

MCAS Alternate Assessment

**Resource Guide to the
Massachusetts *Curriculum Frameworks*
for Students with Disabilities**

MATHEMATICS

Fall 2006

Mathematics

<i>STRAND</i>	Broad Topic Learning Standards for...	Broad Topic Learning Standards for...	Broad Topic Learning Standards for...	Broad Topic Learning Standards for...
<i>Number Sense and Operations</i> (Pages 139 - 163)	Number Sense	Operations	Computation and Estimation	
<i>Patterns, Relations, and Algebra</i> (Pages 164 - 183)	Patterns, Relations, and Functions	Algebra	Relationships and Mathematical Models	Variables and Change
<i>Geometry</i> (Pages 184 - 206)	Characteristics of Geometric Shapes	Spatial Relationships/ Coordinate Geometry	Transformation/ Symmetry	Visualization/ Spatial Reasoning/ Geometric Modeling
<i>Measurement</i> (Pages 207 - 218)	Measurement			
<i>Data Analysis, Statistics, and Probability</i> (Pages 219 - 234)	Data and Statistics	Probability		

The **learning standards** in the Mathematics *Curriculum Framework* are numbered and organized using a unique identifier, such as **4.P.2**, that indicates:

First, the grade level:

K = grades PreK – K
 2 = grades 1 – 2
 4 = grades 3 – 4
 6 = grades 5 – 6
 8 = grades 7 – 8
 10 = grades 9 – 10

Second, the abbreviation of the strand:

N = Number Sense and Operations
 P = Patterns, Relations, and Algebra
 G = Geometry
 M = Measurement
 D = Data Analysis, Statistics, and Probability

Third, the number of each learning standard within each strand.

For example, learning standard **4.P.2** is the second learning standard in the *Patterns, Relations, and Algebra* strand at grades 3-4.

Number Sense and Operations

	Number Sense	Operations	Computation and Estimation
PreK – K	Pages 140 – 141	Pages 140, 142	Pages 140, 143
1 – 2	Pages 144 - 145, 147	Pages 144, 146	Pages 144 – 146
3 – 4	Pages 148, 150	Pages 148, 150, 152	Pages 149, 151, 153
5 – 6	Pages 154 – 155, 157	Pages 154 – 157	Pages 155 – 156
7 – 8	Pages 158 – 159, 161	Pages 158 – 159, 161	Pages 159 – 161
9 – 10	No learning standards for this grade span	Pages 162 – 163	Pages 162 – 163

CONTENT Mathematics
STRAND Number Sense and Operations

Grade Level: PreK – K

Concept	Learning Standard as written		Essence of the Standard(s)
Number Sense	K.N.1	Count by ones to at least 20.	<ul style="list-style-type: none"> ◆ Begin to count, group, sort, and match quantities of numbers ◆ Use the “language” of numbers (both verbal and written) ◆ Identify coins ◆ Learn concepts of whole and half
	K.N.2	Match quantities up to at least 10 with numerals and words.	
	K.N.3	Identify positions of objects in sequences (e.g., first, second) up to fifth.	
	K.N.4	Compare sets of up to at least 10 concrete objects using appropriate language (e.g., none, more than, fewer than, same number of, one more than) and order numbers.	
	K.N.5	Understand the concepts of whole and half.	
	K.N.6	Identify U.S. coins by name.	
Operations	K.N.7	Use objects and drawings to model and solve related addition and subtraction problems to ten.	<ul style="list-style-type: none"> ◆ Represent operations of addition and subtraction concretely
Estimation	K.N.8	Estimate the number of objects in a group and verify results.	<ul style="list-style-type: none"> ◆ Use objects and manipulatives to estimate numbers in group ◆ Compare exact answer with estimates

Possible ENTRY POINTS to Learning Standard(s) (and ACCESS SKILLS embedded in standards-based activities)

← Less Complex

More Complex →

	ACCESS SKILLS The student will:	ENTRY POINTS The student will:	The student will:	The student will:
Number Sense	<ul style="list-style-type: none"> ◆ Point to object as requested ◆ Grasp, manipulate, and release coins of various sizes ◆ Hit switch to activate counting or other math program on computer ◆ Activate an electronic device to access text, communicate with others, or participate in an instructional activity ◆ Turn attention toward another person ◆ Organize instructional materials ◆ Take turns appropriately during classroom discussion ◆ Respond to/initiate contacts with others ◆ Apply rules for appropriate classroom behavior ◆ Use appropriate social greetings ◆ Follow simple directions ◆ Identify self and/or others ◆ Initiate or respond to request for joint attention 	<ul style="list-style-type: none"> ◆ Count up to three objects ◆ Recognize numerals and number words ◆ Demonstrate 1:1 correspondence between objects ◆ Distinguish same/different ◆ Recognize and identify money ◆ Sort U.S. coins using 1:1 correspondence ◆ Distinguish a coin from other objects as something of value ◆ Understand the concept of counting whole numbers ◆ Count by ones to 3 ◆ Identify objects as "whole" vs. "not whole" 	<ul style="list-style-type: none"> ◆ Count up to five objects ◆ Identify objects as coins ◆ Match numbers to groups of objects ◆ Count by ones to 5 ◆ Distinguish between more/less, bigger/smaller, shorter/longer, and other simple comparisons of quantity ◆ Manipulate whole objects to make two objects from one (splitting, dividing, sharing, breaking) ◆ Match U.S. coins to their corresponding numerical value 	<ul style="list-style-type: none"> ◆ Count up to ten objects ◆ Identify U.S. coins by name ◆ Match words and numerals (e.g., 1/one, 2/two) up to 5 ◆ Match words and numbers to small quantities of objects (3 to 5) ◆ Count by ones up to 20 ◆ Compare groups of fewer than ten objects and use appropriate language to describe (none, fewer than) ◆ Manipulate objects to make two equal parts of a whole ◆ Demonstrate whole and half using manipulatives and familiar objects <p style="text-align: right;"><i>Continue to address skills and concepts that approach grade-level expectations in this subject</i></p>

Possible ENTRY POINTS to Learning Standard(s) (and ACCESS SKILLS embedded in standards-based activities)

Less Complex		More Complex	
ACCESS SKILLS <u>The student will:</u>	ENTRY POINTS <u>The student will:</u>	The student will:	The student will:
<p>Operations</p> <ul style="list-style-type: none"> ◆ Place a number of manipulatives into a container ◆ Activate an electronic device to access text, communicate with others, or participate in an instructional activity ◆ Turn attention toward another person ◆ Grasp, manipulate, and materials relevant to the instructional activity ◆ Organize instructional materials ◆ Take turns appropriately during classroom discussion ◆ Respond to/initiate contacts with others ◆ Apply rules for appropriate classroom behavior ◆ Use appropriate social greetings ◆ Follow simple directions ◆ Identify self and/or others ◆ Initiate or respond to request for joint attention 	<ul style="list-style-type: none"> ◆ Add (“put together”) and subtract (“take away”) using the numbers 1, 2, and 3 (with manipulatives and objects) 	<ul style="list-style-type: none"> ◆ Manipulate objects in order to compare sets using concepts (more, less, same, enough, some) ◆ Add (“put together”) and subtract (“take away”) using the numbers 1 – 5 	<ul style="list-style-type: none"> ◆ Manipulate objects to solve addition and subtraction problems ◆ Identify and demonstrate the meanings of “add to” (make more) and “subtract from” (make less) <p style="text-align: center;"><i>Continue to address skills and concepts that approach grade-level expectations in this subject</i></p>

Possible ENTRY POINTS to Learning Standard(s) (and ACCESS SKILLS embedded in standards-based activities)

← Less Complex

More Complex →

	ACCESS SKILLS <u>The student will:</u>	ENTRY POINTS <u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Estimation	<ul style="list-style-type: none"> ◆ Follow simple directions to group objects ◆ Match object to object or quantity to quantity ◆ Turn attention toward another person ◆ Activate an electronic device to access text, communicate with others, or participate in an instructional activity ◆ Grasp, release, and manipulate materials relevant to the instructional activity ◆ Organize instructional materials ◆ Take turns appropriately during classroom discussion ◆ Respond to/initiate contacts with others ◆ Apply rules for appropriate classroom behavior ◆ Use appropriate social greetings ◆ Identify self and/or others ◆ Initiate or respond to request for joint attention 	<ul style="list-style-type: none"> ◆ Estimate using language (a lot, a little, some, few, none, more, less) ◆ Estimate number of objects in a group, check by counting objects ◆ Add objects to a group until it matches a given grouping 	<ul style="list-style-type: none"> ◆ Compare groupings of similar objects ◆ When presented with a model, estimate objects in a similar group 	<ul style="list-style-type: none"> ◆ Choose an accurate estimation given two broadly different options ◆ Determine whether different estimations are too large or too small ◆ Estimate the correct amount without counting <p><i>Continue to address skills and concepts that approach grade-level expectations in this subject</i></p>

CONTENT Mathematics
STRAND Number Sense and Operations

Grade Level: 1 – 2

Concept	Learning Standards as written		Essence of the Standard(s)
Number Sense	2.N.1	Name and write (in numerals) whole numbers to 1000, identify the place values of the digits, and order the numbers.	<ul style="list-style-type: none"> ◆ Manipulate numbers by counting, writing, grouping, sorting, comparing, ordering ◆ Move from concrete to abstract representation of numbers ◆ Use numbers correctly in a variety of formats ◆ Recognize that whole is the sum of parts, and that wholes can be separated into parts ◆ Use and compare common fractions ◆ Recognize place values of numbers ◆ Identify U.S. coins and bills and use correct notation to represent amount
	2.N.2	Identify and distinguish among multiple uses of numbers, including cardinal (to tell how many) and ordinal (to tell which one in an ordered list), and numbers as labels and as measurements.	
	2.N.3	Identify and represent common fractions (1/2, 1/3, 1/4) as parts of wholes, parts of groups, and numbers on the number line.	
	2.N.4	Compare whole numbers using terms and symbols, e.g., less than, equal to, greater than (<, =, >).	
	2.N.5	Identify odd and even numbers and determine whether a set of objects has an odd or even number of elements.	
	2.N.6	Identify the value of all U.S. coins, and \$1, \$5, \$10, and \$20 bills. Find the value of a collection of coins and dollar bills and different ways to represent an amount of money up to \$5. Use appropriate notation, e.g., 69¢, \$1.35.	
Operations	2.N.7	Demonstrate an understanding of various meanings of addition and subtraction, e.g., addition as combination (plus, combined with, more); subtraction as comparison (how much less, how much more), equalizing (how many more are needed to make these equal), and separation (how much remaining).	<ul style="list-style-type: none"> ◆ Demonstrate the meanings of addition and subtraction ◆ Identify when to use addition or subtraction ◆ Use inverse relationship correctly for addition and subtraction
	2.N.8	Understand and use the inverse relationship between addition and subtraction (e.g., $8 + 6 = 14$ is equivalent to $14 - 6 = 8$ and is also equivalent to $14 - 8 = 6$) to solve problems and check solutions.	
Computation and Estimation	2.N.9	Know addition facts (addends to ten) and related subtraction facts, and use them to solve problems.	<ul style="list-style-type: none"> ◆ Estimate using numbers, moving from concrete to abstract ◆ Compare an estimate to a calculated answer
	2.N.10	Demonstrate the ability to add and subtract three-digit numbers accurately and efficiently.	

Computation and Estimation (cont.)	2.N.11	Demonstrate in the classroom an understanding of and the ability to use the conventional algorithms for addition (two 3-digit numbers and three 2-digit numbers) and subtraction (two 3-digit numbers).	◆ Learn the properties and conventions of addition and subtraction
	2.N.12	Estimate, calculate, and solve problems involving addition and subtraction of two-digit numbers. Describe difference between estimates and actual calculations.	

Possible ENTRY POINTS to Learning Standard(s)

← **Less Complex**

More Complex →

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Number Sense	<ul style="list-style-type: none"> ◆ Understand whole and half ◆ Manipulate objects to make two objects from one ◆ Identify U.S. coins <p><i>See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Identify numerals ◆ Identify pennies by value ◆ Identify \$1.00 by value ◆ Identify two U.S. coins by name ◆ Group objects by multiples of two (including coins) ◆ Manipulate whole objects to make two, three, or four parts of a whole ◆ Identify place value to tens 	<ul style="list-style-type: none"> ◆ Count, compare, order, name, and write numerals ◆ Identify nickels and dimes by value ◆ Identify \$5.00 and \$10.00 bills by value ◆ Identify all U.S. coins by name and/or value ◆ Group and count numbers by a variety of multiples, for example, 2, 5, and 10 ◆ Identify odd and even numbers ◆ Use numerals to accurately label groups of up to five objects ◆ Identify multiple uses for numbers, e.g., reporting how many, ordering, labeling, measuring, etc. ◆ Compare numbers using the terms <i>greater than</i>, <i>equal to</i>, and <i>less than</i> ◆ Manipulate up to four parts of an object to assemble a whole ◆ Identify and compare parts of a whole (quarters, thirds, halves) and determine relative size of each ($1/2$, $1/3$, $1/4$) using manipulatives ◆ Identify place value to hundreds <p><i>Continue to address skills and concepts that approach grade-level expectations in this subject</i></p>

Possible ENTRY POINTS to Learning Standard(s) – continued

← **Less Complex**

More Complex →

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
<p>Operations</p> <ul style="list-style-type: none"> ◆ Add and subtract one-digit numbers ◆ Manipulate numbers in a given problem to create the inverse (e.g., $1 + 2 = 3 \rightarrow 3 - 1 = 2$) <p><i>See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Add and subtract two-digit numbers ◆ Demonstrate understanding of the inverse relationship between addition and subtraction, using numbers 1 to 5 ◆ Identify common phrases used to indicate the need for addition and subtraction ◆ Indicate the inverse of a problem ◆ Demonstrate understanding of how the inverse operation is used to check a solution 	<ul style="list-style-type: none"> ◆ Add and subtract three-digit numbers ◆ Identify the appropriate mathematical operation in a simple problem situation, specifically “add vs. subtract.” For example, given a problem situation, the student will be able to identify if more or less (a sum or difference) is needed. <i>Do we need more? Do we have enough? Do we need to add? Do we need to “take away” or subtract?</i> ◆ Understand the inverse relationship between addition and subtraction using numbers 1 to 10 <p><i>Continue to address skills and concepts that approach grade-level expectations in this subject</i></p>	
<p>Computation and Estimation</p> <ul style="list-style-type: none"> ◆ Match numerals with up to five objects <p><i>See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Estimate the number of objects in a group and determine whether the correct answer is more, less, or same ◆ Use manipulatives to “put together” and “take away” 	<ul style="list-style-type: none"> ◆ Estimate, calculate, and solve problems involving addition and subtraction of one-digit numbers ◆ Compare estimations with actual calculations ◆ Make estimations of quantities, measurements, and time using benchmarks e.g., estimate how long it takes to complete a familiar activity) ◆ Evaluate the accuracy of an estimation ◆ Add and subtract one-digit numbers in number sentences and use conventional algorithms ◆ Accurately match numbers with up to five objects <p><i>Continue to address skills and concepts that approach grade-level expectations in this subject</i></p>	

Instructional Idea based on Learning Standard 2.N.1

At grades 1 – 2, students group manipulatives in 1s, 10s, and 100s to learn place value as they count the days of school that have passed from September to June.

