Quick Reference Guide: Common Multiplication and Division Situations

Students explore multiplication in second-grade by pairing and grouping objects in arrays. For example, they work with arranging odd and even numbers of objects and repeated addition. Students begin their formal study of multiplication and division word problems in third-grade and continue this work in fourth-grade. They begin with solving problems involving equal groups and arrays, and progress to solving area and multiplicative comparison problems. **This guide elaborates on *Table 2: Common multiplication and division situations* in the *Massachusetts Curriculum Framework for Mathematics*.**

By learning operations in context, students develop numerical fluency and connect procedural knowledge to real-world situations. As Table 2 shows, the context of *3 x 6 = 18* lends itself to multiple interpretations depending on the position of the unknown.

Grade 3: Multiplication and Division Problems Within 100

Third-graders work with multiplication and division word problems within 100 involving situations from the first two rows of Table 2; *equal groups*, *array*s, and *areas* (3.OA.A.3), using a variable to represent an unknown number. Typically students' first exposure to multiplication involves objects arranged in equal groups, as in row 1 of Table 2. Likewise, division instruction usually begins with understanding equal shares as distributing a known quantity to a particular number of groups (partitive division) and determining the number of groups given a total and number per group (quotative division). In both cases, the concept of equal groups is associated with skip counting and repeated addition or subtraction strategies.

	Unknown Product **3 × 6 = ?**	**Group Size Unknown** (“How many in each group?” Division) **3 × ? = 18 and 18 ÷ 3 = ?**	**Number of Groups Unknown** (“How many groups?” Division) **? × 6 = 18 and 18 ÷ 6 = ?**
Equal Groups	There are three bags with six plums in each bag. How many plums are there in all? *Measurement example*. You need three lengths of string, each six inches long. How much string will you need altogether?	If 18 plums are shared equally into three bags, then how many plums will be in each bag? *Measurement example*. You have 18 inches of string, which you will cut into three equal pieces. How long will each piece of string be?	If eighteen plums are to be packed six to a bag, then how many bags are needed? *Measurement example*. You have 18 inches of string, which you will cut into pieces that are six inches long. How many pieces of string will you have?
Arrays, Area	There are three rows of apples with six apples in each row. How many apples are there? *Area example*. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into three equal rows, how many apples will be in each row? *Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of six apples, how many rows will there be? *Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?

Although *arrays* and *area* situations are both described in row two of Table 2, they involve different levels of understanding of multiplication and division. *Arrays* build on the concepts of equal groups and repeated addition/subtraction by organizing objects into rows and columns but with the focus shifted to the arrangement of the objects. This provides a foundation for more advanced, measurement-based, *area* situations. Conceptually, students transition from arrays of discrete objects to arrays made up of squares that have been pushed together to form an area. Understanding the relationship between the dimensions of a rectangle and its area is vital groundwork for developing the area model algorithm for multiplication (3.MD.C.5-7). Similarly, students understand division as a missing dimension problem.

Grade 4: Multiplicative Comparisons

Multiplicative and additive *compare* situations require students to interpret quantities expressed relative to other quantities. Fourth-grade students begin to distinguish multiplicative comparison from additive comparison (4.OA.A.1 and 2) by learning that one quantity can be expressed as a multiple or factor of another. For example, students interpret comparative language such as “twice as wide” in the context of a situation. They also use this language in their solutions and to check for understanding, saying, for example, “the rubber band is now three times as long.” In addition to solving problems as described in Table 2, fourth-grade students also solve multi-step word problems with whole numbers using all four operations (4.OA.A.3).

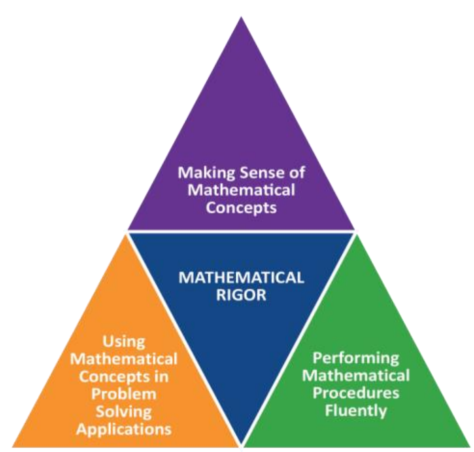
	Unknown Product **3 × 6 = ?**	**Group Size Unknown** (“How many in each group?” Division) **3 × ? = 18 and 18 ÷ 3 = ?**	**Number of Groups Unknown** (“How many groups?” Division) **? × 6 = 18 and 18 ÷ 6 = ?**
Compare	A blue hat costs \$6. A red hat costs three times as much as the blue hat. How much does the red hat cost? *Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be three times as long?	A red hat costs \$18 and that is three times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*. A rubber band is stretched to be 18 cm long and that is three times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? *Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$ and $p \div a = ?$	$? \times b = p$ and $p \div b = ?$

Understanding multiplicative comparisons is vital to a variety of fourth-grade concepts. Fourth-grade students continue to develop a more sophisticated understanding of place value by interpreting digits in terms of relative size (4.NBT.A.1). Additionally, understanding related quantities as multiples or factors of one another is vital to a comprehensive understanding of fraction equivalence and the composition of fractions (4.NF.A and B.4.a-c). Lastly, students apply multiplicative thinking to move between larger and smaller measurement units and create conversion tables (4.MD.A.1).

Balanced Mathematical Instruction

To achieve mathematical understanding, students should be actively engaged in **meaningful mathematics**. The standards focus on developing students’ conceptual understanding, procedural fluency, and problem-solving applications.

By teaching multiplication and division problems in a context, students are able to link their procedural understanding with real-world situations. As they work, students will develop a deep conceptual understanding of decomposing and composing numbers, which is critical to understanding how and why algorithms work.



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