standards, which are outlined in the [Massachusetts Curriculum Frameworks](https://www.doe.mass.edu/frameworks/). To do so, they must have access to educators with strong content knowledge and pedagogical skills, the building blocks of effective instructional practice.

In support of this, the [Subject Matter Knowledge Guidelines](https://www.doe.mass.edu/edprep/domains/instruction/smk-guidelines.docx) set forth the content knowledge expectations for educator licensure in Massachusetts. Through these expectations, the Massachusetts Department of Elementary and Secondary Education (DESE) seeks to ensure that educators entering the workforce have sufficient content knowledge in their licensure area to support students in mastering the Massachusetts Curriculum Frameworks.

While the Curriculum Frameworks serve as an anchor, the intent is not that educators should simply know the content included in the Frameworks. Rather, educators must move beyond basic or functional knowledge to a level of fluency or expertise with the academic standards such that they can teach and support students in mastering the content.

The figure below shows a steady progression, not in the amount of information one knows, but in the depth and ability to use that information for a specific purpose. The boxes below the continuum outline some assessments used to determine varying levels of content knowledge. The depth at which the knowledge and application of content knowledge must be demonstrated is dependent on the stage of development for an individual educator (i.e. Basic, Functional, Fluent, or Expert) and/or license type (Provisional, Initial, or Professional).



This worksheet should be completed for licensure programs with Subject Matter Knowledge expectations in the [Mathematics Curriculum Framework](https://www.doe.mass.edu/frameworks/current.html), including:

Mathematics, 1-6

Mathematics, 5-8

Mathematics, 8-12

**Mathematics Content Progression**

The Mathematics Content Progression outlines the core mathematical knowledge that students should learn from PK through grade 12. To create a strong vertical progression of learning, educators should have the content knowledge to support PK-12 students in mastering prerequisite and advanced content standards. Teachers need to be able to access knowledge from prior grades, and teachers who are aware of later content can make better choices about what to emphasize, what language to use, and what larger contexts to provide for their students. This expectation allows teachers to meet students where they are and prepare them for where they are going.

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| **Instructions*:***Please use the chart below to determine which rows of the matrix should be completed for each license. This content knowledge must either be covered directly through program coursework or screened during the admissions process.  For each relevant grade level, list the numbers/abbreviations/titles of the **sponsoring organization’s required courses where the content knowledge is explicitly targeted and coherently addressed**. Then, **briefly describe where in the syllabus the content is covered** (i.e., unit name, week number, objective number). Course identifiers should match those of submitted syllabi and content knowledge for each grade level should not be spread across too many courses.    The full [Mathematics Curriculum Framework](https://www.doe.mass.edu/frameworks/current.html), including the Guiding Principles which are also available in [Appendix A](#_Appendix_A:_Mathematics) of this document, should be consulted when designing programs to ensure appropriate content coverage and rigor for each licensure field’s grade span. Sponsoring Organizations should prioritize content fluency in the grade span for the license while ensuring functional content knowledge in the two grade levels below and above the grade span. |

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| **License Field and Grade Span** | **Rows to Complete** |
| Mathematics, 1-6 | Pre-kindergarten through grade 8 |
| Mathematics, 5-8 | Grade 3 through grade 10 |
| Mathematics, 8-12 | Grade 6 through grade 12 |