How all students can participate in this activity

Addressing Learning Standard(s) as written for this grade level	Possible Assessment Strategies and Portfolio Products
<p>Jerry counts the days that have passed by grouping and sorting manipulatives by number and place value. He keeps track of the days by (1) adding to his number line, (2) making journal entries using narratives, charts, and drawings, and (3) marking days on a calendar.</p>	<ul style="list-style-type: none"> • Data chart indicating Jerry's ability to group and sort manipulatives by number and place value • Jerry's journal entries that track days that have passed and grouping numbers by place value • Videotape of Jerry's place value activity using manipulatives with his classmates
Addressing Learning Standard(s) at lower levels of complexity (Entry Points)	Possible Assessment Strategies and Portfolio Products
<p>Selma participates in the daily place value activity by counting from 1 to 10 with her peers and grouping and sorting manipulatives to represent place value of 10.</p>	<ul style="list-style-type: none"> • Data chart indicating Selma's ability to count from 1 to 10 using manipulatives • Videotape of Selma counting from 1 to 10 using manipulatives • Selma's graph of her progress in learning to count
Addressing Access Skills embedded in academic instruction	Possible Assessment Strategies and Portfolio Products
<p>Leo participates in the daily place value activity by following one-step directions to add a manipulative to the total number of objects representing days that have passed.</p>	<ul style="list-style-type: none"> • Field data chart indicating Leo's ability to follow one-step directions • Line or bar graph summarizing the field data chart • Videotape of Leo following one-step directions

CONTENT Mathematics
STRAND Number Sense and Operations

Grade Level: 3 – 4

Concept	Learning Standards as written		Essence of the Standard(s)
Number Sense	4.N.1	Exhibit an understanding of the base ten number system by reading, modeling, writing, and interpreting whole numbers to at least 100,000; demonstrating an understanding of the values of the digits; and comparing and ordering the numbers.	<ul style="list-style-type: none"> ◆ Manipulate larger numbers by counting, writing, grouping, sorting, comparing, ordering ◆ Use a variety of numerical forms and classes ◆ Recognize and use decimals ◆ Understand and compare equivalent forms of decimals and fractions
	4.N.2	Represent, order, and compare large numbers (to at least 100,000) using various forms, including expanded notation, e.g., $853 = 8 \times 100 + 5 \times 10 + 3$.	
	4.N.3	Demonstrate an understanding of fractions as parts of unit wholes, as parts of a collection, and as locations on the number line.	
	4.N.4	Select, use, and explain models to relate common fractions and mixed numbers ($\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{10}$, $\frac{1}{12}$, and $1\frac{1}{2}$), find equivalent fractions, mixed numbers, and decimals, and order fractions.	
	4.N.5	Identify and generate equivalent forms of common decimals and fractions less than one whole (halves, quarters, fifths, and tenths).	
	4.N.6	Exhibit an understanding of the base ten number system by reading, naming, and writing decimals between 0 and 1 up to the hundredths.	
	4.N.7	Recognize classes (in particular, odds, evens; factors or multiples of a given number; and squares) to which a number may belong, and identify the numbers in those classes. Use these in the solution of problems.	
Operations	4.N.8	Select, use and explain various meanings and models of multiplication and division of whole numbers. Understand and use the inverse relationship between the two operations.	<ul style="list-style-type: none"> ◆ Identify and describe the meanings of multiplication and division ◆ Represent multiplication and division problems concretely ◆ Use all operations to solve problem situations related to money ◆ Understand commutative properties of addition and multiplication (order can be reversed)
	4.N.9	Select, use, and explain the commutative, associative, and identity properties of operations on whole numbers in problem situations, e.g., $37 \times 46 = 46 \times 37$, $(5 \times 7) \times 2 = 5 \times (7 \times 2)$.	
	4.N.10	Select and use appropriate operations (addition, subtraction, multiplication, and division) to solve problems, including those involving money.	

Computation and Estimation	4.N.11	Know multiplication facts through 12 X 12 and related division facts. Use these facts to solve related multiplication problems and compute related problems, e.g., 3 X 5 is related to 30 X 50, 300 X 5, and 30 X 500.	<ul style="list-style-type: none"> ◆ Understand properties of multiplication and division ◆ Demonstrate knowledge of multiplication and division facts ◆ Add and subtract ◆ Use estimation in appropriate situations ◆ Use rounding strategies ◆ Estimate monetary amounts, measures, and quantities ◆ Compare estimations with exact answers ◆ Perform operations using fractions
	4.N.12	Add and subtract (up to five-digit numbers) and multiply (up to three digits by two digits) accurately and efficiently.	
	4.N.13	Divide up to a three-digit whole number with a single-digit divisor (with or without remainders) accurately and efficiently. Interpret any remainders.	
	4.N.14	Demonstrate in the classroom an understanding of and the ability to use the conventional algorithms for addition and subtraction (up to five-digit numbers), and multiplication (up to three digits by two digits).	
	4.N.15	Demonstrate in the classroom an understanding of and the ability to use the conventional algorithm for division of up to a three-digit whole number with a single-digit divisor (with or without remainders).	
	4.N.16	Round whole numbers through 100,000 to the nearest 10, 100, 1000, 10,000, and 100,000.	
	4.N.17	Select and use a variety of strategies (e.g., front-end, rounding, and regrouping) to estimate quantities, measures, and the results of whole-number computations up to three-digit whole numbers and amounts of money to \$1000, and to judge the reasonableness of the answer.	
	4.N.18	Use concrete objects and visual models to add and subtract common fractions.	

Possible ENTRY POINTS to Learning Standard(s)

Less Complex

More Complex

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
<p>Number Sense</p> <ul style="list-style-type: none"> ◆ Split groups of objects into two, three, and equal parts ◆ Identify coins needed for common purchases <p><i>See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Count, compare, order, name, and write numbers ◆ Identify parts of a whole ($1/2$, $1/4$, $1/3$, $1/8$) written as a fraction ◆ Divide groups of objects into two, three, or four equal parts ◆ Identify parts of a set of fractions in written format ◆ Demonstrate understanding that money written as dollars and cents is the same as using decimals 	<ul style="list-style-type: none"> ◆ Count, compare, order, name, and write numbers to 100,000 ◆ Identify the value of each digit to 100 ◆ Identify, sort, and count U.S. coins in various combinations ◆ Identify, represent, compare, and order common fractions $1/2$, $1/3$, $1/4$, $1/6$, $1/8$ using manipulatives ◆ Write, read, and name decimals to tenths (0.1, 0.2, 0.3, etc.) ◆ Use knowledge of numbers, (e.g., given various classifications, groupings, relationships) when solving problems ◆ Identify and represent mixed numbers ◆ Use bills and coins to represent an amount of money written as a decimal in various combinations <p><i>Continue to address skills and concepts that approach grade-level expectations in this subject</i></p>	
<p>Operations</p> <ul style="list-style-type: none"> ◆ Identify situations in which the order of events makes a difference and situations in which the order does not make a difference (commutative and non-commutative tasks) ◆ Illustrate the concept of multiplication using groups of objects <p><i>See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Identify the commutative property of addition using number sentences (e.g., $3 + 2 = 2 + 3$) ◆ Use objects and manipulatives to demonstrate the commutative property ◆ Illustrate the concept of division using groups of objects ◆ Identify common phrases used to indicate the need for multiplication and division ◆ Match written problems that demonstrate the commutative properties of addition and subtraction (e.g., $2 + 4 = 6 \rightarrow 4 + 2 = 6$) 	<ul style="list-style-type: none"> ◆ Identify the commutative property of addition and multiplication using number sentences ($3 \times 5 = 5 \times 3$) ◆ Identify the required math operation to solve a simple problem e.g., addition, subtraction, multiplication, or division <p><i>Continue to address skills and concepts that approach grade-level expectations in this subject</i></p>	

Less Complex

More Complex



	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Computation and Estimation	<i>See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials</i>	<ul style="list-style-type: none"> ◆ Add and subtract up to 10 objects ◆ Multiply and divide one-digit numbers in number sentences, and using conventional algorithms ◆ Round to the next highest dollar amount (\$2.57 becomes \$3) when making a purchase using dollar bills ◆ Identify multiplication as repeated addition ◆ Identify division as repeated subtraction ◆ In a given fraction, determine which number is the numerator and which is the denominator ◆ In a given fraction, identify which number is manipulated in order to add and subtract 	<ul style="list-style-type: none"> ◆ Add and subtract two-digit numbers, or a like number of objects ◆ Multiply and divide one-digit numbers in number sentences and use conventional algorithms ◆ Select and use addition, subtraction, multiplication, and division tables to solve problems accurately ◆ Use manipulatives to add and subtract common fractions (1/2, 1/4) ◆ Round whole numbers to the nearest 10, 100 ◆ Round to the next highest tenth and twentieth (\$2.37 becomes \$2.40; \$2.44 becomes \$2.45) when making a purchase and counting change ◆ Use rounding strategies to make estimates (including money) ◆ Choose whether an estimate or exact amount is needed in a given situation ◆ Use overestimation and underestimation when appropriate <p><i>Continue to address skills and concepts that approach grade-level expectations in this subject</i></p>

Instructional Idea based on Learning Standards 4.N.10

At grades 3 – 4, students are asked to make “purchases” from a store catalog and spend within their limit by calculating the amount spent and the change that should be received.

How all students can participate in this activity

Addressing Learning Standard(s) as written for this grade level	Possible Assessment Strategies and Portfolio Products
Ricardo participates in a cooperative group activity to make multiple purchases and compare selections, total cost, and change received. Ricardo may only spend the money he has been given.	<ul style="list-style-type: none"> • Data chart showing Ricardo's progress completing math problems involving money • Ricardo's flyer/catalog and work samples of items bought, amount spent, and change received • A quiz chosen by Ricardo for his portfolio
Addressing Learning Standard(s) at lower levels of complexity (Entry Points)	Possible Assessment Strategies and Portfolio Products
Dominique participates by selecting items for purchase and indicating the amount needed by identifying the next highest dollar from the price given. A vertical number line provides Dominique with support so she can participate independently in this activity.	<ul style="list-style-type: none"> • Data chart showing Dominique's progress identifying the “next highest dollar” for a given price • Work samples in which Dominique selected her purchases and indicated the number of dollars needed • Dominique's graph, created with assistance from the teacher, demonstrating her accuracy in identifying the “next highest dollar” for purchases of \$10.00 or less • Videotape of Dominique making purchases in a variety of settings (classroom, cafeteria, school store, drug store) with her number line using the “next highest dollar” method
Addressing Access Skills embedded in academic instruction	Possible Assessment Strategies and Portfolio Products
Alice works with a peer who counts the money needed for each item. Alice follows her peer's one-step directions to put money into corresponding envelopes. Alice exchanges these envelopes when making a purchase.	<ul style="list-style-type: none"> • Data chart showing Alice's ability to assemble money envelopes and exchange correct envelopes when making a purchase • Videotape of Alice making a purchase • Money envelopes selected by Alice for her portfolio

Instructional Idea based on Learning Standards 4.N.12

At grades 3 – 4, students take a field trip into the local community. Prior to their trip, students work in cooperative groups to create maps with landmarks along their route (e.g., school, bank, sub shop, post office, library, park, and school). During their walk, they record the distances between landmarks in feet, yards, and miles, using a pedometer.

