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| Multiplication of Fractions |
| Grade 5 Mathematics(Updated February 2019) |
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| The emphasis of this Grade 5 unit is on developing a conceptual understanding of and proficiency with multiplication of fractions with a particular emphasis on understanding fraction multiplication in the context of using the unit fraction. Students will create visual models to represent multiplication as well as explain how and why the corresponding equations work. They will use what they have previously learned about multiplication to become proficient with *part of whole* and *part of part* multiplication. Students will create and solve word problems which require multiplication of fractions by using both computation and visual models. They will be asked to reason abstractly and quantitatively about fraction multiplication, discuss their reasoning with others, use visual models, and use their understanding of unit fractions to see the structure of multiplying fractions. *These Model Curriculum Units are designed to exemplify the expectations outlined in the MA Curriculum Frameworks for English Language Arts/Literacy and Mathematics incorporating the Common Core State Standards, as well as all other MA Curriculum Frameworks. These units include lesson plans, Curriculum Embedded Performance Assessments, and resources. In using these units, it is important to consider the variability of learners in your class and make adaptations as necessary.* |

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| Stage 1 — Desired Results |
| **ESTABLISHED GOALS G***Standards for Mathematical Practice*SMP2 Reason abstractly and quantitativelySMP3 Construct viable arguments and critique the reasoning of othersSMP4 Model with mathematicsSMP7 Look for and make use of structure*Mathematics Content Standards*5.NF.B.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.5.NF.B.3. Interpret a fraction as division of the numerator by the denominator (*a*/*b* = *a* ÷ *b*). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret ¾ as the result of dividing 3 by 4, noting that ¾ multiplied by 4 equals 3, and that when three wholes are shared equally among four people each person has a share of size ¾. If nine people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*5.NF.B.4a Interpret the product (*a*/*b*) × *q* as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a* × *q* ÷ *b*. *For example, use a* *visual fraction model and/or area model to show (2/3)* × *4 = 8/3, and create a story context for this equation. Do the same with (2/3)* × *(4/5) = 8/15.* (In general, (*a*/*b*) × (*c*/*d*) = *ac*/*bd*.) 5.NF.B.4b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.5.NF.B.5Interpret multiplication as scaling (resizing), by:a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. For example, without multiplying tell which number is greater: 225 or ¾ x 225; 11∕50 or 3∕2 x 11∕50? b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence ***a***/***b*** = **(*n* × *a*)**/**(*n* × *b*)**to the effect of multiplying ***a***/***b*** by 1.5.NF.B.6 Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.ELA:5.SL.1Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on *grade 5 topics and texts*,building on others’ ideas and expressing their own clearly.b. Follow agreed-upon rules for discussions and carry out assigned roles.c. Pose and respond to specific questions by making comments that contribute to the discussion and elaborate on the remarks of others.d. Review the key ideas expressed and draw conclusions in light of information and knowledge gained from the discussions.5.SL.3 Summarize the points a speaker makes and explain how each claim is supported by reasons and evidence.5.RI.4 Determine the meaning of general academic and domain-specific words and phrases in a text relevant to a *grade 5 topic or subject area.* | ***Transfer*** |
| ***Students will be able to independently use their learning to…* T*** interpret and persevere in solving mathematical problems using strategic thinking and expressing answers with a degree of precision appropriate for the problem context
* express appropriate mathematical reasoning by constructing viable arguments, critiquing the reasoning of others, and attending to precision when making mathematical statements
* apply mathematical knowledge to analyze and model mathematical relationships in the context of a situation in order to make decisions, draw conclusions, and solve problems
 |
| ***Meaning*** |
| **UNDERSTANDINGS U*****Students will understand that…***U1 the meaning of the unit fraction can be used to understand fraction multiplicationU2 fraction multiplication shows the relationship between the factors and how those factors are partitioned (or divided)U3 visual models (area model, fraction strip, number line) can be used to represent fraction multiplication.U4 fraction operations follow logical reasoning just as whole number operations do. | **ESSENTIAL QUESTIONS Q**Q1 Why is understanding the unit fraction important?Q2 How is what we understand about multiplication useful when multiplying fractions? Q3 Why are models important to help solve fraction problems? |
| ***Acquisition*** |
| ***Students will know…* K**K1 (a/b) x q = a x q ÷ bK2 (a/b) x (c/d) = ac/ bdK3 visual models (area model, fraction strip, number line) can represent fraction multiplication.K4 how to apply their understanding of unit fractions to more complex fraction multiplicationK5 Targeted Academic Language: unit fraction, factor, product, equal shares, partition, visual model, area model, fraction strip, number line | ***Students will be skilled at…* S**S1 using their understanding of the unit fraction to explain and model more complex multiplication problemsS2 multiplying a whole number by a fraction and a fraction by a fraction (including unit fractions, fractions less than one and fractions greater than one)S3 using visual models to solve problems involving multiplication of fractions.S4 creating and solving real-world problems involving multiplication of fractions**.** |
| Stage 2 — Evidence |
| **Evaluative Criteria** | **Assessment Evidence** |
| * Accurate visual fraction models
* Precise multiplication calculations
* Effectively communicates connections between visual models, calculations and analysis
 | **CURRICULUM EMBEDED PERFOMANCE ASSESSMENT (PERFORMANCE TASKS) PT**iBaby Advertising |
|  | **OTHER EVIDENCE: OE**Exit CardsStation Tasks Cards (Lessons 5 and 7)Station Word Problem Creation (Lesson 5 and 7)Practice Problems (Lesson 10)Any portion of the group work could be used for informal or formal assessment:  |
| Stage 3 — Learning Plan |
| ***Summary of Key Learning Events and Instruction***Although only the first two lessons explicitly mention problems *in context* all lessons should try to use equations, expressions, visual models and context to develop understanding.Lesson 1: Understand a Fraction as Division of the Numerator by the Denominator in Context (5.NF.B.3, 5.NF.B.6)Lesson 2: Understanding Multiplying a Unit Fraction by a Whole Number in Context (5.NF.B.4a, 5.NF. B.6)Lesson 3: Using Visual Models to Multiply a Unit Fraction by a Whole Number in Context (5.NF.B.4a, 5.NF. B.6)Lesson 4: Using Visual Models to Multiply a Non-Unit Fraction Less than 1 by a Whole Number in Context (5.NF.B.4a, 5.NF.B.6)Lesson 5: Practicing Part by Whole Multiplication with Fraction Strip and Number Line Models (5.NF.B.4a, 5.NF.B.6)Lesson 6: Using Fraction Strips, Number Lines, and Area Model to Multiply a Fraction Less than 1 by a Fraction Less than 1 (5.NF.B.4a, 5.NF.B.4b, 5.NF.B.6)Lesson 7: Practicing Part by Whole Multiplication with Fraction Strip, Number Line, and Area Models (5.NF.B.4a, 5.NF.B.4b, 5.NF.B.6)Lesson 8: Using Fraction Strips, Number Lines, and Area Model to Multiply Fractions, Including Those Greater than 1 (5.NF.B.4a, 5.NF.B.4b, 5.NF.B.6)Lesson 9 : Practicing Fraction Multiplication with All Visual Models (Fraction Strip, Number Line, Area Model) (5.NF.B.4a, 5.NF.B.4b, 5.NF.B.6)Lesson 10: Developing the Algorithm (5.NF.B.4a, 5.NF.B.4b, 5.NF.B.6)Lesson 11: Fraction Multiplication and Scaling (5.NF.B.5, 5.NF.B.6)Lesson 12: CEPA - iBaby Advertisement Claim |

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# Teacher Notes

**Domain Progression**

It would be helpful for teachers to read and refer to the Number and Operations Fractions Progression for grades 3–5 prior to teaching this unit.

<http://commoncoretools.files.wordpress.com/2012/02/ccss_progression_nf_35_2011_08_12.pdf>

Teaching these standards requires shifts in thinking about how fractions may have been taught in the past. It is recommended that teachers read through the lessons in this unit in advance to process their own thinking about these shifts before they present them to students. In addition, teachers should spend time constructing their own visual models and equations as they work through the problems prior to students doing so.

**Grouping Students**

Thought should be given to pairings and groupings of students as they work through the problems in this unit. Consider varying groupings so that sometimes they are heterogeneous and sometimes they are homogeneous. Also attend to how English Language Learners are placed into groups by considering both their language ability and their mathematical ability. A student who is an ELL, or a student with language disabilities may have difficulty with language but understand the mathematical concepts and this should be considered when grouping students. In the same way, consider the strengths and needs of students who need more challenge.

**Removing Barriers**

Teachers should be aware of the learning goals for each lesson and remove barriers to learning where necessary. These barriers might include the language of the word problems and this barrier could be addressed in a variety of ways including providing manipulatives, visual images, acting out the problem, small-group previewing, reading problems aloud, and examining multiple meanings of words, among others.

**Pre-assessment**

A pre-assessment is included. It is strongly suggested that this NOT be used as the launch of this unit. It could used as the last task of the previous unit, a homework assignment, or in some other manner. Students could complete it in pairs, but larger groups are not recommended as it may be hard to discern individual student understanding.

**Artifacts and Evidence**

A crucial part of this unit is increasingly building conceptual structures to understand multiplication of fractions. Students will participate in a gallery walk to help shape their understanding of fraction multiplication. Therefore, it is necessary to save and display student work from each lesson as the conceptual development builds so it can be referred to on an ongoing basis throughout the unit. Share this with students so they show their work clearly and legibly as they proceed. This includes all whole class notes and a sampling of individual or pair work which exemplifies relevant concepts.

**Standards for Mathematical Practice**

These are standards in the same sense that content standards are standards and should be explicitly taught and modeled with students. While all eight Standards of Mathematical Practices (SMP) are found in the lessons in this unit, four have been chosen as focus areas. Within each lesson, a subset of these four has been chosen so that teachers can place particular emphasis on those. English Language Learners may need support to understand the language of these mathematical practices, however, it is strongly suggested that the academic vocabulary of the standard is maintained. For example, you may support the student to understand what *precise* means, while not changing the word *precise*.

Pre-Assessment

1. **Show what you know about the expression** $\frac{2}{3}$ **.**

|  |  |  |
| --- | --- | --- |
| What does $\frac{2}{3}$ mean? | Draw a picture or diagram of $\frac{2}{3}$ .Can you draw another picture or diagram that shows $\frac{2}{3}$ in another way?  | Write a word problem that would use $\frac{2}{3}$ . |

1. **Show what you know about the expression** $\frac{1}{4}$ **x 8.**

|  |  |  |
| --- | --- | --- |
| What does $\frac{1}{4}$ × 8 mean? | Draw a picture or diagram of $\frac{1}{4}$ × 8.Draw another picture or diagram thatshows $\frac{1}{4}$ × 8 in another way. | Write a word problem that would use $\frac{1}{4}$ × 8. |

1. **You have 5 pizzas to share among you and two friends. Show how you would share the pizza so the three of you each get an equal share.**

Lesson 1: Understand a Fraction as Division of the Numerator by the Denominator in Context

**Brief Overview of Lesson:** The focus of this unit is for students to understand that a fraction is a division problem and that there is a corresponding multiplication problem which shows the fraction as the product of a unit fraction and a whole number. As you plan, consider the variability of learners in your class and make adaptations as necessary.

**Prior Knowledge Required:**

* Students have already been introduced to fractions and understand that a fraction consists of a numerator and denominator a/b.
* Multiplication of whole numbers
* Multiplication is repeated addition
* Multiplication of whole numbers can be represented by the area model and a number line
* Addition and subtraction of fractions with like and unlike denominators (5.NF.A.1, 5.NF.A.2)
* Concept of commutative, associative and distributive properties of multiplication
* Consistent use of visual models (fraction strip, number line, etc.) and equations to solve word problems

**Estimated Time:** 60 minutes

**Resources for Lesson:** Chart paper (or other means to display student thinking, such as a smart board or document camera), graph paper, fraction strips, fraction manipulatives, markers, and a pack of cards (index or other cards paper clipped or bagged together as a “pack”)

**Content Area/Course:** Mathematics Grade 5

**Lesson 1:** Understand a Fraction as Division of the Numerator by the Denominator in Context

**Time (minutes):** 60

**Standard(s)/Unit Goal(s) to be addressed in this lesson:**

5.NF.B.3. Interpret a fraction as division of the numerator by the denominator (*a*/*b* = *a* ÷ *b*). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret ¾ as the result of dividing 3 by 4, noting that ¾ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size ¾. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

5.NF.B.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

5.NF.B.6 Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

SMP2 Reason abstractly and quantitatively.

SMP3 Construct viable arguments and critique the reasoning of others.

SMP4 Model with mathematics.

**Essential Questions addressed in this lesson:**

How is what we understand about multiplication useful when multiplying fractions?

Why are models important to help solve fraction problems?

**Objectives:** Students will know and be able to:

* Connect previous knowledge of fractions to understand a fraction as a division problem
* Connect the concept of a fraction as a division problem to multiplication of a fraction by a whole number
* Targeted Academic Language**:** factors, product, equal shares, whole

**Instructional tips and strategies**

* Teacher notes are embedded throughout the lesson.