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| **Mathematics Content Progression** | **Course(s) or Screening** |
| *Example Row* | *EDU 101 – Weeks 5-7* |
| **Pre-kindergarten:** Activity time should focus on two critical areas: (1) developing an understanding of whole numbers to 10, including concepts of one-to-one correspondence, counting, cardinality (the number of items in a set), and comparison; and (2) recognizing two-dimensional shapes, describing spatial relationships, and sorting and classifying objects by one or more attributes. Relatively more learning time should be devoted to developing children’s sense of number as quantity than to other mathematics topics. |  |
| **Kindergarten:** Instructional time should focus on two critical areas: (1) representing, relating, and operating on whole numbers, initially with sets of objects; and (2) describing shapes and space. More learning time in kindergarten should be devoted to number than to other topics. |  |
| **Grade 1:** Instructional time should focus on four critical areas: (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; (3) developing understanding of linear measurement and measuring lengths as iterating length units; and (4) reasoning about attributes of, and composing and decomposing geometric shapes |  |
| **Grade 2:** Instructional time should focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes. |  |
| **Grade 3**: Instructional time should focus on four critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes. |  |
| **Grade 4:** Instructional time should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; (3) and understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry. |  |
| **Grade 5:** Instructional time should focus on four critical areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of measurement systems and determining volumes to solve problems; and (4) solving problems using the coordinate plane. |  |
| **Grade 6:** Instructional time should focus on five critical areas: (1) connecting ratio and rate to whole number multiplication and division, and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; (4) developing understanding of statistical thinking; and (5) reasoning about geometric shapes and their measurements. |  |
| **Grade 7:** Instructional time should focus on four critical areas: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples. |  |
| **Grade 8**: Instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; and (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem. |  |
| **Grades 9 and 10:** Instructional time should focus on eight critical areas, (1) extend the laws of exponents to rational exponents; (2) extend understanding of numerical manipulation to algebraic manipulation; (3) create, analyze and solve equations and inequalities involving linear, exponential, and quadratic expressions and functions; (4) apply linear models to data that exhibit a linear trend; (5) establish criteria for congruence based on rigid motions and for similarity of triangles based on dilations and proportional reasoning; (6) prove basic geometric theorems; (7) apply the Pythagorean Theorem to the coordinate plane; and (8) extend work with probability. |  |
| **Grades 11 and 12:** Instructional time should focus on eight critical areas, (1) relate arithmetic of rational expressions to arithmetic of rational numbers; (2) extend work with complex numbers; (3) expand understanding of functions to include polynomial, exponential, rational, trigonometric, logarithmic, and radical functions; (4) use characteristics of polynomial and rational functions to sketch graphs of those functions; (5) expand right triangle trigonometry to include general triangles; (6) consolidate functions and geometry to create models and solve contextual problems; (7) relate data display and summary statistics to probability and explore a variety of data collection methods; and (8) explore and apply concepts of vectors and matrices to model and solve real-world problems. |  |

**Mathematical Rigor**

Students reach fluency by building understanding of mathematical concepts – this lays a strong foundation that prepares them for more advanced math work – and by building automaticity in the recall of basic computation facts, such as addition, subtraction, multiplication, and division. As students apply their mathematical knowledge and skills to solve real-world problems, they also gain an understanding of the importance of mathematics throughout their lives. To achieve mathematical understanding, students should be actively engaged in meaningful mathematics. The content and practice standards focus on developing students in the following areas:

* Conceptual understanding – make sense of the math, reason about and understand math concepts and ideas
* Procedural fluency – know mathematical facts, compute and do the math
* Capacity – solve a wide range of problems in various contexts by reasoning, thinking, and applying the mathematics they have learned.

Educators should have the content knowledge to support PK-12 students in achieving this mathematical understanding.

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| **Instructions:** Initial licensure program candidates must reach a level of fluent content knowledge to be endorsed. They must be able to apply content in a range of contexts and vertically connect content to build students’ knowledge. As such, sponsoring organizations must have at least one course at the fluent level for Mathematical Rigor.  Please list the numbers/abbreviations/titles of the **required course(s) where the elements of Mathematical Rigor are explicitly targeted and coherently addressed**. Course identifiers should match the numbers/abbreviations/titles of submitted syllabi.  Then, **briefly describe how the course(s) teach candidates to understand the content and practice standards using the elements of Mathematical Rigor**. |

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| **Mathematical Rigor** | **Fluent**  *Initial*  *Licensure* |
| *Example Row* | *EDU 101 – Weeks 5-7* |
| * Conceptual understanding – make sense of the math, reason about and understand math concepts and ideas * Procedural fluency – know mathematical facts, compute and do the math * Capacity – solve a wide range of problems in various contexts by reasoning, thinking, and applying the mathematics they have learned. |  |
| *Description:* | |

**Standards for Mathematical Practice**

The Standards for Mathematical Practice describe skills that mathematics educators at all levels should seek to develop in their PK-12 students. They complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

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| **Instructions*:***Initial licensure program candidates must reach a level of fluent content knowledge in order to be endorsed. They must be able to apply content in a range of contexts and vertically connect content to build students’ knowledge. As such, sponsoring organizations must have at least one course covering the practices at the fluent level.  Please list the numbers/abbreviations/titles of the **required courses where practices are explicitly targeted and coherently addressed**. Then, **briefly describe where in the syllabus each practice is covered** (i.e., unit name, week number, objective number). Course identifiers should match those of submitted syllabi and practices should not be spread across too many courses. |