How all students can participate in this activity

Addressing Learning Standard(s) as written for this grade level	Possible Assessment Strategies and Portfolio Products
<p>Harold works in a cooperative group to create a map and record distances between each point. He creates a set of problems based on the map and the distances recorded (e.g., The distance from school to the post office is 520 yards, while the distance from school to the library is 940 yards. How many yards does it take to walk from the post office to the library?). Finally, his group and another group exchange maps and solve corresponding problems using the correct operation.</p>	<ul style="list-style-type: none"> • Data chart showing Harold's ability to create word problems, including homework assignments, quizzes, and in-class work • Harold's homework assignment in which he created a map of his neighborhood, a set of problems involving distances, and their solutions • Harold's map, problems, and solutions • Videotape of Harold's group working on the assignment
Addressing Learning Standard(s) at lower levels of complexity (Entry Points)	Possible Assessment Strategies and Portfolio Products
<p>Ken participates in a cooperative group with Harold. Ken records, writes, and identifies numbers generated during the map and field trip activities. He uses manipulatives to solve one-digit addition and subtraction problems, embedded in more complex problems solved by his peers (e.g., [from the problem above] $4 - 2 = 2$, $9 - 5 = 4$).</p>	<ul style="list-style-type: none"> • Data chart showing Ken's ability to solve one-digit addition and subtraction problems • Ken's graphic organizer in which he solves simple addition and subtraction problems with his peers • Videotape of Ken's participation in and contribution to the group
Addressing Access Skills embedded in academic instruction	Possible Assessment Strategies and Portfolio Products
<p>Jacqui participates in the same cooperative group with Ken and Harold. During the cooperative group activities, Jacqui uses her communication board to select landmarks for the map and to "read" the problems to the group. During the field trip, Jacqui works on identifying landmarks in her local community by pointing to a landmark and then matching it to a topic board.</p>	<ul style="list-style-type: none"> • Field data chart showing Jacqui's ability to identify and match landmarks in her community with corresponding pictures • Line or bar graph summarizing field data chart • Videotape of Jacqui participating in cooperative group activities and in the field trip

CONTENT Mathematics
STRAND Number Sense and Operations

Grade Level: 5 – 6

Concept	Learning Standards as written		Essence of the Standard(s)
Number Sense	6.N.1	Demonstrate an understanding of positive integer exponents, in particular, when used in powers of ten, e.g., 10^2 , 10^5 .	<ul style="list-style-type: none"> ◆ Factor numbers ◆ Identify place value ◆ Represent various types of numbers (i.e., whole, prime, rational) ◆ Demonstrate relationships between numbers by making comparisons and ordering whole numbers, fractions, decimals, percents, exponents, and integers ◆ Demonstrate that a whole can be divided into equal parts that can be put back together to remake a whole ◆ Identify equivalent fractions ($2/4 = 1/2$) ◆ Make comparisons between fractions, integers, and decimals on a number line (both positive and negative) ◆ Order numbers
	6.N.2	Demonstrate an understanding of place value to billions and thousandths.	
	6.N.3	Represent and compare very large (billions) and very small (thousandths) positive numbers in various forms such as expanded notation without exponents, e.g., $9724 = 9 \times 1000 + 7 \times 100 + 2 \times 10 + 4$.	
	6.N.4	Demonstrate an understanding of fractions as a ratio of whole numbers, as parts of unit wholes, as parts of a collection, and as locations on the number line.	
	6.N.5	Identify and determine common equivalent fractions, mixed numbers, decimals, and percents.	
	6.N.6	Find and position integers, fractions, mixed numbers, and decimals (both positive and negative) on the number line.	
	6.N.7	Compare and order integers (including negative integers), and positive fractions, mixed numbers, decimals, and percents.	
	6.N.8	Apply number theory concepts – including prime and composite numbers, prime factorization, greatest common factor, least common multiple, and divisibility rules for 2, 3, 4, 5, 6, 9, and 10 – to the solution of problems.	
Operations	6.N.9	Select and use appropriate operations to solve problems involving addition, subtraction, multiplication, division, and positive integer exponents with whole numbers and with positive fractions, mixed numbers, decimals, and percents.	<ul style="list-style-type: none"> ◆ Use a number line to add and subtract integers ◆ Select and use appropriate operations to solve increasingly complex problems ◆ Apply the Order of Operations for addition, subtraction, multiplication, and division when solving problems ◆ Demonstrate the inverse relationship of addition and subtraction
	6.N.10	Use the number line to model addition and subtraction of integers, with the exception of subtracting negative integers.	

Operations (cont)	6.N.11	Apply the Order of Operations for expressions involving addition, subtraction, multiplication, and division with grouping symbols (+, -, x, ÷).	
	6.N.12	Demonstrate an understanding of the inverse relationship of addition and subtraction, and use that understanding to simplify computation and solve problems.	
Computation and Estimation	6.N.13	Accurately and efficiently add, subtract, multiply, and divide (with double-digit divisors) whole numbers and positive decimals.	<ul style="list-style-type: none"> ◆ Perform accurate numerical operations ◆ Understand the meanings and properties of the numerical operations, given multiple number forms ◆ Estimate results of computations ◆ Simplify fractions
	6.N.14	Accurately and efficiently add, subtract, multiply and divide positive fractions and mixed numbers. Simplify fractions.	
	6.N.15	Add and subtract integers, with the exception of subtracting negative integers.	
	6.N.16	Estimate results of computations with whole numbers, and with positive fractions, mixed numbers, decimals, and percents. Describe reasonableness of estimates.	

Possible ENTRY POINTS to Learning Standard(s)

	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
<p>Number Sense</p> <ul style="list-style-type: none"> ◆ Create and label a number line to order positive numbers ◆ Count, compare, order, name, and write numbers to ten <p><i>See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Count, compare, order, name, and write numbers to 100 and tenths (e.g., 0.1) ◆ Use a number line to order integers and common fractions less than one whole ◆ Write, read, and name decimals to tenths (.1, .2, .3) ◆ Convert from one number form to another to solve a problem ◆ Illustrate the cube of a number with manipulatives (e.g., 10³) 	<ul style="list-style-type: none"> ◆ Count, compare, order, name, and write numbers to 1000 and hundredths (e.g., 0.01), and identify the value of each digit ◆ List prime numbers up to 29 ◆ Convert percents, decimals, and fractions ◆ Identify equivalent forms of common decimals less than one whole and greater than one whole (50% = 1/2) ◆ Create and use a number line to order fractions, mixed numbers, integers, and decimals ◆ Make exact change ◆ Convert from one number form to another to solve a problem ◆ Use number lines to represent negative numbers ◆ Identify prime numbers and numbers that can be factored <p><i>Continue to address skills and concepts that approach grade-level expectations in this subject</i></p>	

Less Complex

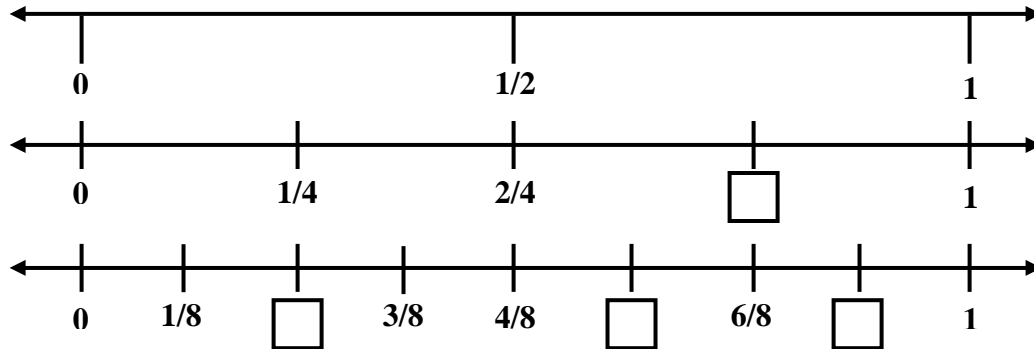
More Complex



	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
<p>Operations</p> <ul style="list-style-type: none"> ◆ Construct a number line demonstrating positive numbers <p><i>See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Select the appropriate operation(s) to solve a given problem ◆ Use the Order of Operations to solve a problem ◆ Construct a number line demonstrating positive and negative numbers 	<ul style="list-style-type: none"> ◆ Use a number line to demonstrate the process of solving a problem ◆ Given a simple equation, use the Order of Operations to obtain the correct answer (e.g., multiplication before addition) <p><i>Continue to address skills and concepts that approach grade-level expectations in this subject</i></p>	
<p>Computation and Estimation</p> <ul style="list-style-type: none"> ◆ Add and subtract 1-digit numbers ◆ Add, subtract, multiply, and divide whole numbers and positive decimals <p><i>See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Add and subtract 2-digit numbers, including fractions ◆ Multiply and divide 1-digit numbers ◆ Use manipulatives to estimate computations that use whole numbers 	<ul style="list-style-type: none"> ◆ Multiply and divide 2-digit numbers ◆ Add and subtract 3-digit numbers ◆ Demonstrate and explain methods used for calculation and estimation ◆ Label proper vs. improper fractions ◆ Identify whole numbers and mixed numbers ◆ Match fractions with appropriate simplified form (e.g., $6/9 = 2/3$) <p><i>Continue to address skills and concepts that approach grade-level expectations in this subject</i></p>	

Instructional Idea based on Learning Standards 6.N.5 and 6.N.9

At grades 5 – 6, students fill in the missing fractions on number lines:



How all students can participate in this activity

Addressing Learning Standard(s) as written for this grade level	Possible Assessment Strategies and Portfolio Products
Julie adds and subtracts fractions with an uncommon denominator (e.g., $1/4 + 3/8 = 5/8$).	<ul style="list-style-type: none"> • Data chart showing Julie's ability to add and subtract fractions with uncommon denominators • A collection of written assignments demonstrating Julie's knowledge of fractions • Julie's responses to open-ended questions on fractions • Videotape of Julie working on fractions
Addressing Learning Standard(s) at lower levels of complexity (Entry Points)	Possible Assessment Strategies and Portfolio Products
Andy uses manipulatives to arrange fractions by size. Andy is also working on finding equivalent fractions once they are ordered.	<ul style="list-style-type: none"> • Data chart showing Andy's progress in his ability to order fractions by size • Work sample in which Andy demonstrates his ability to order fractions and identify equivalent fractions • Videotape in which Andy demonstrates his ability to order fractions
Addressing Access Skills embedded in academic instruction	Possible Assessment Strategies and Portfolio Products
Penny is working on establishing joint attention to the manipulatives used in the fraction lesson for at least 5 seconds per manipulative.	<ul style="list-style-type: none"> • Field data chart indicating Penny's ability to establish and maintain joint attention to the manipulatives for at least 5 seconds each • Line or bar graph summarizing the field data chart • Videotape of Penny establishing and maintaining joint attention to a manipulative for at least 5 seconds

CONTENT Mathematics
STRAND Number Sense and Operations

Grade Level: 7 – 8

Concept	Learning Standards as written		Essence of the Standard(s)
Number Sense	8.N.1	Compare, order, estimate, and translate among integers, fractions and mixed numbers (i.e., rational numbers), decimals, and percents.	<ul style="list-style-type: none"> ◆ Represent and use a variety of numerical forms, including whole, prime, rational, and irrational numbers ◆ Continue to identify relationships between numbers (whole numbers, fractions, decimals, percents, integers, rational, irrational) ◆ Use proportional relationships and rates of change to solve problems
	8.N.2	Define, compare, order, and apply frequently used irrational numbers, such as $\sqrt{2}$ and π .	
	8.N.3	Use ratios and proportions in the solution of problems, in particular, problems involving unit rates, scale factors, and rate of change.	
	8.N.4	Represent numbers in scientific notation, and use them in calculations and problem situations.	
	8.N.5	Apply number theory concepts, including prime factorization and relatively prime numbers, to the solution of problems.	
Operations	8.N.6	Demonstrate an understanding of absolute value, e.g., $ -3 = 3 = 3$.	<ul style="list-style-type: none"> ◆ Use computation skills to simplify equations, using fractions, rules of powers/roots, real numbers ◆ Extend the Order of Operations to include positive integer exponents and square roots ◆ Understand the properties of rational numbers and use them in computations ◆ Select and use appropriate operations to solve problems
	8.N.7	Apply the rules of powers and roots to the solution of problems. Extend the Order of Operations to include positive integer exponents and square roots.	
	8.N.8	Demonstrate an understanding of the properties of arithmetic operations on rational numbers. Use the associative, commutative, and distributive properties; properties of the identity and inverse elements (e.g., $-7+7=0$; $3/4 + 4/3 = 1$); and the notion of closure of a subset of the rational numbers under an operation (e.g., the set of odd integers is closed under multiplication but not under addition).	
	8.N.12	Select and use appropriate operations – addition, subtraction, multiplication, division, and positive integer exponents – to solve problems with rational numbers (including negatives).	

Computation and Estimation	8.N.9	Use the inverse relationships of addition and subtraction, multiplication and division, and squaring and finding square roots to simplify computations and solve problems, e.g., multiplying by $\frac{1}{2}$ or 0.5 is the same as dividing by 2.	<ul style="list-style-type: none"> ◆ Simplify computations using inverse relationships ◆ Make logical estimations and know when it is appropriate to make an educated guess ◆ Judge the validity of an estimation ◆ Explain estimations ◆ Apply estimations to solve problems ◆ Use fractions in computations and estimations
	8.N.10	Estimate and compute with fractions (including simplification of fractions), integers, decimals, and percents (including those greater than 100 and less than 1).	
	8.N.11	Determine when an estimate rather than an exact answer is appropriate and apply in problem situations.	

Possible ENTRY POINTS to Learning Standard(s)

Less Complex

More Complex



	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
Number Sense	<p><i>See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Identify integers (positive and negative) ◆ Demonstrate an understanding of proportional relationships using manipulatives (e.g., recipes) ◆ Illustrate the concept of square root using manipulatives ◆ Classify numbers as rational vs. irrational ◆ Fill in blanks in partially completed factorization trees 	<ul style="list-style-type: none"> ◆ Solve simple problems using equivalent fractions, mixed numbers, decimals, and percents ◆ Identify, sort, and classify integers (positive and negative), fractions, mixed numbers, decimals, and percents ◆ Compare, order, and translate (simplify) fractions, decimals, percents, and whole numbers (positive and negative) ◆ Solve simple problems involving ratios <p><i>Continue to address skills and concepts that approach grade-level expectations in this subject</i></p>
Operations	<ul style="list-style-type: none"> ◆ Demonstrate the concept of the identity element for one and zero using manipulatives ◆ Demonstrate the concept of the associative property using manipulatives ◆ Demonstrate the concept of the commutative property using manipulatives <p><i>See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Illustrate addition of positive and negative numbers on a number line ◆ Demonstrate the concept of the distributive property using manipulatives 	<ul style="list-style-type: none"> ◆ Select the appropriate operations to solve increasingly complex numerical problems ◆ Use the correct order of operations when solving increasingly complex numerical problems <p><i>Continue to address skills and concepts that approach grade-level expectations in this subject</i></p>

Possible ENTRY POINTS to Learning Standard(s)

Less Complex

More Complex



Computation and Estimation

The student will:

- ◆ Use reason to solve problems and identify the outcomes in life situations
- ◆ Add and subtract two-digit numbers
- ◆ Multiply and divide one-digit numbers
- ◆ Add fractions with the same one-digit denominator

See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials

The student will:

- ◆ Add and subtract 3-digit numbers
- ◆ Multiply and divide 2-digit numbers
- ◆ Add, subtract, multiply, and divide fractions with the same denominator

The student will:

- ◆ Multiply and divide 3-digit numbers
- ◆ Add and subtract fractions (simplifying the result), decimals, and positive/negative integers
- ◆ Add, subtract, multiply, and divide fractions with different denominators (with a common factor)
- ◆ Explain/show methods used for calculation/estimation
- ◆ Use manipulatives to compute and estimate multiplication and division of real numbers
- ◆ Identify in relationship to problem situations when it is appropriate to estimate and when an exact answer is necessary when it is appropriate to estimate
- ◆ Use estimation to solve problems; determine accuracy by calculating exact answer

Continue to address skills and concepts that approach grade-level expectations in this subject

Instructional Idea based on Learning Standards 8.N.1, 8.N.12, and 8.N.10

At grades 7 – 8, students are told they have a bank account of \$10,000 to purchase stocks from the New York Stock Exchange. They research a variety of stocks and make selections for purchase. They keep track each day of the percent each stock rises/falls and answer questions involving percentages when given projections made about their stocks (e.g., On day one, stock #1 rises 50% in value; on day two, it falls 50% in value; on day three, it rises 50% in value; on day four, it falls 50% in value. How much is it worth at the end of day 4?). Students keep track of the percent lost/gained per stock, as well as the total amount lost/gained.