**Anticipated Student Preconceptions/Misconceptions**

* Students may not understand the concept of the whole in various contexts, e.g., students may think all of the cards from all of the packs represents the “whole,” rather than understanding a pack of cards is a whole.
* Students may have difficulty representing “easy” calculations (6 ÷ 3) with a visual model.
* Students may not understand sharing when the whole is not divisible by the number of shares needed (6 ÷ 4 vs. 6 ÷ 3).
* Students may have difficulty seeing the connection between the fraction, division expression and multiplication expression ($\frac{5}{3 }= $*5 ÷ 3 = 5 ×* $\frac{1}{3 }$).

**Lesson Sequence**

**Introduction to Unit**

Say to students:

Today we are starting a new unit. We are going to build on our previous work with addition and subtraction of fractions. We will use some of the same visual models that we previously used, such as a number line and fraction strip, but we are adding another model. We previously used an area model for whole numbers, and now we are going to use an area model for fractions. Who remembers what operations we were performing on numbers before when we used an area model? (multiplication) What operation do you think we will be working on with fractions in this unit? (multiplication) Think about the patterns and structure you see as we explore these new concepts so we can eventually develop an efficient method of multiplying fractions. We call this efficient method an algorithm because it is a set of rules which we can always follow to get the answer.

Review and post these Essential Questions: How is what we understand about multiplication useful when multiplying fractions? Why are models important to help solve fraction problems? Refer back to the Essential Questions often in discussion with students.

**Lesson Opening:**

Have students read and reflect on the following problem: Erik has five packs of cards he’d like to share with two friends. Each pack has three cards. How many cards will each person get if the cards are shared equally? What fraction of a pack does each person get? Draw a diagram showing how you solved the problem.

Instructional support: Provide “cards” (index cards paper clipped in threes) to represent a “pack.”

Have students Think-Pair-Share (SMP3 *Construct viable arguments and critique the reasoning of others*).

Note: Establish that Think-Pair-Share means that students will have one or two minutes to think and write quietly about the problem. After that time has passed, they will turn to a partner and share their thoughts for one or two minutes. Each will take a turn speaking while the other listens. The listener should paraphrase or comment on the speaker’s explanation. Students should be explicitly informed that at this stage they should use a critical eye to examine their partner’s work and comment on any misconceptions they see, whether the misconception is on their part or on the part of their partner. They should use evidence from what the partner said or showed in writing to support their comments. The listener then takes a turn speaking and the roles reverse. This should take three to six minutes in total. Times may vary depending on the task at hand. A written product showing what was discussed may or may not be required depending on the task. Pairs may be called upon to share with the whole group.

If this is being established for the first time with students, model a Think-Pair-Share by demonstrating what a conversation around a mathematical problem looks like. If another adult is not available for this role play, show and comment on math work. After modeling this and students have practiced, have a student pair model for the class also. Emphasis should be placed on listening, commenting using mathematical reasoning, rephrasing what the partner said, being polite, and respectfully agreeing or disagreeing. Partners should rotate so students have the opportunity to discuss mathematics with a variety of fellow students. A chart with sentence stems could help students who are new to or have difficulty with these conversations. This is also a scaffold for English Language Learners. Periodically monitor conversations to make sure students are following these guidelines. As students become comfortable with these types of conversations, they may occasionally be called upon to share their conversation with the whole class.

Whole group discussion:

* Student pairs show and describe diagrams used to solve this problem. As different student pairs share their work, be sure these questions are asked and answered of the pairs (if there is a natural place to ask them) or of the whole group:
	+ Describe the strategy you used to solve the problem.
	+ How many packs of cards were there? (five)
	+ How many cards made up a whole pack? (three)
	+ How many shares did you need to make? (three shares because there were three people)
	+ How many cards did each person get? (five)
	+ What fraction of a pack did each person get? ( $\frac{5}{3 }$ )
	+ What division expression shows how to solve this problem? (5 ÷ 3)

Note: Students may have the misconception that the whole is all 15 cards. We are defining the whole as a pack, which is three cards. Therefore, the denominator of the fraction is 3. Defining the whole is an important concept when dealing with fractions (3.NF.A.3d). Each person’s share is five cards, which is shown in the numerator.

**During the Lesson:**

Present another problem: Ani has five pizzas. He and two friends are hungry. He wants to share the five pizzas with his friends. How much pizza will Ani and his two friends each get if the pizza is shared equally? Have students Think-Pair-Share.

Instructional Support: Provide fraction circles, fraction bars, and grid paper for students to choose from to assist them in representing their solutions.

Whole group discussion:

* Student pairs show and describe diagrams used to solve this problem. As different student pairs share their work be sure the following questions are asked and answered of the student pairs if there is a natural place to ask either them or the whole group:
	+ How was this problem the same as or different from the previous problem?
	+ Describe the strategy you used to solve the problem.
	+ How many whole pizzas were there? (5)
	+ How many did you decide to divide up the pizza for sharing? (Possibly three slices per pizza)
	+ How many shares did you need to make? (three shares because there were three people)
	+ How much pizza did each person get? (Possibly five slices, which were each 1/3 of a pizza)
	+ What fraction of the pizza did each person get? ($ \frac{5}{3 } $)
* What division expression shows how to solve this problem? (5 ÷ 3)
* Connect the fraction to the division expression in both the card and the pizza problems to build understanding of 5.NF.B.3:
	+ What share did each person get? ($ \frac{5}{3 } $)
	+ What division expression was used in both cases? (5 ÷ 3)
	+ What can you conclude about what $\frac{5}{3 } $means? (Five divided by three)
	+ How many $\frac{1}{3 }$ pieces are in $\frac{5}{3 }$? (five)
	+ Can you write an addition equation showing that? ( $\frac{1}{3 }$ + $\frac{1}{3 } $+ $\frac{1}{3 }$ + $\frac{1}{3 }$ + $\frac{1}{3 }$ = $\frac{5}{3 }$ )
	+ Can you show this as a multiplication problem? (5 × $\frac{1}{3 }= \frac{5}{3 }$ )
	+ Conclude that 5 ÷ 3 = 5 × $\frac{1}{3 }=$ $\frac{5}{3 }$ ; that five-thirds is five divided by three and also five groups of one third (5.NF.B.3).
	+ Use the National Library of Virtual Manipulatives to develop a shared visual model. A trial version of this resource is free: <http://nlvm.usu.edu/en/nav/frames_asid_274_g_2_t_1.html?open=activities&from=grade_g_2.html>

Next solve: Have students read and reflect on the following problem: Sam has 6 brownies. He and two friends are hungry. He wants to share the 6 brownies with his friends. How many brownies will Sam and his 2 friends each get if the brownies are shared equally?

Instructional support: Provide fraction circles, fraction bars, and grid paper for students to choose from to assist them in representing their solutions.

Give students one or two minutes to think and write quietly about this problem.

As a whole class discuss the following. Write the questions and answers on a chart or board for students to refer to later.

* The answer is two brownies each.
* What would a diagram showing this look like? Have a student share a diagram. Possibly
* What would the equation be? (6 ÷ 3 = 2)
* Which number represents how many brownies Sam started with? (six)

Teacher Note: Understanding a brownie as the whole is important here.

* Which number represents how many shares of brownies we are finding? (3)
* What does the number 2 represent? (The number of brownies each person got.)

Note: Understanding the share as two wholes is important.

* What would a corresponding multiplication problem be? (Some might say 3 × 2 = 6, but have students reflect on the card problem and the pizza problem to see that 6 ÷ 3 = 6 × $\frac{1}{3 }=$ $\frac{6}{3 }$ = 2, which is SMP7 *Look for and make use of structure.*)

**Lesson Closing**

Continue discussion and make connections (SMP7 *Look for and make use of structure*). Wrap up by using the chart below, filling in answers as each question is discussed.

|  |  |
| --- | --- |
| What does the 6 represent/mean? (6 brownies)  | What is this similar to in the pizza problem? (5 slices) |
| What does the 3 represent? (3 groups or shares)  | What is this similar to in the pizza problem? (3 groups or shares) |
| What is the $\frac{1}{3 }$ ? (Each group or share is $\frac{1}{3 } $of the 6 brownies.)  | What is this similar to in the pizza problem? (Each share was $\frac{1}{3 }$ of the 5 pizzas.) |
| What share did each person get? (Each share is 2 brownies which is the same as $\frac{6}{3 }$.)  | What is this similar to in the pizza problem? (Each share is $\frac{5}{3 } $of the pizzas.) |

# Lesson 2: Understanding Multiplying a Unit Fraction by a

# Whole Number in Context

**Brief Overview of Lesson:** In this lesson students develop an understanding of multiplying a unit fraction by a whole number. They begin to learn and use SMP7 by seeing and recording the structure brought out by the word problems that show a fraction, related division problem and related multiplication problem. As you plan, consider the variability of learners in your class and make adaptations as necessary.

**Prior Knowledge Required:**

* Understand a fraction as division of a numerator by the denominator
* A fraction consists of a numerator and denominator a/b
* Multiplication of whole numbers
* Multiplication is repeated addition
* Multiplication of whole numbers can be represented by the area model and a number line
* Addition and subtraction of fractions with like and unlike denominators (5.NF.A.1, 5.NF.A.2)
* Concept of commutative, associative and distributive properties of multiplication
* Consistent use of visual models (fraction strip, number line, etc.) and equations to solve word problems

**Estimated Time:** 60 minutes

**Resources for Lesson:** *Seeing Structure* student handout, chart paper (or other means to display student thinking such as a smart board or document camera), graph paper, fraction strips, fraction manipulatives, fraction circles, and markers

**Content Area/Course:** Mathematics Grade 5

**Lesson 2:** Understanding Multiplying a Unit Fraction by a Whole Number in Context

**Time (minutes):** 60

**Standard(s)/Unit Goal(s) to be addressed in this lesson:**

5.NF.B.4.a. Interpret the product (*a*/*b*) × *q* as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a* × *q* ÷ *b*. *For example, use a visual fraction model to show (2/3)* × *4 = 8/3, and create a story context for this equation. Do the same with (2/3)* × *(4/5) = 8/15 .*

(In general, (*a*/*b*) × (*c*/*d*) = *ac*/*bd* .)

5.NF.B.6 Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

SMP2 Reason abstractly and quantitatively.

SMP3 Construct viable arguments and critique the reasoning of others.

SMP7 Look for and make use of structure.

**Essential Question addressed in this lesson:**

Why is understanding the unit fraction important?

**Objectives:** Students will know and be able to:

* See connections between a fraction, the whole, shares, division expression, a multiplication expression and a written explanation in the context of word problems.
* Understand (*a*/*b*) × *q* as *a* parts of a partition of *q* into *b* equal parts when *q* is a whole number.

**Instructional tips and strategies:** Teacher notes are embedded throughout the lesson.

**Anticipated Student Preconceptions/Misconceptions:**

* Understanding the concept of the whole in various concepts
* Difficulty representing “easy” calculations (6 ÷ 3) with a diagram or picture
* Sharing when the whole is not divisible by the number of shares needed (6 ÷ 4 vs. 6 ÷ 3)
* Difficulty seeing the connection between the fraction, division expression and multiplication expression ($\frac{5}{3 }= $*5 ÷ 3 = 5 ×* $\frac{1}{3 }$)

**Lesson Sequence**

**Lesson Opening:**

As a whole group, begin the lesson by reflecting on what was learned yesterday. Use the following chart to record information about the problems that were solved yesterday. (It is filled in below for reference.)

Start with a blank chart. A handout is included so students can complete their own charts during Turn and Talk. Look for and discuss patterns as they emerge. Have students Turn and Talk (see Teacher Note below) as questions/patterns emerge, then return to the whole group lesson. Draw out the following:

* Relationship between the total and the shares to the division expressions
* Relationship between the total and the shares to the fraction
* Relationship between the fraction and the multiplication problem
* Answer to the problem (During this discussion, be sure to show both the numerical and word answers; see the last row of the chart below.)
* What does the answer mean? What does 5/3 of a pizza mean? (Students will need time to process these difficult connections.)

Note: Establish that Turn and Talk is similar to Think-Pair-Share but does not involve the preliminary “Think” part. Students turn and talk to a partner to explore an idea or question that comes up in a whole-class discussion. In this way, *everyone* in class gets to voice their mathematical thinking instead of a few being called upon in class while others are passively listening. This can be modified by using Turn, Talk, and Write, in which students are required to write down their thinking or a solution. It works particularly well when used in the midst of a whole-class discussion as a way to have students pause, think, and talk about what has been said. Please refer to Think-Pair-Share in Lesson 1 for guidelines for partner discussion.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Erik’s Card Problem** | **Ani’s Pizza Problem** | **Sam’s Brownie Problem** |  |
| What is one whole? | 1 deck | 1 pizza | 1 brownie |  |
| How many wholes did you start with?  | 5 (decks; although each deck had 3 cards, this does not change the whole.) | 5 (pizzas) | 6 (brownies) |  |
| How many shares were needed? | 3 shares | 3 shares | 3 shares |  |
| What was the division expression? | 5 ÷ 3 | 5 ÷ 3 | 6 ÷ 3 |  |
| What fraction is related to that division expression? | $$\frac{5}{3}$$ | $$\frac{5}{3}$$ | $$\frac{6}{3}$$ |  |
| What multiplication expression is related to that division expression? | 5 × $\frac{1}{3}$ | 5 × $\frac{1}{3}$ | 6 × $\frac{1}{3}$ |  |
| What was the answer to the problem? (Reinforce the important concepts here by emphasizing all ways to express the answer. This sometimes requires words.) | * Five $\frac{1}{3}$ portions
* 5/3 of the one whole
* 5/3 of one deck
 | * Five $\frac{1}{3}$ portions
* 5/3 of the one whole
* 5/3 of one pizza
 | * Six $\frac{1}{3}$ portions
* 6/3 of the one whole
* 6/3 or 2 brownies
 |  |

**During the Lesson**

Establish a common understanding of *unit fraction* by writing three unit fractions on the left side of the board or chart paper and three non-unit fractions on the right side. Label the left *Unit Fractions* and the right *Non-Unit Fractions*. Have students Think-Pair-Shard about what they think a unit fraction is. Develop the common understanding of a unit fraction as a fraction with a 1 in the numerator.