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| **Standards for Mathematical Practice** | **Fluent**  *Initial*  *Licensure* |
| *Example Row* | *EDU 101 – Weeks 5-7* |
| **1. Make sense of problems and persevere in solving them.** Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand others’ approaches to solving complex problems and identify correspondences among different approaches. |  |
| **2. Reason abstractly and quantitatively.** Mathematically proficient students make sense of the quantities and their relationships in problem situations. Students bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meanings of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. |  |
| **3. Construct viable arguments and critique the reasoning of others.** Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They can analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. |  |
| **4. Model with mathematics.** Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They can identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose. |  |
| **5. Use appropriate tools strategically.** Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels can identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They can use technological tools to explore and deepen their understanding of concepts. |  |
| **6. Attend to precision.** Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in communicating their own reasoning verbally and/or in writing. In problem solving they state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school, they have learned to examine claims and make explicit use of definitions. |  |
| **7. Look for and make use of structure.** Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression x 2 + 9x + 14, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see 5 – 3(x – y) 2 as 5 minus a positive number times a square, and use that to realize that its value cannot be more than 5 for any real numbers x and y. |  |
| **8. Look for and express regularity in repeated reasoning.** Mathematically proficient students notice if calculations are repeated and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation (y – 2)∕(x – 1) = 3. Noticing the regularity in the way terms cancel when expanding (x – 1)(x + 1), (x – 1)(x 2 + x + 1), and (x – 1)(x 3 + x 2 + x + 1) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results. |  |

# **Appendix A: Mathematics Guiding Principles**

The following principles are philosophical statements that underlie the pre-kindergarten through grade 12 Mathematics Framework standards and resources. These principles should guide the design and evaluation of Mathematics programs in both PK-12 and higher education settings.

**Guiding Principle 1.** Educators must have a deep understanding of the mathematics they teach, not only to help students learn how to efficiently do mathematical calculations, but also to help them understand the fundamental principles of mathematics that are the basis for those operations. Teachers should work with their students to master these underlying concepts and the relationships between them in order to lay a foundation for higher-level mathematics, strengthen their capacity for thinking logically and rigorously, and develop an appreciation for the beauty of math.

**Guiding Principle 2.** To help all students develop a full understanding of mathematical concepts and procedural mastery, educators should provide them with opportunities to apply their learning and solve problems using multiple methods, in collaboration with their peers and independently, and complemented by clear explanations of the underlying mathematics.

**Guiding Principle 3.** Students should have frequent opportunities to discuss and write about various approaches to solving problems, in order to help them develop and demonstrate their mathematical knowledge, while drawing connections between alternative strategies and evaluating their comparative strengths and weaknesses.

**Guiding Principle 4.** Students should be asked to solve a diverse set of real-world and other mathematical problems, including equations that develop and challenge their computational skills, and word problems that require students to design their own equations and mathematical models. Students learn that with persistence, they can solve challenging problems and be successful.

**Guiding Principle 5.** A central part of an effective mathematics curriculum should be the development of a specialized mathematical vocabulary including notations and symbols, and an ability to read and understand mathematical texts and information from a variety of sources.

**Guiding Principle 6.** Assessment of student learning should be a daily part of a mathematics curriculum to ensure that students are progressing in their knowledge and skill, and to provide teachers with the information they need to adjust their instruction and differentiate their support of individual students.

**Guiding Principle 7.** Students at all levels should have an opportunity to use appropriate technological tools to communicate ideas, provide a dynamic approach to mathematic concepts, and to search for information. Technological tools can also be used to improve efficiency of calculation and enable more sophisticated analysis, without sacrificing operational fluency and automaticity.

**Guiding Principle 8.** Social and emotional learning can increase academic achievement, improve attitudes and behaviors, and reduce emotional distress. Students should practice self-awareness, self-management, social awareness, responsible decision-making, and relationship skills, by, for example: collaborating and learning from others and showing respect for others’ ideas; applying the mathematics they know to make responsible decisions to solve problems, engaging and persisting in solving challenging problems; and learning that with effort, they can continue to improve and be successful.