How all students can participate in this activity

Addressing Learning Standard(s) as written for this grade level	Possible Assessment Strategies and Portfolio Products
Paul keeps track of his stock portfolio. He uses stock quotes in the paper to record daily changes and makes written observations. He solves related problems using knowledge of percents.	<ul style="list-style-type: none"> • Data chart showing Paul's progress in understanding and working with percents • Paul's completed quiz on percents of actual stock values • Paul's stock portfolio showing his calculations using percents
Addressing Learning Standard(s) at lower levels of complexity (Entry Points)	Possible Assessment Strategies and Portfolio Products
Nadia participates in a similar activity using percents. She identifies items for purchase from newspaper flyers. She solves problems related to percent discount, using a calculator, and a conversion table she created showing percentages to decimals.	<ul style="list-style-type: none"> • Data chart indicating Nadia's independence performing simple calculations involving percents, using her table and calculator • Table created by Nadia showing her percent-to-decimal conversions • Videotape of Nadia using her newspaper flyers and calculator to complete percent-to-decimal calculations
Addressing Access Skills embedded in academic instruction	Possible Assessment Strategies and Portfolio Products
Greg is working on one-to-one correspondence. Greg places small objects along a number line as part of a counting activity.	<ul style="list-style-type: none"> • Field data chart indicating Greg's ability to place small objects along a number line • Line or bar graph summarizing field data chart • Teacher-scribed work sample charting Greg's ability to place specific items on the number line

CONTENT Mathematics
STRAND Number Sense and Operations

Grade Level: 9 – 10

Concept	Learning Standards as written		Essence of the Standard(s)
Operations	10.N.1	Identify and use the properties of operations on real numbers, including the associative, commutative, and distributive properties; the existence of the identity and inverse elements for addition and multiplication; the existence of n^{th} roots of positive real numbers for any positive integer n ; and the inverse relationship between taking the n^{th} root of and the n^{th} power of a positive real number.	<ul style="list-style-type: none"> ◆ Use properties of operations on real numbers, including <ul style="list-style-type: none"> • associative, commutative, and distributive properties • identity and inverse elements • n^{th} roots, including the inverse relationship between the n^{th} root and the n^{th} power ◆ Simplify expressions involving: <ul style="list-style-type: none"> • positive integer exponents • absolute value
	10.N.2	Simplify numerical expressions, including those involving positive integer exponents or the absolute value, e.g., $3(2^4 - 1) = 45$, $4 3 - 5 + 6 = 14$; apply such simplifications in the solution of problems.	
Computation and Estimation	10.N.3	Find the approximate value for solutions to problems involving square roots and cube roots without the use of a calculator, e.g., $\sqrt{3^2 - 1} \approx 2.8$	<ul style="list-style-type: none"> ◆ Solve problems using approximate value for square roots and cube roots ◆ Use computation skills and estimation to solve problems involving real numbers
	10.N.4	Use estimation to judge the reasonableness of results of computations and of solutions to problems involving real numbers.	

Possible ENTRY POINTS to Learning Standard(s)

Less Complex

More Complex



	<u>The student will:</u>	<u>The student will:</u>	<u>The student will:</u>
<p>Operations</p> <ul style="list-style-type: none"> ◆ Select the appropriate operations to solve simple problems involving integers <p><i>See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Select the appropriate operations to solve increasingly complex numerical problems involving more than one operation and more than one calculation ◆ Use the correct order of operations when solving problems ◆ Given a simple number sentence, demonstrate the associative, commutative, and distributive properties ◆ Define, order, compare, and apply the following: <ul style="list-style-type: none"> • irrational numbers • ratios/proportions 	<ul style="list-style-type: none"> ◆ Solve complex problems using knowledge of the following: <ul style="list-style-type: none"> • fractions and mixed numbers • irrational numbers • ratios and proportions ◆ Define and apply absolute value and rules of powers and roots <p><i>Continue to address skills and concepts that approach grade-level expectations in this subject</i></p>	
<p>Computation and Estimation</p> <ul style="list-style-type: none"> ◆ Add and subtract three-digit numbers ◆ Multiply and divide two-digit numbers ◆ Add, subtract, multiply, and divide fractions with the same denominator ◆ Use estimation to assist in solving simple computations <p><i>See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Multiply and divide three-digit numbers ◆ Add and subtract fractions with different denominators ◆ Use estimation to solve problems ◆ Predict the solution to a problem using estimation, and determine accuracy by calculating the exact answer 	<ul style="list-style-type: none"> ◆ Estimate the answers to problems involving fractions, decimals, or mixed numbers ◆ Explain, identify, and show methods used for estimation ◆ Continue to multiply, divide, add, and subtract increasingly complex problems involving fractions, decimals, and mixed numbers <p><i>Continue to address skills and concepts that approach grade-level expectations in this subject</i></p>	

Patterns, Relations and Algebra

	Patterns, Relations, and Functions	Algebra	Relationships and Mathematical Models	Variables and Change
PreK – K	Pages 165 – 167	No learning standards for this grade span	No learning standards for this grade span	No learning standards for this grade span
1 – 2	Pages 165 – 167	Page 171	Page 176	No learning standards for this grade span
3 – 4	Page 168	Page 171	Pages 177 – 178	Page 182
5 – 6	Page 168	Page 172	Pages 179 – 180	Page 182
7 – 8	Page 169	Pages 173 – 174	Pages 179 – 180	Page 183
9 – 10	Page 170	Page 175	Page 181	No learning standards for this grade span

Mathematics

Strand 2
***Patterns,
Relations, and
Algebra***

Patterns, Relations,
and Functions

PreK – K, 1 – 2

CONTENT Mathematics

STRAND Patterns, Relations, and Algebra

Learning Standards for Patterns, Relations, and Functions

Grade Level: PreK – K, 1 – 2

Learning Standards as written

Essence of the Standard(s)

K.P.1	Identify the attributes of objects as a foundation for sorting and classifying, e.g., a red truck, a red block, and a red ball share the attribute of being red; a square block, a square cracker, and a square book share the attribute of being square shaped.	<ul style="list-style-type: none">◆ Identify, reproduce, create, describe, and extend repeating patterns◆ Identify shared attributes of objects and use as a basis for sorting◆ Count in patterns (skip counting)
K.P.2	Sort and classify objects by color, shape, size, number, and other properties.	
K.P.3	Identify, reproduce, describe, extend, and create color, rhythmic, shape, number, and letter repeating patterns with simple attributes, e.g., ABABAB....	
K.P.4	Count by fives and tens at least up to 50.	
2.P.1	Identify, reproduce, describe, extend, and create simple rhythmic, shape, size, number, color, and letter repeating patterns.	
2.P.2	Identify different patterns on the hundreds chart.	
2.P.3	Describe and create addition and subtraction number patterns, e.g., 1, 4, 7, 10...; or 25, 23, 21....	
2.P.4	Skip count by twos, fives, and tens up to at least 50, starting at any number.	

Possible ENTRY POINTS to Learning Standard(s) (and ACCESS SKILLS embedded in standards-based activities)

← Less Complex

More Complex →

ACCESS SKILLS

The student will:

- ◆ Grasp, release, and manipulate individual objects relevant to the sorting activity
- ◆ Activate switch to participate in an activity
- ◆ Track left to right
- ◆ Follow one-step directions to complete auditory pattern (stomp, clap, stomp...)
- ◆ Turn attention toward another person
- ◆ Activate an electronic device to access text, communicate with others, or participate in an instructional activity
- ◆ Organize instructional materials
- ◆ Take turns appropriately during classroom discussion
- ◆ Respond to/initiate contacts with others
- ◆ Apply rules for appropriate classroom behavior
- ◆ Use appropriate social greetings
- ◆ Follow simple directions
- ◆ Identify self and/or others
- ◆ Initiate or respond to request for joint attention

ENTRY POINTS

The student will:

- ◆ Sort using attributes of objects such as color, shape, size
- ◆ Match objects based on similar attributes (big, red, rough)
- ◆ Demonstrate 1:1 correspondence with descriptive vocabulary (big, red, rough)
- ◆ Identify and sort a small number of objects by common attributes
- ◆ Identify what is missing from a pattern
- ◆ Count by 2s to ten
- ◆ Group objects by 2s
- ◆ Match objects with similar attributes
- ◆ Activate switch to continue/extend a predetermined pattern

The student will:

- ◆ Copy/reproduce a pattern with 1:1 correspondence (using manipulatives, matching a model)
- ◆ Identify and create a pattern using familiar objects
- ◆ Count by 2s to 20
- ◆ Count by 5s to 20
- ◆ Group objects by 2s and 5s
- ◆ Identify and group objects by one attribute

The student will:

- ◆ Identify a pattern in the environment
- ◆ Extend a pattern using pictures or words
- ◆ Group by 2s, 5s, and 10s
- ◆ Identify patterns in a series of numbers
- ◆ Count by 2s, 5s, and 10s to 50
- ◆ Group objects by two attributes (e.g., size and color)

Continue to address skills and concepts that approach grade-level expectations in this subject

Instructional Idea based on Learning Standard K.P.2

At grades PreK-K, pairs of students are each given 25 pattern blocks to sort by attribute. Groups compare and discuss the different ways they sorted their blocks.

How all students can participate in this activity

Addressing Learning Standard(s) as written for this grade level	Possible Assessment Strategies and Portfolio Products
<p>Fred works with his partner to sort pattern blocks by colors, shapes, and sizes, and compares their pile with those of other groups.</p>	<ul style="list-style-type: none"> • Data chart indicating Fred's performance sorting pattern blocks • Videotape of Fred sorting pattern blocks with his peer • Fred's work sample with drawings of each set of sorted blocks based on different attributes (e.g., size and shape)
Addressing Learning Standard(s) at lower levels of complexity (Entry Points)	Possible Assessment Strategies and Portfolio Products
<p>Sami sorts pattern blocks with his partner. He identifies each block by color, using a visual color guide.</p>	<ul style="list-style-type: none"> • Data chart showing Sami's progress sorting the blocks by color • Videotape of Sami identifying pattern blocks by color • Work sample created by Sami identifying colors in a variety of settings (e.g., in math class, at home, on the playground)
Addressing Access Skills embedded in academic instruction	Possible Assessment Strategies and Portfolio Products
<p>Janet works on the pattern block activity with a peer. She works on her fine motor skills to select, hold, and place pattern blocks in the appropriate pile as directed by her peer.</p>	<ul style="list-style-type: none"> • Field data chart indicating Janet's ability to place objects as directed • Line or bar graph summarizing the field data chart • Videotape of Janet placing objects as directed • Teacher-scribed work sample charting Janet's ability to place each item in the appropriate pile

Mathematics

**Strand 2
Patterns,
Relations, and
Algebra**

Patterns, Relations,
and Functions

3 – 4, 5 – 6

CONTENT Mathematics

STRAND Patterns, Relations, and Algebra

Learning Standards for Patterns, Relations, and Functions

Grade Level: 3 – 4, 5 – 6

Learning Standards as written

Essence of the Standard(s)

<p>4.P.1</p>	<p>Create, describe, extend, and explain symbolic (geometric) and numeric patterns, including multiplication patterns like 3, 30, 300, 3000....</p>	<ul style="list-style-type: none"> ◆ Identify, reproduce, create, describe, and extend patterns <ul style="list-style-type: none"> • geometric (symbolic) • numeric (multiplication) ◆ Count in patterns ◆ Determine rules for patterns and progressions
<p>6.P.1</p>	<p>Analyze and determine the rules for extending symbolic, arithmetic, and geometric patterns and progressions, e.g., ABBCCC; 1, 5, 9, 13...; 3, 9, 27....</p>	

Possible ENTRY POINTS to Learning Standard(s)

Less Complex

More Complex



The student will:

- ◆ Identify and complete a pattern, given attributes of size, number, color, and/or shape

See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials

The student will:

- ◆ Extend a pattern using simple addition and subtraction (i.e., adding by 2s, subtracting by 3s)
- ◆ Describe and extend a pattern using numbers, and operations

The student will:

- ◆ Create and explain a pattern using increasingly complex addition and subtraction
- ◆ Identify patterns on a 100's chart

Continue to address skills and concepts that approach grade-level expectations in this subject

Mathematics

Strand 2
Patterns,
Relations, and
Algebra

Patterns, Relations and
Functions

7 – 8

CONTENT Mathematics

STRAND Patterns, Relations, and Algebra

Learning Standards for Patterns, Relations, and Functions

Grade Level: 7 – 8

Learning Standards as written

Essence of the Standard(s)

<p>8.P.1</p>	<p>Extend, represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic expressions. Include arithmetic and geometric progressions, e.g., compounding.</p>	<ul style="list-style-type: none"> ◆ Identify, reproduce, create, describe, and extend patterns, including arithmetic and geometric progressions ◆ Represent linear growth patterns using tables and graphs ◆ Compare rates of change and x- and y- intercepts in different linear patterns
<p>8.P.10</p>	<p>Use tables and graphs to represent and compare linear growth patterns. In particular, compare rates of change and x- and y- intercepts of different linear patterns.</p>	

Possible ENTRY POINTS to Learning Standard(s)

Less Complex

More Complex



The student will:

- ◆ Extend a geometric pattern
- ◆ Extend a pattern using a mathematical operation

See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials

The student will:

- ◆ Create and explain a simple addition or subtraction pattern
- ◆ Extend a simple multiplication or symbolic (geometric) pattern

The student will:

- ◆ Create and explain multiplication or symbolic patterns
- ◆ Use a simple table or graph to describe or show a pattern
- ◆ Identify a linear growth pattern
- ◆ Explain and apply the concept of "rate of change"

Continue to address skills and concepts that approach grade-level expectations in this subject

CONTENT Mathematics

STRAND Patterns, Relations, and Algebra

Learning Standards for Patterns, Relations, and Functions

Grade Level: 9 – 10

Learning Standards as written

Essence of the Standard(s)

10.P.1	Describe, complete, extend, analyze, generalize, and create a wide variety of patterns, including iterative, recursive (e.g., Fibonacci Numbers), linear, quadratic, and exponential functional relationships.	♦ Identify, reproduce, create, and extend increasingly complex patterns (e.g., "What comes next?")
10.P.2	Demonstrate an understanding of the relationship between various representations of a line. Determine a line's slope and x - and y -intercepts from its graph or from a linear equation that represents the line. Find a linear equation describing a line from a graph or a geometric description of the line, e.g., by using the "point-slope" or "slope y -intercept" formulas. Explain the significance of a positive, negative, zero, or undefined slope.	♦ Determine the equation of a line from its graph or from a description of the line ♦ Determine a line's slope and x - and y -intercepts

Possible ENTRY POINTS to Learning Standard(s)

Less Complex

More Complex



The student will:

See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials

The student will:

- ♦ Create and describe patterns, including addition, subtraction, multiplication, symbolic, etc.
- ♦ Describe the rule for a given pattern (e.g., double the number and add five)
- ♦ Plot and connect points on a graph
- ♦ Demonstrate lines with positive, negative, and zero slopes

The student will:

- ♦ Apply understanding of patterns and relationships to solve simple problems
- ♦ Compare and contrast linear and nonlinear graphical representations
- ♦ Explain the rule of a pattern, e.g., $(2n + 5)$
- ♦ Define and explain the slope of a line through representation on a graph

Continue to address skills and concepts that approach grade-level expectations in this subject

CONTENT Mathematics
STRAND Patterns, Relations, and Algebra
Learning Standards for Algebra

Grade Level: 1 – 2, 3 – 4

Learning Standards as written		Essence of the Standard(s)
2.P.5	Construct and solve open sentences that have variables, e.g., $\square + 7 = 10$.	<ul style="list-style-type: none"> ◆ Use symbolic representations for unknown or variable quantities ◆ Determine values of variables in simple equations ◆ Solve algebraic equations with unknown quantities
4.P.2	Use symbol and letter variables (e.g. Δ , x) to represent unknowns or quantities that vary in expressions and in equations or inequalities (mathematical sentences that use =, <, >).	
4.P.3	Determine values of variables in simple equations, e.g., $4106 - \nabla = 37$; $5 = \bigcirc + 3$ and $\square - \bigcirc = 3$.	