Sharing Task: Have students read and reflect on the following problem: Tonya has three large granola bars. She and five friends are hungry for a snack. She wants to share the three granola bars with her friends. How much of a granola bar will Tonya and her five friends each get if each bar is shared equally?Have students Think-Pair-Share. At the end of that discussion, ask the pairs to:

* draw and discuss a diagram showing how the granola bars can be shared.
* draw and discuss a different diagram that shows how the granola bars can be shared.

After about five minutes, discuss as a whole class that this may be an easy problem for some, but the goal is to bring up some thinking about sharing and multiplication and make connections to the patterns observed in the chart above (SMP7 *Look for and make use of structure*). Have student pairs share one diagram with the whole class showing what their thinking was and how they solved this problem. Ask for another pair who has a *different* diagram to show and explain their diagram. Continue until no new diagrams are available. Make sure *three granola bars cut in half, showing six shares* and *three granola bars cut in sixths with sixths shared out to three groups* are shown; if either of these did not surface, show it.

Stop the whole-class discussion and ask pairs to discuss and do the following:

* The answer is $\frac{3}{6}or \frac{1}{2}$ of a granola bar each.
* Write a division equation to that shows what they did to solve the problem. (Note: this will be $3 ÷6=\frac{3}{6}or \frac{1}{2}$ ).
* Write a corresponding multiplication equation (Note: this will be$ 3 × \frac{ 1}{6}$ = $\frac{3}{6}or \frac{1}{2}$ ).
* Which number represents how many granola bars Tonya started with? (3)
* Which number represents how many shares of granola bars we are finding? (6)
* What does the $\frac{3}{6}or \frac{1}{2}$ represent?

Tell students to use the previous (posted) chart with examples of problems where sharing was involved and think about what division equation (or expression) was written and what multiplication equation that was written to help them answer the questions above. The new equations are 3 ÷ 6 = $\frac{1}{2} and 3 × \frac{1}{6} =\frac{3}{6}= \frac{1}{2} . $Think three groups of $\frac{1}{6}$ as $\frac{3}{6} or \frac{1}{2}$ .

Now ask pairs to fill in the last column of the chart with the information from Tonya’s Problem:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Erik’s Card Problem** | **Ani’s Pizza Problem** | **Sam’s Brownie Problem** | **Tonya’s Granola Bar Problem** |
| What is one whole? | 1 deck | 1 pizza | 1 brownie | 1 granola bar |
| How many wholes did you start with?  | 5 (decks. Although each deck had 3 cards this does not change the whole) | 5 (pizzas) | 6 (brownies) | 3 (granola bars) |
| How many shares were needed? | 3 shares | 3 shares | 3 shares | 6 shares |
| What was the division expression? | 5 ÷ 3 | 5 ÷ 3 | 6 ÷ 3 | 3 ÷ 6 |
| What fraction is related to that division expression?  | $$\frac{5}{3}$$ | $$\frac{5}{3}$$ | $$\frac{6}{3}$$ | $$\frac{3}{6}$$ |
| What multiplication expression is related to that division expression? | 5 × $\frac{1}{3}$ | 5 × $\frac{1}{3}$ | 6 × $\frac{1}{3}$ | 3 × $\frac{1}{6}$ |
| What was the answer to the problem? Reinforce the important concepts here by emphasizing all ways to express the answer. This sometimes requires words. | * Five $\frac{1}{3} $portions
* 5/3 of the one whole
* 5/3 of one deck
 | * Five $\frac{1}{3} $portions
* 5/3 of the one whole
* 5/3 of one pizza
 | * Six $\frac{1}{3} $portions
* 6/3 of the one whole
* 6/3 or 2 brownies
 | * Three $\frac{1}{6}$ portions
* $\frac{3}{6}$ of the one whole
* $\frac{3}{6}$ or ½of one granola bar
 |
| State the answer in a sentence that relates the share to the number of wholes—for example, “Five-thirds is one third of five” shows that five thirds is the share of five wholes. | $\frac{\begin{array}{c}\\5\end{array}}{3} is \frac{1}{3} of $\_\_\_\_5\_\_\_\_. | $\frac{\begin{array}{c}\\5\end{array}}{3} is \frac{1}{3} of $\_\_\_\_5\_\_\_\_. | $\frac{\begin{array}{c}\\6\end{array}}{3} is \frac{1}{3} of $\_\_\_\_6\_\_\_\_. | $\frac{\begin{array}{c}\\3\end{array}}{6} is \frac{1}{6} of $\_\_\_3\_\_\_\_\_. |

Students may have difficulty with the expressions in Tonya’s problem. They may see the division problem as 6 ÷ 3 = 2 and see the granola bars in halves vs. sixths. These ideas are relevant to the solution of the problem, but must be distinguished from the answer. The answer is not 2; rather, it is ½ and this may need deep discussion. The numbers involved are selected on purpose to be simple so the focus is not on the calculation but on the underlying concept and the structure compared to previous problems.

Students may need help seeing that the concept illustrated in the last row of the chart. For example, they may need help seeing that five-thirds is one third of five and that this shows that five-thirds is the share each person gets when there are five wholes shared among three people.

Now ask pairs to discuss some new questions:

* How many sixths did each person get? (3)
* Write this as a fraction. ($ \frac{3}{6 } $)
* Write a division equation to that shows what they did to solve the problem. (Note: this will be $3 ÷6= \frac{3}{6 }$).
* Write a corresponding multiplication equation (Note: this will be$ 6 ×\frac{3}{6 }$= 3).

**Closing:** Have students complete the following handout, which may be collected and used as a formative assessment.

Name \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Date \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Jamal’s Bubble Tape

Solve the following problem. Show a multiplication expression and draw a picture.

*Jamal has five feet of bubble tape. He and three friends are going to have a bubble gum blowing contest to see who can make the biggest bubble. How much bubble tape will Jamal and his three friends each get if the tape is shared equally? Draw a diagram and write a multiplication expression.*

|  |  |
| --- | --- |
|  | **Jamal’s Bubble Tape Problem** |
| What is one whole? |  |
| How many wholes did you start with?  |  |
| How many shares were needed? |  |
| What was the division expression? |  |
| What fraction is related to that division expression? |  |
| What multiplication expression is related to that division expression? |  |
| What was the answer to the problem?  |  |

Name \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Date \_\_\_\_\_\_\_\_\_\_\_

Looking for Structure

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Erik’s Card Problem** | **Ani’s Pizza Problem** | **Sam’s Brownie Problem** | **Tonya’s Granola Bar Problem** |
| What is one whole? |  |  |  |  |
| How many wholes did you start with?  |  |  |  |  |
| How many shares were needed? |  |  |  |  |
| What was the division expression? |  |  |  |  |
| What fraction is related to that division expression? |  |  |  |  |
| What multiplication expression is related to it? |  |  |  |  |
| What was the answer to the problem? Use words and numbers. |  |  |  |  |
| Answer in a sentence relating the share to the number of wholes, e.g., 5/3 is 1/3 of five, shows that five thirds is the share of five wholes. | $\frac{}{} is \frac{}{} of $\_\_\_\_\_\_\_\_. | $\frac{}{} is \frac{}{} of $\_\_\_\_\_\_\_\_. | $\frac{}{} is \frac{}{} of $\_\_\_\_\_\_\_\_. | $\frac{}{} is \frac{}{} of $\_\_\_\_\_\_\_\_. |

#

# Lesson 3: Using Visual Models to Multiply a Unit Fraction

# by a Whole Number in Context

**Brief Overview of Lesson:** In this lesson, students will shift focus to multiplication of fractions. Students will multiply a unit fraction by a whole number and use expressions, equations and visual models to explain their thinking. In the next lesson, students will extend this to multiplication of a whole number by a non-unit fraction using equations and visual models. As you plan, consider the variability of learners in your class and make adaptations as necessary.

**Prior Knowledge Required:**

* Understand the meaning of the whole
* Basic understanding of what it means to look for structure in mathematics (SMP7, which was practiced in Lessons 1 and 2)
* Understand a fraction as division of a numerator by the denominator
* Multiplication of whole numbers
* Multiplication is repeated addition
* Multiplication of whole numbers can be represented by the area model and a number line
* Addition and subtraction of fractions with like and unlike denominators (5.NF.A.1, 5.NF.A.2)
* Concept of commutative, associative and distributive properties of multiplication
* Consistent use of visual models (fraction strip, number line, etc.) and equations to solve word problems

**Estimated Time:** 60 minutes

**Resources for Lesson:**

Mini white boards, chart paper (or other means to display student thinking such as a smart board or document camera), graph paper, fraction strips, fraction manipulatives, fraction circles, markers, and paper

**Content Area/Course:** Mathematics Grade 5

**Lesson 3:** Using Visual Models to Multiply a Unit Fraction by a Whole Number in Context

**Time (minutes):** 60

**Standard(s)/Unit Goal(s) to be addressed in this lesson:**

5.NF.B.4.a Interpret the product (*a*/*b*) × *q* as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a* × *q* ÷ *b*. *For example, use a visual fraction model to show (2/3)* × *4 = 8/3, and create a story context for this equation. Do the same with (2/3)* × *(4/5) = 8/15 .*

5.NF.B.6 Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

SMP3 Construct viable arguments and critique the reasoning of others.

SMP4 Model with mathematics.

SMP7 Look for and make use of structure.

**Essential Question addressed in this lesson:**

Why is understanding the unit fraction important?

Why are models important to help solve fraction problems?

**Objectives:** Students will know and be able to:

* Use a fraction strip model to represent fraction multiplication
* Use a number line model to represent fraction multiplication
* Relate the corresponding equation to each model

**Instructional tips and strategies**

* Teacher notes are embedded throughout the lesson.
* Although many students will be able complete the calculations in their head, it is important that they are held accountable for drawing accurate visual models that express their thinking and correlate directly to the equations and the problem.

**Anticipated Student Preconceptions/Misconceptions:**

* Difficulty representing “easy” calculations (6 ÷ 3) with a visual model
* Sharing when the whole is not divisible by the number of shares needed (6 ÷ 4 vs. 6 ÷ 3)
* Difficulty seeing the connection between equation and the visual model
* Difficulty working with a number line larger than one whole

**Lesson Sequence**

**Lesson Opening**

Introduction: We have solved some word problems involving sharing. We have made connections between sharing, fractions, division, and multiplication. Now we will focus on fraction multiplication and the use of two visual models to help our thinking.

Have students read and reflect on the following problem: Anne Marie had four Hershey bars. She ate $\frac{1}{3} $of them. How many bars did she eat?

Draw a diagram and write an expression to show your thinking.

Have students Think-Pair-Share-Write and show their work on mini white boards. Each pair will share with one other pair. There will be no whole group sharing.

Teacher Note: At this point students are NOT expected to know the two models they will be taught today. This opening is intended to get them thinking about multiplication of fractions and representations. Students will be taught how to construct and use a fraction strip model and a number line model in the next section of this lesson through a Teacher Think Aloud.

**During the Lesson**

*Teacher Think Aloud: Solving with Fraction Strips:*Students start by solving the two given problems using what they have previously done by writing an expression and making a drawing or diagram in top row of the attached handout. The teacher then states that they will be learning two specific visual models today: a fraction strip model and a number line model. Model solving the problem above (*Anne Marie had four Hershey bars. She ate* $\frac{1}{3} $*of them. How many bars did she eat?)* by thinking aloud how she would use a fraction strip and a number line. It may go something like this:

Anne Marie has four Hershey bars. I’ll draw a fraction strip with four sections. Each one represents one whole bar.



She ate one third of the bars. Hmm, I cannot split four things evenly into three parts. But I remember when I divided pizzas into equal shares I split each pizza into the number of pieces I needed so I’ll do the same thing with the fraction bars.



Now I need to think about what the problem is asking me. I have each bar divided into thirds. Anne Marie ate 1/3 so she ate one of every three pieces. I’ll shade them to see how much that is.



I have four shaded pieces. Each whole is three parts. That makes three my denominator. Anne Marie must have eaten $\frac{4}{3}$ of a bar.

* This is more than one whole bar: $\frac{4}{3}$ is 1/3 of 4.$ $The equation is $\frac{4}{3}$ = 1/3 × 4.
* $\frac{4}{3}$ is also one share when four things are divided into three parts each. $\frac{4}{3}$ is the share.
* 1/3 is the size of each part. Four is the number of wholes.

Now have students solve this problem individually: Karl and Kandice have 7 feet of ribbon. They are going to use 1/5 of the ribbon to make award pins for the sports banquet. How much of the total amount of ribbon will they be using? Write an equation and draw a fraction strip model.