Possible ENTRY POINTS to Learning Standard(s) (and ACCESS SKILLS embedded in standards-based activities)

Less Complex		More Complex	
<p>ACCESS SKILLS The student will</p> <ul style="list-style-type: none"> ◆ Match pictures or objects ◆ Identify symbols used in common situations (e.g., stop sign, information, telephone) ◆ Use the terms “same” and “different” ◆ Grasp, release, and manipulate materials relevant to the instructional activity ◆ Take turns during activities ◆ Initiate or respond to request for joint attention ◆ Activate an electronic device to access text, communicate with others, or participate in an instructional activity ◆ Follow simple directions ◆ Turn attention toward another person ◆ Apply rules for appropriate classroom behavior 	<p>ENTRY POINTS The student will:</p> <ul style="list-style-type: none"> ◆ Match/manipulate pictures and objects to create sets, make comparisons between sets, and determine missing objects or quantities within sets ◆ Identify the terms and corresponding symbols for addition (+) and subtraction (-) 	<p>The student will:</p> <ul style="list-style-type: none"> ◆ Represent and solve simple addition and subtraction number sentences, using pictures, objects, and/or manipulatives ◆ Create sets to demonstrate the meaning of “equal to” 	<p>The student will:</p> <ul style="list-style-type: none"> ◆ Solve problems involving variables in simple one- and two-digit number sentences ◆ Find or identify the missing component in a number sentence, using symbols or objects ◆ Demonstrate an understanding of equalities and inequalities in number sentences ◆ Use appropriate terms and corresponding symbols for equal to, less than, and greater than <p><i>Continue to address skills and concepts that approach grade-level expectations in this subject</i></p>

CONTENT Mathematics
STRAND Patterns, Relations, and Algebra
Learning Standards for Algebra

Grade Level: 5 – 6

Learning Standards as written

Essence of the Standard(s)

6.P.2	Replace variables with given values and evaluate/simplify, e.g., $2(\bigcirc) + 3$ when $\bigcirc = 4$.	<ul style="list-style-type: none"> ◆ Evaluate and simplify number sentences by replacing variables with given values ◆ Use properties of equality (e.g., apply the same operation to each side of an equation) to solve problems
6.P.3	Use the properties of equality to solve problems, e.g., if $\square + 7 = 13$, then $\square = 13 - 7$, therefore $\square = 6$; if $3 \times \square = 15$, then $1/3 \times 3 \times \square = 1/3 \times 15$, therefore $\square = 5$.	

Possible ENTRY POINTS to Learning Standard(s)

Less Complex

More Complex

The student will:

- ◆ Solve problems that use variables in simple one- and two-digit number sentences

See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials

The student will:

- ◆ Write and solve a simple number sentence with a variable
- ◆ Replace variables with given values in simple mathematical sentences (e.g., solve $2\square$, if $\square = 2, 3, \text{ or } 4$)

The student will:

- ◆ Identify the value of an unknown variable in a simple mathematical sentence (e.g., $2\square = 4$)
- ◆ Solve mathematical sentences containing variables
- ◆ Given a mathematical sentence with an unknown variable, solve the equation by determining the value of the variable

Continue to address skills and concepts that approach grade-level expectations in this subject

CONTENT Mathematics
STRAND Patterns, Relations, and Algebra
Learning Standards for Algebra

Grade Level: 7 – 8

Learning Standards as written

Essence of the Standard(s)

8.P.2	Evaluate simple algebraic expressions for given variable values, e.g., $3a^2 - b$ for $a = 3$ and $b = 7$.	<ul style="list-style-type: none"> ◆ Evaluate and simplify algebraic expressions ◆ Create and use symbolic expressions ◆ Solve linear equations and inequalities with one or two variables
8.P.3	Demonstrate an understanding of the identity $(-x)(-y) = xy$. Use this identity to simplify algebraic expressions, e.g., $(-2)(-x + 2) = 2x - 4$.	
8.P.4	Create and use symbolic expressions and relate them to verbal, tabular, and graphical representations.	
8.P.6	Identify the roles of variables within an equation, e.g., $y = mx + b$, expressing y as a function of x with parameters m and b .	
8.P.7	Set up and solve linear equations and inequalities with one or two variables, using algebraic methods, models, and/or graphs.	

Suggested ENTRY POINTS to Learning Standard(s)

Less Complex

More Complex

The student will:

- ◆ Demonstrate the concept of inequality and use the corresponding symbol correctly ($<$, $>$)

See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials

The student will:

- ◆ Simplify algebraic expressions using symbols
- ◆ Write an algebraic expression based on a word problem
- ◆ Read and interpret numbers written with the exponent of two (e.g., x^2)
- ◆ Identify the value of an unknown variable in a simple mathematical sentence (e.g., $2x = 4$)

The student will:

- ◆ Calculate the square of a number
- ◆ Write and solve algebraic expressions
- ◆ Solve linear equations with variables

Continue to address skills and concepts that approach grade-level expectations in this subject

Instructional Idea based on Learning Standard 8.P.6

At grades 7 – 8, students simplify and solve complex algebraic equations to determine the value(s) of one or more unknown variables.

How all students can participate in this activity

Addressing Learning Standard(s) as written for this grade level	Possible Assessment Strategies and Portfolio Products
Jamie simplifies and solves algebraic equations and calculates the values of unknown variables.	<ul style="list-style-type: none"> • Data chart indicating Jamie's ability to solve algebraic equations • Jamie's completed homework assignment in which he solved algebraic equations • Jamie's test solving algebraic equations
Addressing Learning Standard(s) at lower levels of complexity (Entry Points)	Possible Assessment Strategies and Portfolio Products
Meredith works with a group of peers to identify the values of unknown variables in simple mathematical sentences involving addition and subtraction (e.g., $2 + \square = 4$). She uses manipulatives to solve each problem independently.	<ul style="list-style-type: none"> • Data chart indicating Meredith's ability to solve mathematical sentences for unknown variables • Videotape of Meredith using manipulatives to identify the variables in simple number sentences • Meredith's worksheet solving similar problems involving unknown variables in everyday situations
Addressing Access Skills embedded in academic instruction	Possible Assessment Strategies and Portfolio Products
Thomas works with Meredith's group as they identify the values of unknown variables. He chooses manipulatives to be used and assigns them to the appropriate person, using his communication board.	<ul style="list-style-type: none"> • Field data chart indicating Thomas's ability to identify peers by name during mathematics activities • Line or bar graph summarizing the field data chart • Videotape of Thomas assigning manipulatives to the appropriate person on cue • Teacher-scribed notation in a section of Thomas's communication notebook indicating his ability to identify the appropriate person during the activity

CONTENT Mathematics
STRAND Patterns, Relations, and Algebra
Learning Standards for Algebra

Grade Level: 9 – 10

Learning Standards as written		Essence of the Standard(s)
10.P.3	Add, subtract, and multiply polynomials. Divide polynomials by monomials.	<ul style="list-style-type: none"> ◆ Solve quadratic equations by factoring, completing the square, or using the quadratic formula and demonstrate their equivalence ◆ Demonstrate the symbolic manipulation of polynomial and rational expressions
10.P.4	Demonstrate facility in symbolic manipulation of polynomial and rational expressions by rearranging and collecting terms; factoring (e.g., $a^2 - b^2 = (a + b)(a - b)$, $x^2 + 10x + 21 = (x + 3)(x + 7)$, $5x^4 + 10x^3 - 5x^2 = 5x^2(x^2 + 2x - 1)$); identifying and canceling common factors in rational expressions; and applying the properties of positive integer exponents.	
10.P.5	Find solutions to quadratic equations (with real roots) by factoring, completing the square, or using the quadratic formula. Demonstrate an understanding of the equivalence of the methods.	
10.P.6	Solve equations and inequalities including those involving absolute value of linear expressions (e.g., $ x - 2 > 5$), and apply to the solution of problems.	

Possible ENTRY POINTS to Learning Standard(s)

Less Complex		More Complex
<p><u>The student will:</u></p> <ul style="list-style-type: none"> ◆ Identify the value of an unknown variable in a simple mathematical sentence (e.g., $2x = 4$) <p><i>See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials</i></p>	<p><u>The student will:</u></p> <ul style="list-style-type: none"> ◆ Add, subtract, and multiply numbers and variables to solve linear equations ◆ Use properties of equality (applying the same operation to both sides of an equation and canceling out) to simplify equations ◆ Demonstrate an understanding of absolute value 	<p><u>The student will:</u></p> <ul style="list-style-type: none"> ◆ Solve increasingly complex linear and nonlinear expressions containing variables, using knowledge of <ul style="list-style-type: none"> • operations • property of equality • common factors ◆ Solve everyday problems using knowledge of algebra, including factoring, completing the square, or the quadratic formula <p><i>Continue to address skills and concepts that approach grade-level expectations in this subject</i></p>

CONTENT Mathematics

STRAND Patterns, Relations, and Algebra

Learning Standards for Relationships and Mathematical Models

Grade Level: 1 – 2

Learning Standards as written

Essence of the Standard(s)

<p>2.P.6</p>	<p>Write number sentences using +, -, <, =, and/or > to represent mathematical relationships in everyday situations.</p>	<ul style="list-style-type: none"> ◆ Represent mathematical relationships through number sentences ◆ Use and apply equivalents of measurement
<p>2.P.7</p>	<p>Describe functions related to trading, including coin trades and measurement trades, e.g., five pennies make one nickel or four cups make one quart.</p>	

Possible ENTRY POINTS to Learning Standard(s) (and ACCESS SKILLS embedded in standards-based activities)

← Less Complex		More Complex →	
<p>ACCESS SKILLS <u>The student will:</u></p> <ul style="list-style-type: none"> ◆ Grasp, manipulate, and release materials relevant to instructional activity ◆ Initiate or respond to request for joint attention ◆ Follow simple directions ◆ Use appropriate social greetings ◆ Activate an electronic device to access text, communicate with others, or participate in an instructional activity ◆ Turn attention toward another person ◆ Apply rules for appropriate classroom behavior ◆ Respond to/initiate contacts with others ◆ Take turns appropriately during classroom discussion 	<p>ENTRY POINTS <u>The student will:</u></p> <ul style="list-style-type: none"> ◆ Demonstrate the concept of trading equal amounts ◆ Identify sets containing more/bigger, less/fewer, and same ◆ Use informal equivalents (e.g., one homework assignment every night, three choices for leisure time, etc.) ◆ Identify coins/bills ◆ Identify measurement tools (e.g., cups, spoons) 	<p><u>The student will:</u></p> <ul style="list-style-type: none"> ◆ Define the terms "greater than," "less than," "equal to" ◆ Create and compare groupings of objects, using terms like "more," "bigger," "fewer," "less," "same" ◆ Identify common measurement equivalents (i.e., money, quantity, size) and trade appropriately ◆ Group and compare familiar objects (e.g., coins, quantities) 	<p><u>The student will:</u></p> <ul style="list-style-type: none"> ◆ Use conventional mathematical signs and symbols to represent mathematical relationships ◆ Describe and represent the relationships between groupings of objects using symbols (=, >, <) ◆ Write correct number for equivalent number of objects (e.g., 5 pennies = 1 nickel) <p><i>Continue to address skills and concepts that approach grade-level expectations in this subject</i></p>

Mathematics

Strand 2
*Patterns, Relations,
and Algebra*

Relationships and
Mathematical Models

3 – 4

CONTENT Mathematics
STRAND Patterns, Relations, and Algebra
Learning Standards for Relationships and Mathematical Models

Grade Level: 3 – 4

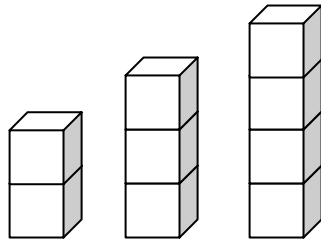
Learning Standards as written		Essence of the Standard(s)
4.P.4	Use pictures, models, tables, charts, graphs, words, number sentences, and mathematical notations to interpret mathematical relationships.	<ul style="list-style-type: none"> ◆ Represent mathematical relationships in a variety of formats ◆ Use proportional relationships to solve problems <ul style="list-style-type: none"> • unit pricing • map interpretation
4.P.5	Solve problems involving proportional relationships, including unit pricing (e.g., four apples cost 80¢, so one apple costs 20¢) and map interpretation (e.g., one inch represents five miles, so two inches represent ten miles).	