Students may show a fraction strip with 7 wholes divided into five pieces each and one piece of every whole shaded. Or they could shade one whole out of five wholes. Then they would have two remaining wholes to divide into fifths and shade one fifth each. In either case, they will end up with $\frac{7}{5}$ shaded.

*Teacher Think Aloud Solving with a Number Line:*You may choose to use online resources to create a virtual number line. One example is: <http://www.glencoe.com/sites/common_assets/mathematics/ebook_assets/vmf/VMF-Interface.html>. The Teacher Think Aloud may go something like this: We already solved the Hershey bar problem with your own diagram and a fraction strip. We know the answer to the problem is that Anne Marie ate $\frac{4}{3}$ of the Hershey bars. Models are used as tools to help us solve a problem, but also used to help us show our thinking about how we solved a problem. Many of these problems are easy and might be done mentally. This is on purpose so we can focus our efforts on the models and thinking process. Now I am going to show how I would think about solving this problem with a number line model.

Anne Marie has four Hershey bars. I’ll draw a number line from zero to four because I have four wholes. You will notice that I am being as accurate as possible and making my wholes equally spaced. Each one represents one whole bar.



She ate one third of the bars. Hmm. I cannot split four things evenly into three parts. But I remember when I divided pizzas into equal shares I split each pizza into the number of pieces I needed so I’ll do the same thing with the fraction bars.



Now I need to think about what the problem is asking me. I have each whole divided into thirds on my number line. Anne Marie ate 1/3 so she ate one of every three pieces. I’ll mark them to see how much that is.



I have four marked pieces. Each whole is sectioned into three parts. That means the denominator is three. Anne Marie must have eaten $\frac{4}{3}$ of a bar.

This is more than one whole bar; $\frac{4}{3}$ is 1/3 of 4.$ $That means that 1/3 of four is 4/3. Let me think about that. How could I show that? I’m going to move my pieces together to show one third of four.



Now I’m going to show that much two more times because if four-thirds is one third of four, there should be three four-thirds pieces in four, right? Turn and Talk to a partner about that. (Wait briefly.) I’m going to change the color so I can see it.

**

Wow! There are three pieces. Each piece is worth 4/3. So 1/3 of 4 wholes is 4/3. I wonder if that is always true? Let’s try the Karl and Kandice ribbon problem with the number line and see.

Now have students solve the Karl and Kandice problem again but use the number line this time: Karl and Kandice have 7 feet of ribbon. They are going to use 1/5 of the ribbon to make award pins for the sports banquet. How much of the total amount of ribbon will they be using? Write an equation and draw a fraction strip model.

When they finish, have students complete the Exploring Visual Models: Fraction Strip and Number Line sheet with a partner.

Be sure to save students’ work and post it around the room for students to refer to. In particular, save the Teacher Think Aloud diagrams.

**Lesson Closing**

*Exit Ticket***:** Have students complete the following problem and collect: Sally had to walk 12 miles during the week. On Monday she walked 1/8 of the miles. How far did she walk on Monday?

Exploring Visual Models: Fraction Strip and Number Line

Name \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Date \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

|  |  |  |
| --- | --- | --- |
|  | Milo’s Dog FoodMilo got into the dog food and ate $\frac{1}{4}$ of it. Before he ate any, there were five pounds. How much did he eat? How much was left? Your answer should be in fractions of a pound. | Write and solve a problem with your partner. |
| Fraction Strip |  |  |
| Number Line |  |  |

# Lesson 4: Using Visual Models to Multiply a

# Non-Unit Fraction by a Whole Number in Context

**Brief Overview of Lesson:** Students will build upon their work with unit fractions as they multiply whole numbers by fractions with a numerator greater than one. Students will make use of the unit fraction structure they examined in Lessons 1–3 in order to build conceptual understanding. As you plan, consider the variability of learners in your class and make adaptations as necessary.

**Prior Knowledge Required:**

* Conceptual understanding of the unit fraction and the ability to use visual models to explain a whole number multiplied by a unit fraction
* Addition and subtraction of fractions with like and unlike denominators (5.NF.A.1, 5.NF.A.2)
* Concept of commutative, associative and distributive properties of multiplication
* Consistent use of visual models (fraction strip, number line, etc.) and equations to solve word problems

**Estimated Time:** 60

**Resources for Lesson**

Manipulatives (fraction strips, fraction circles) available for students to use, chart paper, and index cards

**Content Area/Course:** Mathematics Grade 5

**Lesson 4:** Using Visual Models to Multiply a Non-Unit Fraction by a Whole Number in Context

**Time (minutes):** 60

**Standard(s)/Unit Goal(s) to be addressed in this lesson:**

5.NF.B.4a. Interpret the product (*a*/*b*) × *q* as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a* × *q* ÷ *b*. *For example, use a visual fraction model to show (2/3)* × *4 = 8/3, and create a story context for this equation. Do the same with (2/3)* × *(4/5) = 8/15 .*

(In general, (*a*/*b*) × (*c*/*d*) = *ac*/*b* .)

5.NF.B.6 Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

SMP3 Construct viable arguments and critique the reasoning of others.

SMP4 Model with mathematics.

SMP7 Look for and make use of structure.

**Essential Question addressed in this lesson:**

Why is understanding the unit fraction important?

Why are models important to help solve fraction problems?

**Objectives:** Students will know and be able to:

* Multiply a non-unit fraction less than one by a whole and demonstrate that understanding using visual models and equations
* Targeted Academic Language (review)**:** equal shares, partition, unit fraction, factor, visual model (fraction strip, number line)

**Instructional tips and strategies**

* Teacher notes are embedded throughout the lesson.
* This is not the place to develop an algorithm. This lesson is still guided discovery of the underlying concepts.

**Anticipated Student Preconceptions/Misconceptions**

* Difficulty in the flexible thinking needed to see 5/6 as a number in itself and seeing it also as a sum of five 1/6ths.

**Lesson Sequence**

**Lesson Opening**

Discuss with students that by now they have had a lot of practice with multiplying unit fractions by whole numbers. Now they will be solving some problems with non-unit fractions. Refer back to Lesson 2 to make sure students remember what a unit fraction is.

Post the following problem and have students work with a partner. Make sure fraction strips and fraction manipulatives are available. Give partners large chart paper because their solutions may be posted.

Tell students to look back to yesterday’s teacher Think Aloud work on Anne Marie’s Hershey Bar problem if they are struggling. This may be a place to mention SMP1 *Make sense of problems and persevere in solving them* so students understand that struggling with a problem and persevering through it is OK. This is about the process of thinking about fractions and the focus is not on answer getting. Circulate and observe partners as they work.

Problem: Marcello has four ice cream sandwiches. He ate 2/3 of them. How many ice cream sandwiches did he eat? Write an equation and use either a fraction strip or number line visual model.

**During the Lesson**

Bring students back to a whole group discussion of the Marcello problem. Call on a couple of pairs to share their work. Choose these pairs in advance by observing their work as you circulated around the room while they were solving the problem. Try to choose an example of a fraction strip model solution and a number line model solution. If the shared student work is understandable and organized with the correct models, post an example of a solution with a number line model and another of a fraction strip model. If not, teacher charts these examples so they can be compared to yesterday's work.

As a class, discuss:

* What represented a whole in Anne Marie’s problem? (a Hershey bar)
* What about in Marcello’s problem? (an ice cream sandwich)
* How many wholes did Anne Marie have? (four)
* How many wholes did Marcello have? (four)
* What fractional part of the four wholes did Anne Marie eat? (1/3)
* What fractional part of the four wholes did Marcello eat? (2/3)
* Let’s look at the visual models from Anne Marie’s problem. In Anne Marie’s problem we were finding a unit fraction (1/3) of the Hershey bars. In Marcello’s problem we are finding 2/3 of the ice cream sandwiches. How can using a unit fraction help us solve Marcello’s problem?
* Once students understand the unit fraction connection, have them think about yesterday’s statement: “There are three pieces. Each piece is worth 4/3. So 1/3 of 4 wholes is 4/3.”



* Have students make connections between this model and using it to find 1/3 and using it now to find 2/3. (*Students should see that if one-third is 4 pieces, then two-thirds is eight pieces.)* This corresponds to two sections of the number line above. Try to lead them to see the following: *If 1/3 of 4 is 4/3, then 2/3 of 4 is 8/3.* Do NOT emphasize the algorithm yet.
* Have students do these problems to practice their new understanding of multiplying by a non-unit fraction:
* Melodie was watching a You-Tube clip that was six minutes long. The progress bar showed that she had watched 2/5 of the clip. How many minutes had she watched? How many minutes were left?
* Marcus had a video clip that was two minutes shorter than Melodie's. He watched 3/5 of the clip. How many minutes did he watch?
* Who watched for more minutes?

Note: Both answers will be twelve fifths (12/5). This raises the question of what the whole is. Is 12/5 the same amount in both cases? The answer is yes because both refer to parts of one minute and one minute is the whole. See stacked fraction strip model below:

**Next: We have been solving lots of problems taking part of a whole. Let’s look at a problem like this:

* Catherine walks her dog 2/3 of a mile each day. How far does she walk in a week if she walks every day? How would I model this using a fraction strip model? I’ll model this for you (Model solving this.)
* Now try a problem like this on your own: Gustavo practices the flute ¾ of an hour each day. How many hours does he practice every two weeks if he practices each day? This shows two-fifths of six wholes and three-fifths of four wholes both equal 12 fifths of one whole. Imagine moving the pieces around so they shape matches up. I’ll shade the ones I moved orange so you can see them…

**

**Lesson Closing**

Have students complete an exit ticket (index card) as a formative assessment of their understanding thus far. This will be important data to help guide your small-group choices for the next day’s lesson.

On one side, have them answer questions about their understanding, such as: What are you feeling confident about in regards to fraction multiplication? Writing an expression? Solving the equation? Creating visual models? Which visual model do you prefer to work with? Number line or fraction strip?

On the other side, have students complete the following problem: Zander had 5 quarts of azure blue paint. He used 2/3 of the paint. How many total quarts did he use? Write an equation and show a visual model.

#

# Lesson 5: Practicing Part by Whole Multiplication with Fraction Strip and Number Line Models in Context

**Brief Overview of Lesson:** Students will have time to practice what they have learned about part by whole multiplication including working on writing their own contextualized word problems for other students to solve. This also gives the teacher time to pull in students for small group instruction while students are involved in math learning stations. As you plan, consider the variability of learners in your class and make adaptations as necessary.

**Prior Knowledge Required:**

* Students have already been introduced to fractions and understand that a fraction consists of a numerator and denominator a/b
* Multiplication of whole numbers can be represented by the area model and a number line
* Addition and subtraction of fractions with like and unlike denominators (5.NF.A.1, 5.NF.A.2)
* Concept of commutative, associative and distributive properties of multiplication
* Consistent use of visual models (fraction strip, number line, etc.) and equations to solve word problems

**Estimated Time:** 60 minutes

**Resources for Lesson:** Manipulatives (fraction strips, fraction circles) available for students to use, baggies with index cards, math station worksheets, and any materials required for small-group work based on student need

**Content Area/Course:** Mathematics Grade 5

**Lesson 5:** Practicing Part by Whole Multiplication with Fraction Strip and Number Line Models in Context

**Time (minutes):** 60 minutes

**Standard(s)/Unit Goal(s) to be addressed in this lesson:**

5.NF.B.4.a Interpret the product (*a*/*b*) × *q* as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a* × *q* ÷ *b*. *For example, use a visual fraction model to show (2/3)* × *4 = 8/3, and create a story context for this equation. Do the same with (2/3)* × *(4/5) = 8/15.* (In general, (*a*/*b*) × (*c*/*d*) = *ac*/*bd*.)

5.NF.B.6 Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

SMP3 Construct viable arguments and critique the reasoning of others.

SMP4 Model with mathematics.

SMP7 Look for and make use of structure.

**Essential Question addressed in this lesson:**

Why is understanding the unit fraction important?

Why are models important to help solve fraction problems?

**Objectives:** Students will know and be able to:

* Multiply a part by a whole and demonstrate that understanding using visual models and equations
* Targeted Academic Language (review)**:** equal shares, partition, unit fraction, factor, visual model (fraction strip, number line)

**Instructional tips and strategies:**

See teacher notes embedded throughout the lesson.

**Anticipated Student Preconceptions/Misconceptions:**

* As this is a review lesson, students’ preconceptions/misconceptions should have been cleared up to this point.
* Students who still may be struggling should be given small-group intervention support during this lesson.

**Lesson Sequence**

**Lesson Opening**

Say: We are going to be reviewing our work over the past few days and giving our brains time to practice the new information. I am also going to be pulling small groups to do some additional multiplication of fractions work. It is important that we work together to be focused on our tasks for the day. At the end of the day today, everyone should feel comfortable multiplying a whole number by a fraction (unit fraction and non-unit fraction).

Note: If this is students’ first time working with math stations or working independently in “center-like” rotations, time will need to be spent building appropriate routines and modeling acceptable/unacceptable behavior. All of the rotations could be used as seat work if needed. You may also chose a more interactive opener if needed. However, the purpose of a quick and brief opening is to give as much time possible to small-group work and math station work.

**During the Lesson**

Students rotate around to the various math stations outlined below. During this time the teacher also pulls small groups as needed. It is important that all materials are readily available and accessible to students in order to minimize distraction. For this activity, it is suggested that you group by like ability so that students can choose the level of task appropriate.

*Rotation One*

Individual Work: Creation of individual story problem. This task “levels” itself as students more confident with this skill choose to select elements from each category to write more challenging problems. (See accompanying word problem sheet.)

*Rotation Two*

Pair Work: Students work together in groups of two (or three if needed) to complete the task cards. It is suggested that student teams self-select the task cards so that they may choose problems that they feel comfortable completing. You may choose to glue the tasks on index cards and laminate them for future use.

*Rotation Three*

Pair Work: Students work together (preferably in pairs but groups of three would work as well) to create Memory Game cards to be used in another practice lesson in this unit. Student pairs will each be given a baggie of 12 index cards. Each person writes three problems and three corresponding visual models.

Note: You may choose to add additional rotations if time allows. Additional rotations may include:

* Practice problems (although it is suggested that you limit worksheet practice to only one rotation)
* Math games already established in your room as math review
* Extension work may be creating anchor charts or writing a lesson as if students were the teacher

**Closing:**

Talk with students on how the session went. If this is the first time they were involved in station work, the debrief may include discussing student behavior as well. Are there particular aspects they find challenging? Are there aspects that still need review? This is an important part of the conversation as it may guide future small-group work.

If time allows have a student or a couple of students share the word problems they created. These could be copied and distributed or projected using a document camera for the entire class to view.

Word Problem Creation: Lesson 5, Rotation 1

Choose from each of the categories to create your own word problem. Select one item from each category in the box below. Complete the following for your word problem: expression, fraction strip visual model, number line visual model:



**Name \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Date \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

Task Card Recording Sheet: Lesson 5, Rotation 2

Record your answers to the task cards below. Make sure to note the number of the task card. Use space on the back if needed.

|  |  |
| --- | --- |
| Task # \_\_\_\_\_\_\_\_ | Task # \_\_\_\_\_\_\_\_ |
| Task # \_\_\_\_\_\_\_\_ | Task # \_\_\_\_\_\_\_\_ |
| Task # \_\_\_\_\_\_\_\_ | Task # \_\_\_\_\_\_\_\_ |

Task Cards

|  |  |
| --- | --- |
| Task #1 Write a number line model for $\frac{2}{5}$ of 4  | Task #2Peter the parakeet ate $\frac{1}{3}$ of 12 ounces of birdseed. How many ounces did Peter eat? Show how you arrived at your answer. |
| Task #3Write a number line model and fraction strip model for $\frac{2}{5}$ of 9.  | Task #4Write a fraction strip model for $\frac{2}{3}$ of 6. |
| Task #5Kylee had 5 cupcakes left over. She ate $\frac{3}{4}$ of the cupcakes. Draw a number line model of fraction strip model with accompanying equation.  | Task #6The answer is 1 $\frac{1}{3}.$ What is the question? |
| Task #7Write an expression and construct a number line to represent the image below.14=14X5 | Task #8The visual fraction model is below. What is the question and accompanying expression? |

Memory Game: Lesson 5, Rotation 3

Name \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Date \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Your task is to create six memory card pairs (12 cards) for a memory game to be used later in the unit. Each partner teams needs to create 6 sets. One card in each set should contain the word problem. The other card should be either the expression or a visual model. Please vary your choices. An example is given below:



# Lesson 6: Using Fraction Strips, Number Lines, and Area Model to Multiply a Fraction Less than 1 by a Fraction Less than 1

**Brief Overview of Lesson:** Students build on their prior experiences in this unit by multiplying a fraction by a fraction. Prior to this they have learned how to use visual models to multiply a fraction by a whole number. This lesson begins with multiplying a unit fraction by another unit fraction using the two models students have already learned (fraction strip and number line) and add the last visual model for this unit, the area model. After they understand these three models with unit fractions, they will extend this to multiplying other fractions less than one by fractions less than 1. As you plan, consider the variability of learners in your class and make adaptations as necessary.

**Prior Knowledge Required:**

* Multiplying a whole number by a unit and non-unit fraction
* Familiarity with area model for multiplying whole numbers
* Multiplication of whole numbers can be represented by the area model and a number line
* Addition and subtraction of fractions with like and unlike denominators (5.NF.A.1, 5.NF.A.2)
* Concept of commutative, associative and distributive properties of multiplication
* Consistent use of visual models (fraction strip, number line, etc.) and equations to solve word problems

**Estimated Time:** 60 minutes

**Resources for Lesson:**

Manipulatives (fraction strips, fraction circles) available for students to use, math station worksheets, any materials needed for small group work based on student need

National Library of Virtual Manipulates (<http://nlvm.usu.edu/>) for area model, fraction strip, number line, etc.

Annenberg Learner: Models for the Multiplication and Division of Fractions, Session 9, part A <http://www.learner.org/courses/learningmath/number/session9/part_a/index.html>

Grid for recording the process of moving from problem to representation to algorithm

Two-sided counters, tiles or squares

Triangles that can be combined to form a square the same size as the tiles

Fraction models such as squares and bars (*It may also be helpful to have paper with pre-drawn squares that could serve as outlines for fraction models, which could be used with an option of layering fraction bar models.)*

Paper for folding (prepared ahead to a particular size, such as 6" x 4")

Prepared transparencies of the same-size squares or rectangles to develop layers of halves, thirds, fourths, sixths, eighths, and twelfths

C**ontent Area/Course:** Mathematics Grade 5

**Lesson 6:** Using Fraction Strips, Number Lines, and Area Model to Multiply a Fraction Less than 1 by a Fraction Less than 1

**Time (minutes):** 60 minutes

**Standard(s)/Unit Goal(s) to be addressed in this lesson:**

5.NF.B.4.a Interpret the product (*a*/*b*) × *q* as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a* × *q* ÷ *b*. *For example, use a visual fraction model to show (2/3)* × *4 = 8/3, and create a story context for this equation. Do the same with (2/3)* × *(4/5) = 8/15 .*

(In general, (*a*/*b*) × (*c*/*d*) = *ac*/*bd*.)

5.NF.B.5.b Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence *a*/*b* = (*n* × *a*)/(*n* × *b*)to the effect of multiplying *a*/*b* by 1.

5.NF.B.6 Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

SMP4 Model with mathematics.

**Essential Question addressed in this lesson:**

Why is understanding the unit fraction important?

Why are models important to help solve fraction problems?

**Objectives**: Students will know and be able to:

* Multiply a unit fraction by a unit fraction with accompanying visual models – fraction strip, number line, and area model
* Multiply any two fractions less than 1 with accompanying visual models – fraction strip, number line, and area model
* Develop an understanding for showing fraction multiplication with an area model

**Lesson Sequence**

**Lesson Opening**

Introduce the lesson by saying: Who can tell me what two visual fraction models we have been working with? (fraction strip and number line). The last model we will be working on is an area model. You may be familiar with this from when you used it to multiply whole numbers. Remember that fractions are numbers too and what works with whole numbers works with fractions. We will begin by exploring area models by folding paper.

Students develop a concrete representation to show multiplying unit fractions. Lead students in folding a 6" x 6" sheet into an area model. Have paper already cut into squares. Smaller paper is not advised since the resulting partitions may be too small. This design can be used to represent a number of examples.

Folding directions: Holding the sheet horizontally, fold it into three columns:

 

Keeping paper folded into thirds, fold in half one more time vertically, creating six columns.

 

Open the paper and begin folding in the *opposite* direction. Fold the sheet in half so that it has 12 sections, then fold it in half once more to create an area model that shows sixths on one side and fourths on the other.

Have students Turn and Talk to discuss:

* How does this model $\frac{1}{6}$ × $\frac{1}{4}$ ?
* What is the answer to $\frac{1}{6}$ × $\frac{1}{4}$ ?
* What is the resulting equation? ($\frac{1}{6}$ × $\frac{1}{4}= \frac{1}{12}$ )
* What do you notice about the size of the partitions? (Reinforce the fact that the sixths are smaller portions than the fourths, given the same whole —the side length. Students should understand that when we discuss fraction comparisons—the fourths are larger than the sixths—we need to be referring to the same whole. This is why we are using square shapes to start the area model.)



**During the Lesson**

Now have students draw an area model on paper without folding. Demonstrate 1/3 × ¼. Remind students that, when they draw an area model, the whole (1) should be the same on both sides of their square. Provide the sheet at end of lesson with squares for students to use in showing area models. Have students use the first sheet, which has room for using all three models on two problems. They will show the other visual models in the next section of this lesson. Have them copy the model you drew as an example. See diagram below for 1/3 × 1/4:



Now ask students to try to solve another problem without folding. Students may shade one column (1/4) and one row (1/6) in two colors to show the overlap shown below as the answer. In the model below the green (overlap of yellow and blue) is the answer.

 1/6 x ¼ = 1/24



Next, student pairs practice the area model with additional unit fraction problems, such as ½ × ¼ and ½ × 1/3 and 1/5 × 1/3 on paper provided. Provide two different color writing implements for shading purposes. Have students share with a partner to check their work before they move on. Circulate and observe.

Show students an example of how to solve these problems using the fraction strip model and number line model that they previously learned. What is new here is that students are not multiplying by a whole number.

*Teacher Think Aloud* *#1*:

I am going to show multiplying 1/3 × ¼ on a fraction strip model. First, I label my fraction strip with zero on the left and 1 on the right because my fractions are both less than 1.



Then I partition my fraction strip into fourths and shade one fourth.

**

Next, I partition each fourth into thirds and shade one third of one fourth. This double shaded area is my answer (shown in purple below.) My fraction strip shows 1 part when a whole is partitioned into 12ths. The answer is 1/12.



Now I will show a number line model for 1/3 × ¼. First I will draw a number line and label zero and 1.



I will partition my whole into fourth and show one fourth.



Now I will partition each fourth into thirds and show one of those. This double-shaded area is my answer (shown below in yellow.) My fraction strip shows 1 part when a whole is partitioned into 12ths. The answer is 1/12.



On the same sheet they have been using, have students show these two problems: 1/3 × ¼ and 1/6 × ¼. Circulate to assist.

*Teacher Think Aloud #2*

Building on the previous example showing 1/3 × ¼, think aloud:

I can use what I know about shading to solve other fraction problems. Let me think about 1/3 × ¾. My denominators tell me that my whole is divided the same way as it was in the other problem. Three partitions on one side and four on the other. I’m still going to shade one of the three partitions to show 1/3. Next I’m going to shade 3 of the four partitions to show ¾. Notice how this is like shading ¼ three times. My overlapping shaded area is still my numerator and the number of parts in one whole is still my denominator. It is important to know that my denominator is the number of parts in one entire whole, not just those that are shaded.

**

 ¼ ¼ ¼ ¼

Have students practice with several examples, such as 3/6 × ¾ , ¾ × 5/6, 2/4 × 3/6, etc. After they complete these additional area model problems, model how to show these non-unit fraction multiplication problems with a fraction strip model and a number line model, similar to Think Aloud above.

*Teacher Think Aloud #3*

I am going to show multiplying 1/3 × 3/4 on a fraction strip model. First I label my fraction strip with zero on the left and 1 on the right because my fractions are both less than 1.



Then, I partition my fraction strip into fourths and shade three fourths this time.



Next, I partition each fourth into thirds and shade one third of each of three fourths. This is because I am multiplying by three fourths this time so I need to shade a third of each of the three fourths I am working with. The double shaded part is my answer, which is three out of 12 partitions or 3/12. My fraction strip shows 3 parts when a whole is partitioned into 12ths. The answer is 3/12.



Oh, I can also show that three twelfths is equal to one fourth by rearranging the shaded pieces.



Now I will show a number line model for 1/3 × 3/4. First I will draw a number line and label zero and 1.



I will partition my whole into fourth and show three fourths.



Next, I partition each fourth into thirds and show one third of each of three fourths. This is because I am multiplying by three fourths this time so I need to show a third of each of the three fourths I am working with. The double shaded part is my answer, which is three out of 12 partitions or 3/12. My fraction strip shows three parts when a whole is partitioned into 12ths. The answer is 3/12.



Oh, again I can also show that three twelfths is equal to one fourth by rearranging the shaded pieces.



**Closing**

Students complete the following Exit Ticket: Explain why 1/2 × 3/4 is less than 1. Draw a visual model to help justify your answer

Multiplying Fractions Less Than 1 Using Area Model, Number Line Model, and Fraction Strip Model

Multiplying Fractions Less than 1 Using Area Model

# Lesson 7: Fraction Strips, Number Lines, and Area Models to Multiply a Fraction Less than 1 by a Fraction Less than 1

**Brief Overview of Lesson:** Students will have time to practice what they have learned about part by part multiplication involving fractions less than one including working with the area model and playing the memory game created in lesson five. This also gives the teacher time to pull in struggling students for small group intervention while students are involved in math learning stations. As you plan, consider the variability of learners in your class and make adaptations as necessary.