Possible ENTRY POINTS to Learning Standard(s)

<p style="font-size: 1.2em; font-weight: bold;">← Less Complex</p> <p style="text-align: center; font-weight: bold;">The student will:</p> <ul style="list-style-type: none"> ◆ Create and compare groupings of objects using language (more, bigger, fewer, less, same) ◆ Demonstrate common measurement equivalents (e.g., coin, quantity, size) through trading ◆ Define the terms “greater than,” “less than,” “equal to” <p><i>See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials</i></p>	<p style="font-size: 1.2em; font-weight: bold;">← More Complex</p> <p style="text-align: center; font-weight: bold;">The student will:</p> <ul style="list-style-type: none"> ◆ Use a variety of measurement equivalents (e.g., coin, quantity, size) to solve simple problems ◆ Write number sentences using +, -, <, =, and/or > to represent mathematical relationships in everyday situations ◆ Describe functions related to trading, including coin trades and measurement trades (e.g., five pennies make one nickel; four cups make one quart) 	<p style="font-size: 1.2em; font-weight: bold;">← More Complex</p> <p style="text-align: center; font-weight: bold;">The student will:</p> <ul style="list-style-type: none"> ◆ Use proportional relationships to solve everyday problems (e.g., 25 pennies = 1 quarter = 5 nickels; 1 quart = 2 pints = 4 cups) ◆ Create simple charts and graphs to demonstrate common mathematical relationships (e.g., inches vs. feet) ◆ Use a variable to represent the change in quantity for two proportional relationships ◆ Solve simple problems involving unit pricing (e.g., 2 apples cost 1 dollar, therefore 1 apple costs 50¢) <p><i>Continue to address skills and concepts that approach grade-level expectations in this subject</i></p>
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Instructional Idea based on Learning Standard 4.P.4

At grades 3 – 4, students participate in a “tower of cubes” activity and answer the following questions: “How many squares make up the surface of each tower of cubes (including the top and bottom)? As the tower gets taller, how does this number change?” They record their observations on the table below.



Number of Cubes (N)	Number of Squares on the Surface
1	6
2	10
3	14
4	18

How all students can participate in this activity

Addressing Learning Standard(s) as written for this grade level	Possible Assessment Strategies and Portfolio Products
Armen works with Unifix cubes as he participates in the “tower of cubes” activity. He answers each question by counting the squares on the surface of the tower after adding each additional cube. He identifies and records the pattern as the number increases.	<ul style="list-style-type: none"> • Data chart showing Armen’s ability to record the pattern correctly • Armen’s record of the pattern he observed • Work sample in which Armen translates his observations into a formula to determine a pattern for any number of cubes
Addressing Learning Standard(s) at lower levels of complexity (Entry Points)	Possible Assessment Strategies and Portfolio Products
Sharlene counts and records the number of cubes as she adds each cube to the tower. She makes comparative observations about the size of the tower as cubes are added.	<ul style="list-style-type: none"> • Data chart showing Sharlene’s ability to correctly identify the number of cubes by adding and counting • Sharlene’s work sample depicting the number of cubes in the tower at intervals • Videotape showing Sharlene identifying a general pattern (“As I add a cube, the tower gets bigger.”)
Addressing Access Skills embedded in academic instruction	Possible Assessment Strategies and Portfolio Products
Kris participates by selecting appropriate materials (Unifix cubes) to be used for the activity from a range of materials, and handing the next cube to her partner as the tower is built.	<ul style="list-style-type: none"> • Field data chart showing Kris’s ability to choose the cube and give it to her partner • Line or bar graph summarizing the field data chart • Videotape of Kris selecting, grasping, and passing cubes to her partner

Mathematics

Strand 2
***Patterns, Relations,
and Algebra***

Relationships and
Mathematical Models

5 – 6, 7 – 8

CONTENT Mathematics
STRAND Patterns, Relations, and Algebra
Learning Standards for Relationships and Mathematical Models

Grade Level: 5 – 6, 7 – 8

Learning Standards as written	Essence of the Standard(s)
6.P.4 Represent real situations and mathematical relationships with concrete models, tables, graphs, and rules in words and with symbols, e.g., input-output tables.	<ul style="list-style-type: none"> ◆ Represent real situations and mathematical relationships graphically and symbolically ◆ Use linear equations to solve, model, and analyze problems involving proportional relationships ◆ Apply technology when representing mathematical relationships ◆ Solve linear equations concretely and graphically ◆ Graph relationships between two variables in real life
6.P.5 Solve linear equations using concrete models, tables, graphs, and paper-pencil methods.	
6.P.6 Produce and interpret graphs that represent the relationship between two variables in everyday situations.	
8.P.9 Use linear equations to model and analyze problems involving proportional relationships. Use technology as appropriate.	

Possible ENTRY POINTS to Learning Standard(s)

Less Complex		More Complex
<p style="text-align: center;"><u>The student will:</u></p> <ul style="list-style-type: none"> ◆ Graph, plot, or mark numbers on a number line ◆ Use pictures, models, tables, charts, graphs, words, number sentences, and mathematical notations to represent and/or interpret mathematical relationships ◆ Solve problems involving proportional relationships, including unit pricing (e.g., four apples cost 80 cents, so one apple costs 20 cents) and map interpretation (one inch represents five miles, so two inches represent ten miles) <p><i>See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials</i></p>	<p style="text-align: center;"><u>The student will:</u></p> <ul style="list-style-type: none"> ◆ Graph ordered pairs on a grid ◆ Use symbolic expression to describe the rule of a pattern, e.g., $2n + 5$ ◆ Create tables, graphs, number charts 	<p style="text-align: center;"><u>The student will:</u></p> <ul style="list-style-type: none"> ◆ Interpret real-world data using tables, charts, graphs, and/or algebraic equations ◆ Match the appropriate equation or inequality to its graph ◆ Use increasingly complex proportional relationships to solve problems <p><i>Continue to address skills and concepts that approach grade-level expectations in this subject</i></p>

Instructional Idea based on Learning Standards 6.P.4 and 6.P.6

At grades 5 – 6, students each plant a seed and record the growth of the resulting plant on a chart for 20 days. Each student's chart includes the dates, the daily height of the student's plant, and the daily rate of change in the plant's growth. Based on the raw data, each student creates a graph depicting the growth of the plant over time, and develops a formula to predict future growth.

How all students can participate in this activity

Addressing Learning Standard(s) as written for this grade level	Possible Assessment Strategies and Portfolio Products
Banji plants and records the growth of her seed. In a report, she describes the relationship among the three variables of time, height, and rate of growth.	<ul style="list-style-type: none"> • Data chart showing Banji's ability to record the relationship between two variables, time and height • Banji's graph of plant growth in height (cm) vs. time (days) • Banji's formula predicting the future relationship between height and time for her plant
Addressing Learning Standard(s) at lower levels of complexity (Entry Points)	Possible Assessment Strategies and Portfolio Products
Alvin works with a partner to grow a plant from a seed and record whether it has gotten bigger, smaller, or stayed the same. He records his data in a table (grew/stayed the same/got smaller vs. number of days)	<ul style="list-style-type: none"> • Data chart showing Alvin's ability to track two variables • Table completed by Alvin graphing size (smaller, same, bigger) vs. day of the week • Alvin's written interpretation of his results
Addressing Access Skills embedded in academic instruction	Possible Assessment Strategies and Portfolio Products
Patrick helps his partners graph their data. He identifies the day of the week on his calendar and matches the correct day on his peers' graphs.	<ul style="list-style-type: none"> • Field data chart showing Patrick's ability to identify the correct day • Line or bar graph summarizing field data chart • Teacher-scribed work sample indicating how Patrick's responses match calendar to day of the week

Mathematics
Strand 2
***Patterns, Relations,
and Algebra***

Relationships and
Mathematical Models

9 – 10

CONTENT Mathematics

STRAND Patterns, Relations, and Algebra

Learning Standards for Relationships and Mathematical Models

Grade Level: 9 – 10

Learning Standards as written

Essence of the Standard(s)

10.P.7	Solve everyday problems that can be modeled using linear, reciprocal, quadratic, or exponential functions. Apply appropriate tabular, graphical, or symbolic methods to the solution. Include compound interest, and direct and inverse variation problems. Use technology when appropriate.	<ul style="list-style-type: none"> ◆ Solve everyday problems using linear, reciprocal, quadratic, or exponential functions, including compound interest ◆ Use graphs and tables to represent mathematical solutions ◆ Solve problems that can be modeled using systems of equations, including solve mixture, rate and work problems ◆ Use linear equations to solve everyday problems
10.P.8	Solve everyday problems that can be modeled using systems of linear equations or inequalities. Apply algebraic and graphical methods to the solution. Use technology when appropriate. Include mixture, rate, and work problems.	

Possible ENTRY POINTS to Learning Standard(s)

<p style="font-size: 1.5em; font-weight: bold;">← Less Complex</p> <p style="font-weight: bold; text-align: center;">The student will:</p> <ul style="list-style-type: none"> ◆ Represent real situations and mathematical relationships on a table, graph, or using words, and/or symbols ◆ Solve linear equations using models, tables, graphs, and/or paper and pencil ◆ Produce and interpret tables and/or graphs that represent the relationship between two variables in everyday situations <p><i>See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials</i></p>	<p style="font-weight: bold; text-align: center;">The student will:</p> <ul style="list-style-type: none"> ◆ Use linear equations to model and analyze problems involving proportional relationships ◆ Use technology to represent, model, and demonstrate a mathematical relationship 	<p style="font-size: 1.5em; font-weight: bold;">More Complex →</p> <p style="font-weight: bold; text-align: center;">The student will:</p> <ul style="list-style-type: none"> ◆ Define the following mathematical functions: linear, reciprocal, quadratic, and exponential ◆ Use tables, graphs, and symbols to solve increasingly complex problems ◆ Identify the following concepts: interest, compound interest, direct variation, inverse variation ◆ Recognize and solve mixture, rate, and work problems ◆ Continue to use increasingly complex proportional relationships, models, representations, and functions to solve mathematical problems <p><i>Continue to address skills and concepts that approach grade-level expectations in this subject</i></p>
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CONTENT Mathematics

STRAND Patterns, Relations, and Algebra

Learning Standards for Variables and Change

Grade Level: 3 – 4, 5 – 6

Learning Standards as written

Essence of the Standard(s)

<p>4.P.6</p>	<p>Determine how change in one variable relates to a change in a second variable, e.g., input-output tables.</p>	<ul style="list-style-type: none"> ◆ Describe the concept of change using input-output tables ◆ Recognize that altering one variable results in change in another ◆ Identify and describe the relationship between two variables with constant rate of change and compare with those whose rate of change are not constant
<p>6.P.7</p>	<p>Identify and describe relationships between two variables with a constant rate of change. Contrast these with relationships where the rate of change is not constant.</p>	

Possible ENTRY POINTS to Learning Standard(s)

Less Complex

More Complex

ACCESS SKILLS

The student will:

- ◆ Activate an electronic device to access text, communicate with others, or participate in an instructional activity
- ◆ Turn attention toward another person
- ◆ Grasp, release, and manipulate materials relevant to the instructional activity
- ◆ Take turns appropriately during classroom discussion
- ◆ Respond to/initiate contacts with others
- ◆ Apply rules for appropriate classroom behavior
- ◆ Use appropriate social greetings
- ◆ Follow simple directions
- ◆ Identify self and/or others
- ◆ Initiate or respond to request for joint attention

ENTRY POINTS

The student will:

- ◆ Give examples to show how routines sometimes change due to changes in a variable over which we have no control
- ◆ Give examples to show how individuals can manipulate situations by changing certain variables (student can control/affect change)
- ◆ Construct sets to show cause and effect
- ◆ Give examples of changes that can be controlled
- ◆ Use input-output tables to demonstrate simple relationships

The student will:

- ◆ Identify and describe how a change in a variable affects something concrete in an individual's surroundings, routine, or social interactions
- ◆ Provide examples showing how changes in one variable result in changes to another variable in an equation
- ◆ Define "constant rate of change" and give an example
- ◆ Use input-output tables to demonstrate relationships between variables

The student will:

- ◆ Explain how change in one variable affects another variable when the variables are related
- ◆ Use models, pictures, and/or graphs to show how a change in one variable affects the change in another in a functional relationship
- ◆ Identify and describe how varying the rate of change in a variable affects the outcome
- ◆ Use an input-output table
- ◆ Define and demonstrate examples of rates of change that are constant and not constant
- ◆ Use increasingly complex input-output tables to demonstrate relationships

Continue to address skills and concepts that approach grade-level expectations in this subject

CONTENT Mathematics
STRAND Patterns, Relations, and Algebra
Learning Standards for Variables and Change

Grade Level: 7 – 8

Learning Standards as written

Essence of the Standard(s)

8.P.5	Identify the slope of a line as a measure of its steepness and as a constant rate of change from its table of values, equation, or graph. Apply the concept of slope to the solution of problems.	<ul style="list-style-type: none"> ◆ Identify slope of a line as a constant rate of change in steepness, and that can be applied to solve problems ◆ Recognize that a change in one variable results in a change in another variable in functional relationships ◆ Represent and compare patterns of linear growth
8.P.8	Explain and analyze—both quantitatively and qualitatively, using pictures, graphs, charts, or equations—how a change in one variable results in a change in another variable in functional relationships, e.g., $C = \pi d$, $A = \pi r^2$ (A as a function of r), $A_{\text{rectangle}} = lw$ ($A_{\text{rectangle}}$ as a function of l and w).	
8.P.10	Use tables and graphs to represent and compare linear growth patterns. In particular, compare rates of change and x- and y-intercepts of different linear patterns.	