**Prior Knowledge Required:**

* Concept of commutative, associative, and distributive properties of multiplication
* Consistent use of visual models (fraction strip, number line, area model) and equations to solve word problems

**Estimated Time:** 60 minutes

**Resources for Lesson:**

Manipulatives (fraction strips, fraction circles) available for students to use, baggies with index cards for memory game, math station worksheets, and any materials required for small-group work based on student need

**Content Area/Course:** Mathematics Grade 5

**Lesson 7** Practicing Fraction Strips, Number Lines, and Area Model to Multiply a Fraction Less than 1 by a Fraction Less than 1

**Time (minutes):** 60

**Standard(s)/Unit Goal(s) to be addressed in this lesson:**

5.NF.B.4.a Interpret the product (*a*/*b*) × *q* as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a* × *q* ÷ *b*. *For example, use a visual fraction model to show (2/3)* × *4 = 8/3, and create a story context for this equation. Do the same with (2/3)* × *(4/5) = 8/15.* (In general, (*a*/*b*) × (*c*/*d*) = *ac*/*bd*.)

5.NF.B.5.b Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence *a*/*b* = (*n* × *a*)/(*n* × *b*)to the effect of multiplying *a*/*b* by 1.

5.NF.B.6 Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem

SMP3 Construct viable arguments and critique the reasoning of others.

SMP4 Model with mathematics.

SMP7 Look for and make use of structure.

**Essential Question addressed in this lesson:**

Why is understanding the unit fraction important?

Why are models important to help solve fraction problems?

**Objectives:** Students will know and be able to:

* Multiply a part by a part (fractions less than one) and demonstrate that understanding using visual models and equations.
* Targeted Academic Language (review)**:** equal shares, partition, unit fraction, factor, visual model (fraction strip, number line, area model).

**Anticipated Student Preconceptions/Misconceptions:**

* Students who still may be struggling should be given small group intervention support during this lesson.

**Lesson Sequence**

**Lesson Opening:**

Talk to students, saying:

We are going to be reviewing our work over the past few days and giving our brains time to practice the new information, just as we did a couple of days ago. Why do you think it might be important to practice before we move on? (elicit responses from class) I am also going to be pulling small groups to do some additional multiplication of fractions work. It is important that we work together to be focused on our tasks for the day. At the end of the day today, everyone should feel comfortable multiplying a whole number by a fraction (unit fraction and non-unit fraction)

Note: If this is students second time working with math stations or working independently in rotations, they will need time to reflect on how rotations went during Lesson 5 and build appropriate routines and model acceptable/unacceptable behavior. You may also choose a more interactive opener if needed. However, the purpose of a quick and brief opening is to give as much time possible to small group work and math station work.

**During the Lesson**

Students rotate around the various math stations (outlined below). During this time, you can pull small groups as needed. All materials should be readily available and accessible to students to minimize distraction. For this activity, grouping by like ability enables students to choose the level of task.

Note: Notice that this set of rotations is parallel to Lesson 5. This supports struggling students and English Language Learners. The problems have changed but the overall structure is the same.

*Rotation One*

Individual work: Creating individual story problems. This task “levels” itself as students who are more confident with this skill will choose to select elements from each category to write more-challenging problems (see accompanying word problem sheet). If time allows, students can solve each other’s problems as well.

*Rotation Two*

Pair work: Students work in groups of two (or three if needed) to complete the task cards. Student teams can self-select the task cards so that they may choose problems that they feel comfortable completing. You may choose to glue the tasks on index cards and laminate them for future use.

*Rotation Three*

Pair work: Students use the Memory Cards from Lesson 5 to play Memory. There should be enough cards to divide the deck into two or three separate games. Give students 15 to 20 minutes for the game, with the winner being the person with the most matches. A reporting sheet is included to add an additional layer of student accountability.

Note: Add additional rotations if time allows. Additional rotations may include:

* Practice problems (although it is suggested that you limit worksheet practice to only one rotation)
* Math games already established in your room as math review
* Extension work may be creating anchor charts or writing a lesson as if students were the teacher

**Closing:**

Talk with students about how the session went. Debrief “station work”: Are there particular aspects they find challenging? Aspects that still need review?

If time allows have a student or a pair share the word problems they created. These could be copied and distributed or projected using a document camera for the entire class to view.

Word Problem Creation: Lesson 7, Rotation 1

Choose from each of the categories below to create your own word problem. Select one item from each category in the box below. Complete the following for your word problem: expression, fraction strip visual model, number line visual model

|  |
| --- |
| Word Problem |
| Fraction Strip Visual Model |
| Number Line Visual Model |
| Area Model |
| Expression |

Task Card Recording Sheet: Lesson 7, Rotation 2

Student Name \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_Date \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Record your answers to the task cards below. Make sure to note the number of the task card. Use space on the back if needed.

|  |  |
| --- | --- |
| Task # \_\_\_\_\_\_\_\_ | Task # \_\_\_\_\_\_\_\_ |
| Task # \_\_\_\_\_\_\_\_ | Task # \_\_\_\_\_\_\_\_ |
| Task # \_\_\_\_\_\_\_\_ | Task # \_\_\_\_\_\_\_\_ |

Task Cards: Lesson 7

|  |  |
| --- | --- |
| Task # 1Draw a fraction strip model for $\frac{2}{5}$ of $\frac{3}{4}$. | Task # 2Alicia had $\frac{3}{8}$ of a watermelon left over from the picnic. She ate $\frac{3}{4}$ of the leftover melon for a snack. How much of the whole watermelon did she eat for a snack? Draw an accompanying visual model and equation. |
| Task # 3Draw an area model for $\frac{2}{5}$ of $\frac{3}{4}$. | Task # 4Draw a number line model for $\frac{2}{3}$ of $\frac{1}{3}$. |
| Task # 5Draw an area model for $\frac{1}{6}$ of $\frac{1}{5}$. | Task # 6Draw a number line model and a fraction strip model for $\frac{1}{6}$ of $\frac{1}{5}$. |
| Task # 7Makala was $\frac{3}{4}$ of a mile away from the baseball field. She ran $\frac{2}{3}$ of the way, then skipped the rest. How far did she run? | Task # 8The visual fraction model is below. What is the question and accompanying expression?area model showing 2/3 x 1/4 |

Memory Game: Lesson 7, Rotation 3

Student Name \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Date \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

You will be playing Memory with three other members of your group, using the game cards you made a couple of days ago. Set the timer for 15 minutes. The person with the most matches at the end of that time wins. Record three of your best matches below.

****

# Lesson 8: Using Fraction Strips, Number Lines, and Area Model to Multiply Fractions, Including Those Greater than 1

**Brief Overview of Lesson:** Students build on their prior experiences in Lesson 6 by exploring the area model for multiplying fractions greater than 1. This lesson begins with a folding activity in which students see how the area model can be used to show multiplication of fractions greater than one. They then connect this work to models they previously learned: fraction strip and number line. As you plan, consider the variability of learners in your class and make adaptations as necessary.

**Prior Knowledge Required:**

* Understanding of area model for multiplying fractions less than 1.
* Concept of commutative, associative, and distributive properties of multiplication
* Consistent use of visual models (fraction strip, number line, area model) and equations to solve word problems

**Estimated Time:** 60 minutes

**Resources for Lesson**

Manipulatives (fraction strips, fraction circles) available for students to use, baggies with index cards for memory game, math station worksheets, any materials needed for small group work based on student need

**Content Area/Course:** Mathematics Grade 5

**Lesson 8:** Using Fraction Strips, Number Lines, and Area Model to Multiply Fractions Including Those Greater than 1

**Time (minutes):** 60

**Standard(s)/Unit Goal(s) to be addressed in this lesson:**

5.NF.B.4.a. Interpret the product (*a*/*b*) × *q* as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a* × *q* ÷ *b*. *For example, use a visual fraction model to show (2/3)* × *4 = 8/3, and create a story context for this equation. Do the same with (2/3)* × *(4/5) = 8/15 .*

(In general, (*a*/*b*) × (*c*/*d*) = *ac*/*bd*.)

5.NF.B.5.b Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence *a*/*b* = (*n* × *a*)/(*n* × *b*)to the effect of multiplying *a*/*b* by 1.

5.NF.B.6 Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

SMP4 Model with mathematics.

**Essential Question addressed in this lesson:**

Why is understanding the unit fraction important?

Why are models important to help solve fraction problems?

**Objectives**: Students will know and be able to:

* Multiply fractions greater than 1 with accompanying visual models—fraction strip, number line, and area model
* Deepen their understanding for showing fraction multiplication with an area model

**Lesson Sequence**

**Lesson Opening:**

Introduce the lesson by saying: We already practiced using the area model for multiplying fractions. The factors we used in those fraction problems were each less than one whole. Who remembers what a factor is? (Factors are the numbers that are multiplied in a multiplication problem.) We also solved these problems with a number line model and a fraction strip model. What did we notice about the answers when we solved the same problem using three different models? (The answer is always the same. The model shows different ways to get the answer and to represent the answer.) (This entire lesson focused on SMP4 Model with mathematics.)

Today we will follow a similar process to learn how to multiply fractions greater than 1. Who can name a fraction greater than 1? (Solicit several answers and write them on the board. Discuss that they are greater than 1 because the numerator shows how many parts there are and it is larger than the denominator, which shows the number of parts in a whole. Examples are 5/3, 7/2, 4/2, 4/3, 4/1.)

When we folded paper in the previous lesson, the entire paper represented one whole. Now we will fold a paper and it will represent more than one whole.

Students will develop a concrete representation to show multiplying unit fractions. Lead them in folding a 8" x 8" piece of paper into an area model. This design can be used to represent a number of examples. You should have paper already cut into squares. Smaller paper is not advised since the resulting partitions may be too small. Help students to fold as follows:

Holding the sheet horizontally, fold it in half and half again. This creates four equal squares. Each one will represent one whole so the paper represents four wholes.

Label 0, 1, and 2 as shown. Have students highlight one whole so they do not lose sight of what 1 whole is.

Tell students: We are going to model multiplying ¾ X 5/3. This means that one whole is partitioned into four on one side and three on the other side.

Draw a model of ¼ × 1/3 if necessary to reinforce this. Explain: Because we are partitioning *one* whole into four parts, we fold the paper in half so one whole is on one side and then fold this into four parts. When we unfold, the paper will show two wholes across the bottom, each of which is partitioned into four parts. A total of eight parts will be across the bottom because there are two wholes. (See diagram at right.)

Next, students fold the paper in half in the other direction, then fold this into thirds. This results in each whole being partitioned into fourths on one side and thirds on the other. Students may label the fourths across the bottom (1/4, 2/4, 3/4, 4/4, 5/4, 6/4, 7/4, 8/4) and the thirds up the side (1/3, 2/3, 3/3, 4/3, 5/3, 6/3).

Ask students: How many parts make one whole? Turn and Talk to a partner about this question.

In a whole-class discussion, solicit the answer (12 parts make one whole) and discuss so all understand.

Now ask students to shade ¾. Explain that they need to shade ¾ all the way from top to bottom of the paper because our other fraction (5/3) is greater than 1, so the ¾ shading goes higher than the one whole.

Then have students shade 5/3. They should go across in their shading to the edge. The overlapped shading is their answer. The overlap is 15 parts; because it has been established that one whole is 12 parts, so the answer is 15/12.

Alternatively, you could ask students to shade the area bounded by the ¾ line going up and the 5/3 line going across. This results in only the 12/15 being shaded:

Think Pair Share:

Give students a few minutes to think and write about the following questions:

* How many wholes are on the paper we folded?
* How many parts make up one whole?
* We modeled the answer to 3/4 × 5/3. How are these numbers shown in the shading we did?
* Draw a diagram showing the same thing you showed by folding. (Provide the sheet below for these drawings.)

Say: Turn to a partner and share your answers to and thoughts about the questions. Share as a whole group.

**During the Lesson**

Now students have drawn one area model on paper without folding. Have students practice with a few more problems.

* 4/3 × 2/3 4/3 × 3/2 6/5 × 3/2

Have students share their work with the whole class. Bring examples that show that a fraction greater than 1 multiplied by another fraction greater than one (e.g., 4/3 × 3/2) follows the same process as practiced earlier. Ask students what 4/3 × 3/3 would look like. Ask: Can someone draw this on the board or on paper and share with a document camera? Discuss why this equals 4/3 (because 3/3 = 1 and anything multiplied by 1 equals itself).

Show students an example of how to solve these problems using the fraction strip model and number line model that they previously learned. What is new is that students are not multiplying by a whole number.

*Teacher Think Aloud*

Say: I am going to show multiplying 3/4 × 5/3 on a fraction strip model. First, I label my fraction strip with zero on the left and 2 on the right because one of my fractions is more than 1. Then I partition each whole on my fraction strip into thirds and shade five of them.



Next, I partition each third into fourths and shade three fourths of each shaded part. This double shaded area is my answer, shown in green below. My fraction strip shows that each whole is partitioned into 12ths, and 15 of the parts are double shaded (shaded green). The answer is 15/12.



Now I will show a number line model for 3/4 X 5/3. First I draw a number line and label zero, 1and 2 because I have a fraction greater than 1. My number line must be longer than the largest fraction I am multiplying. Then I partition each whole on my number line into thirds and shade five of them.



Next I partition each third into fourths and shade three fourths of each shaded part. This double shaded area is my answer, shown in green below. My number line shows that each whole is partitioned into 12ths. 15 of the parts are double shaded (shaded green). The answer is 15/12.

**Closing**

Have students show their earlier problems (4/3 × 2/3, 4/3 × 3/2, and 6/5 × 3/2) with a fraction strip model and a number line model on the same sheet they have been using. Circulate and assist.