Possible ENTRY POINTS to Learning Standard(s)

Less Complex

More Complex

The student will:

- ◆ Given two graphical representations, identify the rate of change that is constant and the rate of change that is not constant
- ◆ Contrast relationships between two variables with constant rate of change with those whose rates of change are not constant
- ◆ Describe the effect of on one variable of changing the other variable in simple problems

See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials

The student will:

- ◆ Describe the effect that changing one variable has on another
- ◆ Define slope related to rates of change
- ◆ Identify and graphically represent constant rate of change
- ◆ Identify and graphically represent rate of change that is not constant
- ◆ Solve problems involving rates of change

The student will:

- ◆ Use increasingly complex models, pictures, tables, and graphs to show how a change in one variable affects the outcome in a real-world situation
- ◆ Identify and describe or portray rates of change using a variety of models and representations
- ◆ Identify linear and nonlinear growth patterns (e.g., chart the growth of a plant, animal, or person and demonstrate type/rate of change)
- ◆ Set up and solve linear equations
- ◆ Demonstrate how changing one variable affects another variable in the equation, using pictures, graphs, charts, or equations

Continue to address skills and concepts that approach grade-level expectations in this subject

Geometry

	Characteristics of Geometric Shapes	Spatial Relationships/ Coordinate Geometry	Transformation/ Symmetry	Visualization/ Spatial Reasoning/ Geometric Modeling
PreK – K	Page 185	Page 194	No learning standards for this grade span	No learning standards for this grade span
1 – 2	Pages 186 – 187	No learning standards for this grade span	Page 197	Page 203
3 – 4	Pages 188 – 189	Page 195	Page 198	No learning standards for this grade span
5 – 6	Page 190	Page 195	Pages 199 – 200	Page 204
7 – 8	Page 191	No learning standards for this grade span	Page 201	Page 205
9 – 10	Pages 192 – 193	Page 196	Page 202	Page 206

CONTENT Mathematics
STRAND Geometry

Learning Standards for Characteristics of Geometric Shapes

Grade Level: PreK – K

Learning Standards as written

Essence of the Standard(s)

K.G.1	Name, describe, sort, and draw simple two-dimensional shapes.	<ul style="list-style-type: none"> ◆ Identify, describe, draw, and compare simple two-dimensional shapes ◆ Identify and compare three-dimensional shapes
K.G.2	Describe attributes of two-dimensional shapes, e.g., number of sides, number of corners.	
K.G.3	Name and compare three-dimensional shapes.	

Possible ENTRY POINTS to Learning Standard(s) (and ACCESS SKILLS embedded in standards-based activities)

Less Complex

More Complex



ACCESS SKILLS
The student will:

- ◆ Follow simple direction to match shape to shape
- ◆ Grasp, release, and manipulate a variety of shapes related to instructional activity
- ◆ Demonstrate 1:1 correspondence by matching shapes
- ◆ Activate an electronic device to access text, communicate with others, or participate in an instructional activity
- ◆ Initiate or respond to request for joint attention
- ◆ Respond to/initiate contacts with others
- ◆ Turn attention toward another person
- ◆ Apply rules for appropriate classroom behavior

ENTRY POINTS
The student will:

- ◆ Match simple two-dimensional shapes
- ◆ Identify geometric shapes in the environment

The student will:

- ◆ Sort simple two-dimensional and three-dimensional shapes with similar attributes (number of sides or faces, corners, etc.)
- ◆ Match similar shapes of different sizes and colors
- ◆ Draw simple two-dimensional shapes

The student will:

- ◆ Identify simple shapes (e.g., circle, square, triangle) and match to familiar objects (e.g., circle → ball)
 - ◆ Sort shapes by attribute and describe attributes used
 - ◆ Match simple three-dimensional shapes
- Continue to address skills and concepts that approach grade-level expectations in this subject*

CONTENT Mathematics

STRAND Geometry

Learning Standards for Characteristics of Geometric Shapes

Grade Level: 1 – 2

Learning Standards as written

Essence of the Standard(s)

2.G.1	Describe attributes and parts of two- and three-dimensional shapes, e.g., length of sides, and number of corners, edges, faces, and sides.	<ul style="list-style-type: none"> ◆ Describe, draw, name, and compare two-dimensional shapes, such as polygons and curved figures (circles) ◆ Describe, draw, name, and compare three-dimensional shapes ◆ Describe attributes of polygons ◆ Recognize congruent shapes
2.G.2	Identify, describe, draw, and compare two-dimensional shapes, including both polygonal (up to six sides) and curved figures such as circles.	
2.G.3	Recognize congruent shapes.	

Possible ENTRY POINTS to Learning Standard(s)

Less Complex

More Complex



The student will:

- ◆ Match shapes with corresponding symbols, pictures, and shapes in the environment (e.g., stop sign)

See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials

The student will:

- ◆ Identify and compare two- and three-dimensional shapes
- ◆ Identify the differences between shapes
- ◆ Sort simple shapes

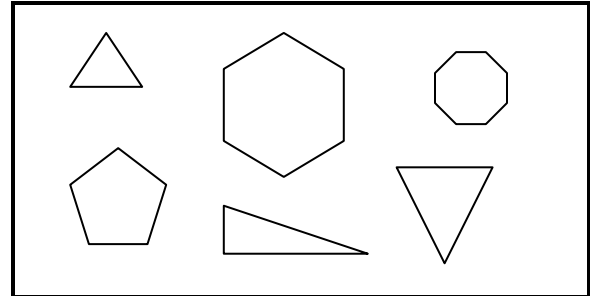
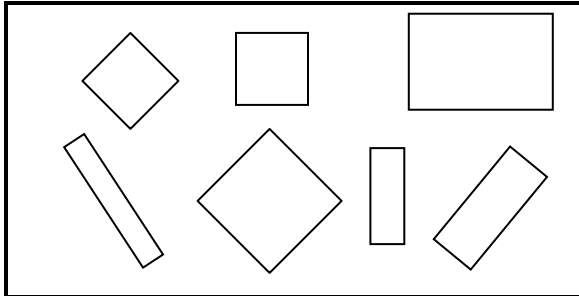
The student will:

- ◆ Sort more complex two-dimensional shapes (such as rectangles and half-circles)
- ◆ Draw and compare more complex shapes (in addition to circles, squares, and triangles), such as rectangle
- ◆ Match and sort simple congruent shapes, e.g., congruent triangles
- ◆ Identify more complex shapes

Continue to address skills and concepts that approach grade-level expectations in this subject

Instructional Idea based on Learning Standards 2.G.1 and 2.G.2

At grades 1 – 2, students answer the following question based on the sets shown below: What rule was used to sort the shapes into two groups?



How all students can participate in this activity

Addressing Learning Standard(s) as written for this grade level	Possible Assessment Strategies and Portfolio Products
Mika determines the rule to sort the shapes organized above – shapes with four sides and shapes without four sides. He uses manipulatives to create his own problems for peers to solve at his classroom math center.	<ul style="list-style-type: none"> • Data chart showing Mika's knowledge of the properties of geometric shapes • Mika's mathematics journal on sorting/grouping of geometric shapes by different properties • Work samples of sorting problems Mika created for his peers (with corresponding correct answers)
Addressing Learning Standard(s) at lower levels of complexity (Entry Points)	Possible Assessment Strategies and Portfolio Products
During this activity, Kelly matches shapes that are the same (e.g., square – square, circle – circle). She identifies each shape by name and finds an example of each in the classroom.	<ul style="list-style-type: none"> • Data chart showing Kelly's ability to match geometric shapes • Videotape of Kelly matching shapes • Teacher-scribed work sample showing the shapes Kelly matched and the name of each
Addressing Access Skills embedded in academic instruction	Possible Assessment Strategies and Portfolio Products
Walter manipulates tools and materials correctly and safely (e.g., glue, scissors) to help peers create sets of shapes to compare in the classroom mathematics center.	<ul style="list-style-type: none"> • Data chart showing Walter's ability to use a variety of tools and materials safely • Videotape of Walter making sets of shapes • Sample shapes created by Walter

CONTENT Mathematics

STRAND Geometry

Learning Standards for Characteristics of Geometric Shapes

Grade Level: 3 – 4

Learning Standards as written

Essence of the Standard(s)

4.G.1	Compare and analyze attributes and other features (e.g., number of sides, faces, corners, right angles, diagonals, and symmetry) of two- and three-dimensional geometric shapes.	<ul style="list-style-type: none"> ◆ Compare and analyze attributes and features of shapes (e.g., diagonals, faces, symmetry) ◆ Draw, describe, compare, and classify two- and three-dimensional shapes ◆ Recognize similar shapes (e.g., similar pairs of triangles) ◆ Describe and draw lines (parallel, intersecting, perpendicular) and angles created by intersecting lines (acute, right, obtuse)
4.G.2	Describe, model, draw, compare, and classify two- and three-dimensional shapes, e.g., circles, polygons—especially triangles and quadrilaterals—cubes, spheres, and pyramids.	
4.G.3	Recognize similar figures.	
4.G.4	Identify angles as acute, right, or obtuse.	
4.G.5	Describe and draw intersecting, parallel, and perpendicular lines.	

Possible ENTRY POINTS to Learning Standard(s)

Less Complex

More Complex

The student will:

- ◆ Identify parts of shapes (e.g., lines, angles, curves)
- ◆ Sort and compare two- and three-dimensional shapes by attributes, e.g., length of sides, number of sides, etc.
- ◆ Recognize geometric shapes in the environment

The student will:

- ◆ Sort and compare shapes by two or more attributes
- ◆ Identify, describe, draw and/or trace, and compare two-dimensional shapes, e.g., polygons, circles, squares, triangles, rectangles
- ◆ Match right angles with familiar objects in the environment

The student will:

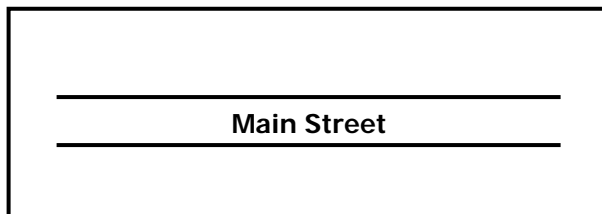
- ◆ Identify complex shapes as similar, regardless of other attributes (e.g., two triangles of different size but with the same angles)
- ◆ Compare shapes by their attributes and other features
- ◆ Identify an angle as part of a shape (the space between two intersecting lines)
- ◆ Identify two lines that never meet as parallel

See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials

Continue to address skills and concepts that approach grade-level expectations in this subject

Instructional Idea based on Learning Standard 4.G.5

At grades 3 – 4, students learn about the relationships among various lines (intersecting, parallel, and perpendicular), and use their knowledge to answer questions and extend the map (below). For their final project, students create and present their maps.



Final Project Directions:

1. Draw Broadway Street *parallel* to Main Street. Write the name Broadway on this street.
2. Draw Birch Street *perpendicular* to Main Street. Write the name Birch on this street.
3. Draw Walnut Street so that it *intersects* Main Street but is not perpendicular to Main Street. Write the name Walnut on this street.

How all students can participate in this activity

Addressing Learning Standard(s) as written for this grade level	Possible Assessment Strategies and Portfolio Products
Boris follows the directions and completes his map. He identifies lines on the map, then creates his own poster-size map, labels it, and presents it to the class.	<ul style="list-style-type: none"> • Data chart showing Boris's correct identification of lines • Boris's work sample, map, and answers to questions • Photograph of Boris's poster-size map • Homework assignment for which Boris identified and drew lines
Addressing Learning Standard(s) at lower levels of complexity (Entry Points)	Possible Assessment Strategies and Portfolio Products
For her presentation, Danica answers questions using a peer's map. She uses a picture vocabulary board to match and identify streets that cross (intersect) and streets that are next to one another (parallel).	<ul style="list-style-type: none"> • Data chart showing Danica's progress demonstrating her knowledge of vocabulary related to lines • Danica's homework assignment relating to types of lines • Videotape of Danica identifying streets on her map
Addressing Access Skills embedded in academic instruction	Possible Assessment Strategies and Portfolio Products
Jen completes her map using computer software with assistance from a peer, Ryan, who helps Jen make choices of lines and streets for her map. He helps Jen advance slides for a PowerPoint presentation to the class.	<ul style="list-style-type: none"> • Field data chart indicating Jen's ability to make choices for her map using the computer • Line or bar graph summarizing field data chart • Videotape of Jen making choices with assistance from Ryan in order to present her map project

CONTENT Mathematics

STRAND Geometry

Learning Standards for Characteristics of Geometric Shapes

Grade Level: 5 – 6

Learning Standards as written

Essence of the Standard(s)

<p>6.G.1</p>	<p>Identify polygons based on their properties, including types of interior angles, perpendicular or parallel sides, and congruence of sides, e.g., squares, rectangles, rhombuses, parallelograms, trapezoids, and isosceles, equilateral, and right triangles.</p>	<ul style="list-style-type: none"> ◆ Identify the properties of polygons and three-dimensional shapes, such as squares, rhombuses, parallelograms, cubes, cones, and pyramids ◆ Identify relationships among points, lines, and planes (e.g., a line is perpendicular to a plane, two planes are parallel, etc.)
<p>6.G.2</p>	<p>Identify three-dimensional shapes (e.g. cubes, prisms, spheres, cones, and pyramids) based on their properties, such as edges and faces.</p>	
<p>6.G.3</p>	<p>Identify relationships among points, lines, and planes, e.g., intersecting, parallel, and perpendicular.</p>	

Possible ENTRY POINTS to Learning Standard(s)

Less Complex

More Complex



The student will:

- ◆ Reproduce a shape
- ◆ Draw different angles and categorize as obtuse, right, or acute

See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials

The student will:

- ◆ Identify polygons by name
- ◆ Identify, describe, draw and/or trace, and compare two- and three-dimensional shapes (e.g., cubes, cones, polygons, spheres)

The student will:

- ◆ Identify, describe, draw and/or trace, and compare two- and three-dimensional shapes (e.g., cubes, cones, polygons, spheres)
- ◆ Identify and draw lines that are intersecting, perpendicular, parallel, and categorize/describe
- ◆ Identify points, lines, and planes

Continue to address skills and concepts that approach grade-level expectations in this subject

CONTENT Mathematics
STRAND Geometry

Learning Standards for Characteristics of Geometric Shapes

Grade Level: 7 – 8

Learning Standards as written

Essence of the Standard(s)

8.G.1	Analyze, apply, and explain the relationship between the number of sides and the sums of the interior and exterior angle measures of polygons.	<ul style="list-style-type: none"> ◆ Explain the relationship between the number of sides of a polygon and the sums of its interior and exterior angles ◆ Classify similar shapes and apply information to solve problems ◆ Compare relationships among angles formed by intersecting lines ◆ Understand and apply the Pythagorean theorem ◆ Identify 3-dimensional figures by spatial relationships (e.g., a cube has 6 faces, and each face is perpendicular to its adjacent face)
8.G.2	Classify figures in terms of congruence and similarity, and apply these relationships to the solution of problems.	
8.G.3	Demonstrate an understanding of the relationships of angles formed by intersecting lines, including parallel lines cut by a transversal.	
8.G.4	Demonstrate an understanding of the Pythagorean theorem. Apply the theorem to the solution of problems.	
8.G.7	Identify three-dimensional figures (e.g., prisms, pyramids) by their physical appearance, distinguishing attributes, and spatial relationships such as parallel faces.	