Non-unit Fraction X Non-unit Fraction

with 3 Visual Models

# Lesson 9: Practicing Fraction Multiplication with Fractions Greater than One with All Visual Models

**Brief Overview of Lesson:** Students will practice multiplying fractions greater than one by fractions greater or less than one with accompanying visual models (fraction strip, number line, area model) and equations. As you plan, consider the variability of learners in your class and make adaptations as necessary.

**Prior Knowledge Required:**

* Multiplication of a fraction greater than one by a fraction greater or less than one
* Multiplication of whole numbers can be represented by the area model and a number line.
* Addition and subtraction of fractions with like and unlike denominators (5.NF.A.1, 5.NF.A.2)
* Concept of mixed number equal to a fraction for example 5/4 = 1 1/4
* Concept of commutative, associative, and distributive properties of multiplication
* Consistent use of visual models (fraction strip, number line, area model) and equations to solve word problems

**Estimated Time:** 60 minutes

**Resources for Lesson**

Manipulatives (fraction strips, fraction circles) available for students to use, area model paper from Lessons 6 and 8, chart paper, markers (scented markers may add additional “fun” factor”), and rulers

**Content Area/Course:** Mathematics Grade 5

**Lesson 9** Practicing Fraction Multiplication with Fractions Greater than One with Visual Models

**Time (minutes):** 60

**Standard(s)/Unit Goal(s) to be addressed in this lesson:**

5.NF.B.4.a Interpret the product (*a*/*b*) × *q* as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a* × *q* ÷ *b*. *For example, use a visual fraction model to show (2/3)* × *4 = 8/3, and create a story context for this equation. Do the same with (2/3)* × *(4/5) = 8/15 .*

(In general, (*a*/*b*) × (*c*/*d*) = *ac*/*bd*.)

5.NF.B.5.b Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence *a*/*b* = (*n* × *a*)/(*n* × *b*)to the effect of multiplying *a*/*b* by 1.

5.NF.B.6 Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem

SMP3 Construct viable arguments and critique the reasoning of others.

SMP4 Model with mathematics.

SMP7 Look for and make use of structure.

**Essential Question addressed in this lesson:**

Why are models important to help solve fraction problems?

**Objective:**

* + Students will know and be able to multiply a fraction greater than one by a fraction greater or less than one.

**Anticipated Student Preconceptions/Misconceptions**

* Students who still may be struggling should be given small group intervention support during this lesson.

**Lesson Sequence**

**Lesson Opening**

The opening should be short and brief to allow most time for the day’s activities. Have students examine a page of a textbook (or something else that lays out how to solve a problem or illustrates the multiple steps involved in solving a problem).

Have students Turn and Talk about what they notice. Answers may include: clearly laid out, crisp images/diagrams, consistent layout, outlines each step of solving the problem, and appropriate labels.

Tell students that they are going to be creating “textbook” anchor charts for various problems today as a review and to prepare for the next day’s lesson. Making the diagrams are clear and easily legible is important. Outline each step. You could refer to an anchor chart that’s posted from a previous lesson. Point out how it is clearly visible.

**During the Lesson**

Pose the problems below to students. Give groups chart paper with the problem already written on it and labeled with a designated model. Groups should not be larger than four. Students work out their answers in “textbook” format on chart paper. Thin line markers and rulers would be helpful for groups to improve visibility and neatness. Reinforce that you want them to make their thinking visible and detail out each step.

|  |  |  |  |
| --- | --- | --- | --- |
| ***Problem*** | Rafael was watching a You-Tube clip that he and his friends made the night before. The video was 5 $\frac{1}{2}$ minutes long and he has watched 2/3 of it. How many minutes has he watched? | Cyrinah is designing a large wall mural for an anti-bullying project at school. She needs to know the area of the wall she has been given. It is 3 $\frac{2}{3}$ yards long by 2 $\frac{1}{4}$ yards wide. What is the area? | Oscar is making his outrageous oatmeal cookies. For each batch he needs 2 $\frac{1}{3}$ cups of raisins. He is going to make 1 $\frac{4}{5}$ batches in order to have enough for everyone. How many cups of raisins does Oscar need? |
| ***Problem*** | Carlos measured his Game Boy screen and found that it was 3 $\frac{1}{3}$ inches long by 3 $\frac{1}{5}$ inches wide. What is the area of his screen? | Emilia bought $\frac{4}{3}$ of a pound of candy at the Willy Wonka Factory. She ate $\frac{5}{7}$ of what she bought. How many pounds did she eat? | Chan wants to drink more water each week. He has set a goal of drinking 3 $\frac{1}{2}$ gallons of water during the week. He reaches $\frac{2}{3}$ of his goal. How many gallons did he drink? |
| ***Visual Models*** | * Group A/D = number line
* Group B/E = area model
* Group C/F = fraction strip
* Everyone = equation
 | * Group A/D = fraction strip
* Group B/E = number line
* Group C /F= area model
* Everyone = equation
 | * Group A/D = area model
* Group B/E = fraction strip
* Group C/F = number line
* Everyone = equation
 |

Note: Modifications of this activity could include using a document camera or transparency sheets. Circulate and work with groups or work more closely with a group of students that is struggling.

**Closing**

Leave time for this Lesson Closing. If additional time is needed, this lesson can be split into two days.

Gather all students together and review the work completed. Verify the accuracy of the visual models, and continue to make connections among the models. You may give students time to look at each other’s work. Have them carry a clipboard to note any questions or wonderings.

Save all of the charts used in this lesson as part of the gallery walk tomorrow.

Discuss with students which visual models were easier to construct. Which were more of a struggle?

# Lesson 10: Developing the Algorithm

**Brief Overview of Lesson:** The focus is for students to explore, discover, and use the multiplication of fractions algorithm (in general, (*a*/*b*) × (*c*/*d*) = *ac*/*bd)*. Students draw upon their prior learning of algorithms for whole number operations and addition and subtraction of fractions as well as their conceptual understanding of fractions to gain an understanding of the multiplication of fractions algorithm and its efficiency for solving problems.As you plan, consider the variability of learners in your class and make adaptations as necessary.

**Prior Knowledge Required:**

* Addition and subtraction of fractions with like and unlike denominators (5.NF.A.1, 5.NF.A.2)
* Concept of a mixed number equal to a fraction for example 5/4 = 1 1/4
* Concept of commutative, associative, and distributive properties of multiplication
* Consistent use of visual models (fraction strip, number line, area model) and equations to solve word problems

**Estimated Time:** 60 minutes

**Resources for Lesson**

Chart paper, markers (scented markers may add additional “fun” factor”)

**Content Area/Course:** Mathematics Grade 5

**Lesson 10:** Developing the Algorithm

**Time (minutes):** 60

**Standard(s)/Unit Goal(s) to be addressed in this lesson:**

5.NF.B.4.a. Interpret the product (*a*/*b*) × *q* as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a* × *q* ÷ *b*. *For example, use a visual fraction model to show (2/3)* × *4 = 8/3, and create a story context for this equation. Do the same with (2/3)* × *(4/5) = 8/15 .*

(In general, (*a*/*b*) × (*c*/*d*) = *ac*/*bd*.)

5.NF.B.6 Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem

SMP4 Model with mathematics.

SMP7 Look for and make use of structure.

**Essential Question addressed in this lesson:**

Why are models important to help solve fraction problems?

**Objective:**

* Students will know and be able to discover and use the multiplication of fractions algorithm.

**Anticipated Student Preconceptions/Misconceptions**

* Students may think that when they multiply two mixed numbers using the algorithm, they only need to multiply the two whole numbers and the two fractions of the two mixed numbers in order to find the product (e.g., 2 ½ x 3 1/3= 2 x 3 = 6 and ½ x 1/3 = 1/6, yielding a final answer of 6 1/6).

**Lesson Sequence**

**Lesson Opening**

Have students Turn and Talk with a partner to brainstorm the algorithms that they know. Answers may include:

* the addition of whole numbers
* partial products
* addition of fractions with unlike denominators
* subtraction of fractions with like denominators
* long division of whole numbers

Ask: How we have solved fraction multiplication thus far in this unit? (with visual models) If we are able to solve the problems with a visual model, why have an algorithm? Why is an algorithm important?

Try to get to the key understanding that algorithms often help us compute with greater speed and accuracy. Think of the multiplication expression 12356.7 × 456.7. Using a visual model or even partial products would be time consuming and leave open many chances for error. Following the steps of an algorithm is helpful once we have the conceptual understanding to guide our thinking.

Say: Today we are going to be connecting our learning and visual models to a multiplication of fraction algorithm.

**During the Lesson**

Have a gallery walk of the work done in the unit. A gallery walk is a learning walk, during which students quietly observe the work posted, often with a specific objective or purpose.

In this lesson, the purpose is to have students “derive” the multiplication algorithm. Set this purpose with students by saying: As you walk around the room, jot down your thinking and any patterns you might see. Remember our discussion about algorithms. We are looking to derive the multiplication of fractions algorithm. What steps can I take for example to multiply $\frac{2}{3}$ × $\frac{3}{4}$ or $\frac{8}{9}$ × $\frac{1}{3}$ ?

Put these two problems on the board as anchor problems. Say: As you walk around the room be thinking about these two problems. We are going to come back to the whole group in 10 minutes. If you think you have discovered the algorithm, try it out with a few of the problems you see. Does your answer match the visual models?

Give students a clipboard and pencil and have them begin. Walk around with students to informally assess their thinking.

Note: If you are worried that there are students who won’t see the pattern, shadow them as they walk around and probe their thinking, using guiding questions such as, what patterns do you see? How might one get that denominator without drawing pictures? Or that numerator?Resist the urge to give them the answer or directly point out the algorithm.

After about 10 minutes gather students back together. Have them share their thinking. Use two problems on the board as discussion points. You might also focus their attention on a particular chart from the wall. If students do not readily understand the algorithm, refer back to the anchor charts and make a list of the expressions with equations. Once students have realized the algorithm, ask them if they can generalize it for any fraction. You are working to lead them to (*a*/*b*) × (*c*/*d*) = *ac*/*bd*.

Note: This may present a challenge for students who are not used to working with variables. Do not get stuck here -- at this point you might give them a few quick practice problems on mini white boards if they took substantial time to generate the algorithm. If they readily derived the algorithm, move on to mixed numbers and hold the practice for the next portion of the lesson.

Post the following mixed number expression on the board: 2 $\frac{3}{4}$ × 1 $\frac{2}{3}$ . Have students Think-Pair-Share on how to solve this. Students may come up with the idea of partial products (2 + $\frac{3}{4}$ ) × (1 + $\frac{2}{3}$ ). Explore this option with students by computing (2 × 1) + (2 × $\frac{2}{3}$ ) + ( $\frac{3}{4}$ × 1 ) + ( $\frac{3}{4}$ × $\frac{2}{3}$ ). Discuss how this works but is also time consuming. If no one comes up with the idea of changing the mixed number into an improper fraction, lead the discussion in that direction by asking if there might be a different way of expressing a mixed number that would be helpful. At this point, students should readily come up with the idea of changing 2 $\frac{3}{4}$ × 1 $\frac{2}{3}$ to $\frac{11}{4}$ × $\frac{5}{3}$. Students are likely to grasp the algorithm. Complete enough examples as a class so that you feel comfortable with the majority of the students practicing on their own. Have students complete the Multiplication Algorithm worksheet independently.

Note: Students should complete the worksheet independently, since majority of the work in this unit involves partners or groups, giving you a chance to assess student understanding in a more formal fashion.

**Closing:**

Have students discuss the following question with a partner and then share out: Why do you think we waited until the end of this until to discuss the algorithm? Students might discuss how easy the algorithm is, but had they known earlier they could just multiply across, they would not have built such a strong understanding of fractions.

Developing the Multiplication Algorithm (handout)

Complete the problems below. For each problem, include an equation. Choose three of the problems to include a visual model.

|  |  |  |
| --- | --- | --- |
| **Problem** | **Visual Model** | **Equation** |
| $\frac{2}{5}$ of a carton of ice cream is left. You can have $\frac{1}{3}$ for dessert. What fraction of the carton will you get? |  |  |
| You are running on the track at school. The track is 1 $\frac{2}{3}$ of a mile long. If you have run $\frac{3}{4}$ of the distance around, what fraction of a mile have you run? |  |  |
| You ran $\frac{3}{7}$ of a mile for 1 week. How many miles did you run after the 1 week? |  |  |
| Dante has $\frac{6}{5}$ bags of oranges. He ate $\frac{5}{ 6}$ of the oranges. How much of the bags did he eat? |  |  |
| Michael weighs 3 $\frac{2}{3}$ times as much as his younger cousin Alfred. Alfred weighs 43 $\frac{1}{2} $pounds. How many pounds does Michael weigh? |  |  |
| The recipe calls for ¾ of a teaspoon of salt. You are going to use 1/3 of the recipe. How much salt do you need? |  |  |

# Lesson 11: Fraction Multiplication and Scaling

**Brief Overview of Lesson:** This lesson serves as an introduction to scaling, which will be further explored as students work with ratios and proportional reasoning. Students reason abstractly by connecting to their understanding of whole-number multiplication as they explore what happens to the product depending on the relative size of the factors. As you plan, consider the variability of learners in your class and make adaptations as necessary.

**Prior Knowledge Required:**

* Concept of mixed number equal to a fraction for example 5/4 = 1 1/4
* Concept of commutative, associative, and distributive properties of multiplication
* Consistent use of visual models (fraction strip, number line, area model) and equations to solve word problems

**Estimated Time:** 60 minutes

**Resources for Lesson**

Noticing station sheets, chart paper prepared in advance matching noticing station sheets, sentence frame strips

**Content Area/Course:** Mathematics Grade 5

**Lesson 11 Multiplication and Scaling**

**Time (minutes):** 60

**Standard(s)/Unit Goal(s) to be addressed in this lesson:**

5.NF.B.5 Interpret multiplication as scaling (resizing), by:

a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. For example, without multiplying tell which number is greater: 225 or ¾ x 225; 11∕50 or 3∕2 x 11∕50?

b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence *a*/*b* = (*n* × *a*)/(*n* × *b*)to the effect of multiplying *a*/*b* by 1.

5.NF.B.6 Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

SMP7 Look for and make use of structure.

**Essential Question addressed in this lesson:**

How is what we understand about multiplication useful when multiplying fractions?

**Objectives:** Students will know and be able to:

* Compare the size of the product to one factor based on the size of the other factor without completing the calculation
* Reason that a given number times a number greater than one has a product greater than the given number
* Reason that a given number times a number less than one has a product smaller than the given number

**Anticipated Student Preconceptions/Misconceptions:**

* Students may still think that multiplying always yields a larger number.

**Lesson Sequence**

**Lesson Opening:**

Tell students: We are going to be constructing a few generalizations in regards to multiplication of fractions. We are going to pull everything we have learned in this unit together as we think about patterns we find regarding factors and products when we multiply.

Pose the problem 3 × 2 = 6. Have students Turn and Talk about what they notice about the size of the factors and the product. After giving students a minute or two to discuss, begin the activity below.

Note: It may be tempting to explore the concept further at this point or lead students to greater understandings. However, the purpose of the following activity is for students to derive the generalizations through guided discovery.

**During the Lesson**

Tell the students that they are going to be visiting four stations with a partner. Each station has a half a sheet of paper with a group of like problems.

Note: Prepare a chart-size version of the station sheets ahead of time to use during the lesson closing. Establish with the students that the equations are correct. This is not about the solutions but rather about discovering the patterns involved in multiplying by fractions greater or less than one.

For each station ask students to think about the following with the partner:

* What do you notice about the size of the factors?
* What do you notice about the size of the product?
* Are fractions involved in any of the equations? If so, what do you notice about the size of the fractions? Greater than one? Less than one?

Tell students to record what they notice and what they think about. They can move at their own pace but reinforce that, if possible, they should draw a conclusion about what they see.

After 20 to 25 minutes, gather the whole group together to share what they noticed and see what generalizations can be made.

Have students bring their “noticing” sheet to a whole group gathering. Post the chart paper on the board and lead a discussion about each station. Discuss the following questions. (A teacher version of Generalizations/Noticings are below).

* What do you notice about the size of the factors?
* What do you notice about the size of the product?
* Are fractions involved in any of the equations? If so, what do they notice about the size of the fractions? Greater than one? Less than one?

In general students should draw the following conclusions:

When working with whole number × whole number:

* The larger the factors, the larger the product.
* The smaller the factors, the smaller the product.
* The product is larger than both factors.

When working with fractions:

* If a given number is multiplied by a fraction greater than one, the product is greater than the given number.
* If a given number is multiplied by a fraction less than one, the product is less than the given number.

After charting these generalizations, talk about reasoning about the size of the product without doing the calculation:

* Put 34 × 234 up on the board. Ask: What can we reason about the size of the product? (It is going to be greater than 234 or 34.)
* Put 34 × $\frac{1}{2}$ up on the board. Ask: What can we reason about the size of the product? (It is going to be less than 34.) How much smaller? (half as small)
* Put 34 × $\frac{1}{3}$ up on the board. Ask: What can we reason about the size of the product? (It is going to be less than 34.) How much smaller? (third as small)
* Put $\frac{1}{2}$ × $\frac{1}{3}$ up on the board. Ask: What can we reason about the size of the product? (It is going to be less than $\frac{1}{2}$.) How much smaller? (third as small)

Have students work with partners and mini white boards to continue to prove or disprove the generalizations (as time allows).

**Closing**

Draw students back together once they have had time to try to “disprove” the generalizations. Clarify confusion or questions if any have surfaced. See whether any students were able to find a non-example.

Ask students to Turn and Talk with a partner about the following statement: Mrs. Chenowski told her students that when you multiply two numbers together you get a larger number.

Once students have had time to process with a partner, process as an entire group*.* Ask: Why would Mrs. Chenowski say this? It is true for whole numbers? Is this statement true? (No, it is not true for fractions.)

Present the following sentence frames:

When a given number is multiplied by a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ the product is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ the given number.

When a given number is multiplied by a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ the product is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ the given number.

When a given number is multiplied by a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ the product is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ the given number.

Have them complete the frames on an exit card with a partner.

Using their work and the generalizations students will arrive at the following:

* When a given number is multiplied by a *whole number* the product is larger than both the factors.
* When a given number is multiplied by a *fraction greater than a one* the product is *larger* than the given number.
* When a given number is multiplied by a *fraction smaller than one* the product is smaller than the given number.

Multiplication as Scaling: Student Page

**Directions:** Visit each of the four stations. At each of the stations, take a paper and examine the set of given equations. Think about the following:

* What do you notice about the size of the factors? Make sure to note if the size of one of the factors is always greater than or less than one.
* What do you notice about the size of the product in relation to the factors?

Write down your team’s noticing and any conclusions you are drawing.

|  |  |  |
| --- | --- | --- |
| **Station One** | **Station Two** | **Station Three** |
| **Noticings:** | **Noticings:** | **Noticings:** |
| **Conclusions:** | **Conclusions:** | **Conclusions:** |

Lesson 11: Stations

|  |  |  |
| --- | --- | --- |
| **Station One****3 × 6 = 18****3 × 5 = 15****3 × 4 = 12****and****3 × 6 = 18****3 × 7 = 21****3 × 8 =24** | **Station Two**$\frac{1}{2}$ **× 1** $\frac{1}{2}$ **=** $\frac{3}{4}$ **5 × 1** $\frac{1}{2}$ **= 7** $\frac{1}{2}$$ \frac{1}{2} $ **× 1** $\frac{1}{4}$ **=** $\frac{5}{8}$ **4 ×** $\frac{6}{5}$ **= 4** $\frac{4}{5}$$\frac{1}{2}$ **×** $\frac{9}{8}$ **=** $\frac{9}{16}$ | **Station Two**$\frac{1}{2}$ **×** $\frac{1}{2}$ **=** $\frac{1}{4}$ **3 ×** $\frac{1}{2}$ **= 1** $\frac{1}{2}$$\frac{1}{2}$ **×** $\frac{1}{4}$ **=** $\frac{1}{8}$ **4 ×** $\frac{3}{4}$ **= 3**$\frac{1}{2}$ **×** $\frac{1}{8}$ **=** $\frac{1}{16}$ |

Answer Page (Lesson 11)

|  |  |  |
| --- | --- | --- |
| **Station One** | **Station Two** | **Station Three** |
| **Noticings:*** All whole numbers
* Product are getting larger as one of the factors is getting larger
* Product are getting smaller as one of the factors is getting smaller
 | **Noticings:*** One factor is always greater than one
* Product is greater than the given number (other factor)
 | **Noticings:*** One factor is always less than one
* Product is smaller than the given number (other factor)
 |
| **Conclusions:**When multiplying *whole* numbers, for a given number the product increases as the factor increasesWhen multiplying *whole* numbers, for a given number the product decreases as the factor decreasesProduct is always greater than the given number | **Conclusions:**When multiplying a given number *by a fraction greater than one*, the product is greater than the given number. | **Conclusions:**When multiplying a given number *by a fraction less than one*, the product is less than the given number. |

\* This only applies to positive integers

# Curriculum Embedded Performance Assessments (CEPA)

**Goal:** You have been asked to evaluate the claims made by Brad’s Electronics in regards to his tablet, iBaby. Your group will be evaluating both the area and cost of the new tablet in relation to a recent advertisement. Consumer beware!

**Role:** You are asked to be savvy consumers while using what you have learned about the multiplication of fractions.

**Audience**: Your fellow classmates

**Situation:** The challenge involves dealing with the following scenario: The iTablet is the hottest electronic item on the market. Brad’s Electronics is trying to capitalize on the demand for tablets and is advertising its own iBaby. As a savvy consumer, you are going to take a closer look at their advertising claims. Your job is to answer the questions and decide if Brad’s Electronics is telling the truth with their claim of half the screen area at less than half the cost. You need to convince your fellow classmates with your analysis, using visual models, equations, and written analysis.

**Product/Performance and Purpose:** You will create a visual presentation in order to communicate your team’s findings including calculations, visual models, and analysis of advertising claims.

**Criteria for Success:** Your work will be judged by the CEPA Rubric with the following three categories: visual models, calculations, and communication.

**iBaby**

Half the screen area at less than half the cost!!

Brad’s Electronics

Our tablet is half the screen area of that popular iTablet, but at less than half the cost. The original tablet is $400, but ours is a deal at only $199.

**iTablet… Our iBaby…**

7 $\frac{3}{4}$ inches long, and

5 inches wide $400 $199

CEPA: Fraction Multiplication

Name \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Date \_\_\_\_\_\_\_\_

The iTablet is the hottest item on the electronics market. Brad’s Electronics is trying to capitalize on the demand for tablets and is advertising their very own iBaby. As a savvy consumer, you are going to take a closer look at their advertising claims. Your job is to answer the following questions and decide if Brad’s Electronics is telling the truth or not with their claim of “half the screen area at less than half the cost.”

* Find the area of the iTablet
* The Consumer Protection Group found that the iBaby is half as long and half as wide as the iTablet.
	+ What is the iBaby’s length? Show your work by solving an equation and a fraction strip model.
	+ What is the iBaby’s width? Show your work by solving an equation and a number line model.
	+ What is the iBaby’s area? Show your work by solving an equation and an area model.
* Evaluate the advertisement’s claim, ”half the screen area at less than half the cost,” using the answers to the questions above.
* You will present your findings in a three- to five-minute presentation that includes an explanation of your reasoning, accompanying visual models, and final analysis of Brad’s Electronics’ claim.

CEPA Rubric Fraction Multiplication

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| --- | --- | --- | --- | --- | --- |
| **Presentation** | **4** | **3** | **2** | **1** | **0** |
| **Visual Models**  | All three visual models are present and accurate. | All three visual models are present and two visual models are accurate. | All three visual models are present and one visual model is accurate. | All three visual models are present, but none are accurate. | Did not include visual models.  |
| **Calculations** | All equations correctly model the situations and are solved accurately.  | All equations correctly model the situation, but some are not solved accurately.  | Some equations incorrectly model the situation but are solved accurately.  | Some equations incorrectly model the situation, and some are not solved accurately. | Did not include equations. |
| **Communication** | Connections between the visual models, equations, and verbal description of problem are clearly shown and explained. | Connections between the visual models, equations and verbal description of the problem are clearly shown but explanation is unclear. | Connection between the visual models and equations are limited or incorrect. | Unable to match the visual model and the equation. Shows little understanding of the task. | Did not complete assignment. |

Unit Resources

**Lesson 1**

Chart paper (or other means to display student thinking, such as a smart board or document camera); graph paper

Fraction strips; fraction manipulatives

Markers

Pack of cards (index or other cards paper clipped or bagged together as a “pack”)

**Lesson 2**

Seeing Structure student handout

Chart paper (or other means to display student thinking; graph paper

Fraction strips, fraction manipulatives, fraction circles

Markers

**Lesson 3**

Mini white boards

Chart paper (or other means); graph paper

Fraction strips, fraction manipulatives, fraction circles

Markers and paper

**Lesson 4**

Manipulatives (fraction strips, fraction circles)

Chart paper

Index cards

**Lesson 5**

Manipulatives (fraction strips, fraction circles)

Baggies with index cards

Math station worksheets

**Lesson 6**

Manipulatives (fraction strips, fraction circles)

Math station worksheets

National Library of Virtual Manipulates (<http://nlvm.usu.edu/>) for area model, fraction strip, number line, etc.

Annenberg Learner: Models for the Multiplication and Division of Fractions, Session 9, part A <http://www.learner.org/courses/learningmath/number/session9/part_a/index.html>

Grid for recording the process of moving from problem to representation to algorithm

Two-sided counters, tiles or squares

Triangles that can be combined to form a square the same size as the tiles

Fraction models such as squares and bars

Paper for folding (prepared ahead to a particular size, such as 6" x 4")

Prepared transparencies of the same-size squares or rectangles to develop layers of halves, thirds, fourths, sixths, eighths, and twelfths

**Lesson 7 and 8**

Manipulatives (fraction strips, fraction circles)

Baggies with index cards for Memory game

Math station worksheets

**Lesson 9**

Manipulatives (fraction strips, fraction circles)

Area model paper from Lessons 6 and 8

Chart paper; markers; rulers

**Lesson 10**

Chart paper; markers

**Lesson 11**

Noticing station sheets (after the lesson)

Chart paper prepared in advance, matching noticing station sheets

Sentence frame strips