Possible ENTRY POINTS to Learning Standard(s)

Less Complex

More Complex

The student will:

- ◆ Describe and/or reproduce a three-dimensional figure using a variety of materials
- ◆ Identify polygons based on their properties (e.g., angles, number of sides, congruence of sides)

See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials

The student will:

- ◆ Identify relationships among lines, and among planes
- ◆ Identify, describe, draw and/or trace, and compare three-dimensional shapes

The student will:

- ◆ Identify and measure interior angles of a polygon (e.g., triangle, parallelogram)
- ◆ Describe and compare properties of a pair of lines, or a line and a plane, or two planes
- ◆ Solve simple geometric problems by applying geometric properties and relationships
- ◆ Solve problems using the Pythagorean theorem (e.g., determine the length of a side of a triangle)

Continue to address skills and concepts that approach grade-level expectations in this subject

CONTENT Mathematics
STRAND Geometry
Learning Standards for Characteristics of Geometric Shapes

Grade Level: 9 – 10

Learning Standards as written		Essence of the Standard(s)
10.G.1	Identify figures using properties of sides, angles, and diagonals. Identify the figures' type(s) of symmetry.	<ul style="list-style-type: none"> ◆ Identify figures by properties and determine types of symmetry ◆ Draw congruent and similar figures using a variety of tools ◆ Recognize and solve problems associated with radii, chords, and arcs ◆ Use congruence and similarity to find missing quantities in geometric figures; justify answers and prove results
10.G.2	Draw congruent and similar figures using a compass, straightedge, protractor, and other tools such as computer software. Make conjectures about methods of construction. Justify the conjectures by logical arguments.	
10.G.3	Recognize and solve problems involving angles formed by transversals of coplanar lines. Identify and determine the measure of central and inscribed angles and their associated minor and major arcs. Recognize and solve problems associated with radii, chords, and arcs within or on the same circle.	
10.G.4	Apply congruence and similarity correspondences (e.g., $\triangle ABC \cong \triangle XYZ$) and properties of the figures to find missing parts of geometric figures, and provide logical justification.	

Possible ENTRY POINTS to Learning Standard(s)

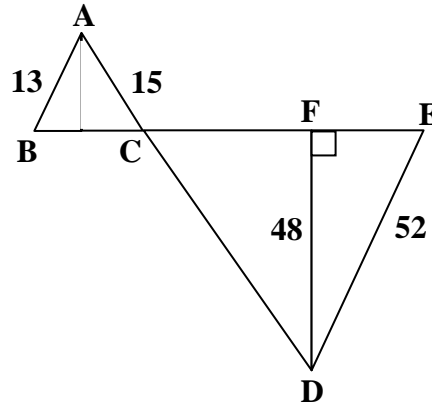
Less Complex	↔	More Complex
<p>The student will:</p> <ul style="list-style-type: none"> ◆ Recognize shapes and parts of shapes that can be created using a compass and straightedge ◆ Draw two- and three-dimensional geometric shapes using tools, such as a protractor or computer software <p><i>See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials</i></p>	<p>The student will:</p> <ul style="list-style-type: none"> ◆ Describe complex three-dimensional shapes by attributes and relationships of lines/angles ◆ Identify an appropriate sequence to solve a problem based on geometric properties and shapes ◆ Match congruent and similar figures ◆ Define the Pythagorean theorem and its use 	<p>The student will:</p> <ul style="list-style-type: none"> ◆ Solve geometric problems by applying geometric properties and relationships (including concepts of congruence and similarity) ◆ Describe the steps taken and the answer obtained in solving a problem involving geometric properties and relationships <p><i>Continue to address skills and concepts that approach grade-level expectations in this subject</i></p>

Instructional Idea based on Learning Standards 10.G.4

At grades 9 – 10, students solve geometric problems, like the one below, requiring deduction and proof.

Problem:

In this figure, line AB is parallel to line DE and line DF is perpendicular to line CE. Determine the perimeter of triangle CDE. Explain how you found your answer and how you know it is correct.



How all students can participate in this activity

Addressing Learning Standard(s) as written for this grade level	Possible Assessment Strategies
Emily solves problems in her geometry class and presents her reasoning for each process, using an overhead projector and slides.	<ul style="list-style-type: none"> • Data chart showing Emily's ability to solve geometry problems • Emily's quiz in which she solves problems • Emily's homework assignments that include geometric problems • Videotape of Emily's presentation justifying her solutions
Addressing Learning Standard(s) at lower levels of complexity (Entry Points)	Possible Assessment Strategies and Portfolio Products
Jonas works on identifying geometric shapes and measures the sides and angles of each figure.	<ul style="list-style-type: none"> • Data chart showing Jonas' ability to use tools to gather information (ruler/protractor to measure lines/angles) • Videotape of Jonas measuring the sides and angles of a shape • A completed homework assignment in which Jonas measures angles/sides of objects in his home
Addressing Access Skills embedded in academic instruction	Possible Assessment Strategies and Portfolio Products
Valerie helps Emily (a peer) give her presentation by passing objects to Emily for her to measure as part of her presentation.	<ul style="list-style-type: none"> • Field data chart indicating Valerie's ability to follow one-step directions • Line or bar graph summarizing the field data chart • Teacher-scribed work sample listing the objects Valerie was asked to pass to Emily • Videotape of Valerie handing the identified objects to Emily when directed to do so

CONTENT Mathematics
STRAND Geometry
Learning Standards for Spatial Relationships/
Coordinate Geometry

Grade Level: PreK – K

Learning Standards as written Essence of the Standard(s)

K.G.4	Identify positions of objects in space, and use appropriate language (e.g., beside, inside, next to, close to, above, below, apart) to describe and compare their relative positions.	♦ Identify, describe, and compare the relative positions of objects
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Possible ENTRY POINTS to Learning Standard(s) (and ACCESS SKILLS embedded in standards-based activities)

Less Complex

More Complex



ACCESS SKILLS
The student will:

- ♦ Tolerate a variety of body positions and movement during geometry lessons
- ♦ Grasp, release, and manipulate materials relevant to the instructional activity
- ♦ Initiate or respond to request for joint attention
- ♦ Take turns appropriately during classroom discussion
- ♦ Respond to/initiate contacts with others
- ♦ Activate an electronic device to access text, communicate with others, or participate in an instructional activity
- ♦ Follow simple directions
- ♦ Use appropriate social greetings
- ♦ Turn attention toward another person
- ♦ Apply rules for appropriate classroom behavior

ENTRY POINTS
The student will:

- ♦ Given a direction and a demonstration of a simple positional concept, student will respond appropriately (e.g., put your binder on your desk; put the cube behind the globe)
- ♦ Match pictures representing common positional concepts

The student will:

- ♦ Demonstrate positions of objects in space (e.g., up, down, over, under, near, far)
- ♦ Follow a direction involving a higher-level positional concept (e.g., turn left at the library)

The student will:

- ♦ Identify an object's location in space from a given point of view
- ♦ Identify an object's location in space relative to another object, using appropriate language
- ♦ Follow a given direction involving a combination of positional concepts (e.g., Put your name in the upper right-hand corner of your paper)
- ♦ Choose appropriate vocabulary to describe relative positions of objects

Continue to address skills and concepts that approach grade-level expectations in this subject

Mathematics

**Strand 3
Geometry**

Spatial Relationships/
Coordinate Geometry

3 – 4, 5 – 6

CONTENT Mathematics
STRAND Geometry
Learning Standards for Spatial Relationships/
Coordinate Geometry

Grade Level: 3 - 4, 5 – 6

Learning Standards as written		Essence of the Standard(s)
4.G.6	Using ordered pairs of numbers and/or letters, graph, locate, identify points, and describe paths (first quadrant).	<ul style="list-style-type: none"> ◆ Locate, identify, and interpret ordered pairs ◆ Graph coordinates of points ◆ Find distance between two points on a number line
6.G.4	Graph points and identify coordinates of points on the Cartesian coordinate plane (all four quadrants).	
6.G.5	Find the distance between two points on horizontal or vertical number lines.	

Possible ENTRY POINTS to Learning Standard(s)

Less Complex	More Complex
<p>The student will:</p> <ul style="list-style-type: none"> ◆ Identify numbers on a horizontal number line ◆ Identify numbers on a vertical number line <p><i>See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials</i></p>	<p>The student will:</p> <ul style="list-style-type: none"> ◆ Place numbers correctly on a number line ◆ Determine how many numbers separate two points on a number line <p><i>Continue to address skills and concepts that approach grade-level expectations in this subject</i></p>
<p>The student will:</p> <ul style="list-style-type: none"> ◆ Portray information on distance between points on a number line either graphically or in verbal or written form ◆ Plot and recognize points in a single quadrant of a Cartesian coordinate (graph) plane 	

CONTENT Mathematics
STRAND Geometry
Learning Standards for Spatial Relationships/
Coordinate Geometry

Grade Level: 9 – 10

Learning Standards as written		Essence of the Standard(s)
10.G.7	Using rectangular coordinates, calculate midpoints of segments, slopes of lines and segments, and distances between two points, and apply the results to the solutions of problems.	<ul style="list-style-type: none"> ◆ Perform calculations to determine <ul style="list-style-type: none"> • midpoints of segments • slopes of lines/segments • distances between two points ◆ Solve problems using above calculations ◆ Find linear equations representing lines parallel or perpendicular to a given line and through a given point
10.G.8	Find linear equations that represent lines either perpendicular or parallel to a given line and through a point, e.g., by using the “point-slope” form of the equation.	

Possible ENTRY POINTS to Learning Standard(s)

Less Complex

More Complex



The student will:

- ◆ Find the distance between two points on a horizontal or vertical number line

See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials

The student will:

- ◆ Plot points on a number line

The student will:

- ◆ Graph points and identify coordinates of points on the Cartesian coordinate plane (all four quadrants)
- ◆ Determine lengths and midpoints of horizontal and vertical lines and figures on a plane (either given to or drawn by student)
- ◆ Measure the slope of a line on a coordinate plane
- ◆ Demonstrate and describe concepts of parallel, perpendicular, midpoint, and slope of a line

Continue to address skills and concepts that approach grade-level expectations in this subject

CONTENT Mathematics
STRAND Geometry
Learning Standards for Transformation/Symmetry

Grade Level: 1 – 2

Learning Standards as written		Essence of the Standard(s)
2.G.4	Identify shapes that have been rotated (turned), reflected (flipped), translated (slid), and enlarged. Describe direction of translations, e.g., left, right, up, down.	<ul style="list-style-type: none"> ◆ Identify shapes that are turned, flipped, slid, enlarged ◆ Identify symmetry in two-dimensional shapes
2.G.5	Identify symmetry in two-dimensional shapes.	

Possible ENTRY POINTS to Learning Standard(s) (and ACCESS SKILLS embedded in standards-based activities)

Less Complex		More Complex	
<p>ACCESS SKILLS The student will:</p> <ul style="list-style-type: none"> ◆ Grasp and manipulate two- and three-dimensional objects ◆ Activate an electronic device to access text, communicate with others, or participate in an instructional activity ◆ Turn attention toward another person ◆ Respond to/initiate contacts with others ◆ Initiate or respond to request for joint attention ◆ Follow simple directions ◆ Use appropriate social greetings ◆ Turn attention toward another person ◆ Apply rules for appropriate classroom behavior 	<p>ENTRY POINTS The student will:</p> <ul style="list-style-type: none"> ◆ Complete puzzles by matching shapes ◆ Use manipulatives to turn and flip two- and three-dimensional objects ◆ Match identical and similar shapes ◆ Identify shapes that have been rotated, reflected, translated, and enlarged 	<p>The student will:</p> <ul style="list-style-type: none"> ◆ Use manipulatives to demonstrate movement of a shape, e.g., follow directions to rotate it, move it left, right, up, and down ◆ Use manipulatives to move a variety of objects left, right, up, and down 	<p>The student will:</p> <ul style="list-style-type: none"> ◆ Demonstrate movement of an object or shape along a straight line ◆ Identify symmetries (about an axis) of simple two-dimensional shapes ◆ Use manipulatives and technology to manipulate a variety of objects in order to demonstrate rotation, reflection, translation, and increase/reduction in size <p><i>Continue to address skills and concepts that approach grade-level expectations in this subject</i></p>

CONTENT Mathematics
STRAND Geometry
Learning Standards for Transformation/Symmetry

Grade Level: 3 – 4

Learning Standards as written		Essence of the Standard(s)
4.G.7	Describe and apply techniques such as reflections (flips), rotations (turns), and translations (slides) for determining if two shapes are congruent.	<ul style="list-style-type: none"> ◆ Determine congruency of shapes by turning, flipping, and sliding ◆ Determine symmetry of two-dimensional shapes ◆ Separate, fold, and combine two-dimensional shapes
4.G.8	Identify and describe line symmetry in two-dimensional shapes.	
4.G.9	Predict and validate the results of partitioning, folding, and combining two- and three-dimensional shapes.	

Possible ENTRY POINTS to Learning Standard(s)

Less Complex

More Complex



The student will:

- ◆ Demonstrate correct and appropriate movement (translation) of a shape or object along a flat surface using the language *left, right, up, down*
- ◆ Fold objects
- ◆ Combine and/or separate two objects

See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials

The student will:

- ◆ Demonstrate and describe movement of an object through space on a surface
- ◆ Fold objects to demonstrate symmetry
- ◆ Combine and/or separate two or more objects

The student will:

- ◆ Separate, fold, and combine two-dimensional shapes, using manipulatives
- ◆ Identify shapes that are symmetrical, using manipulatives

Continue to address skills and concepts that approach grade-level expectations in this subject

CONTENT Mathematics
STRAND Geometry
Learning Standards for Transformation/Symmetry

Grade Level: 5 – 6

Learning Standards as written		Essence of the Standard(s)
6.G.6	Predict, describe, and perform transformations on two-dimensional shapes, e.g., translations, rotations, and reflections.	<ul style="list-style-type: none"> ◆ Predict, describe, and perform transformations on two-dimensional shapes ◆ Identify line and rotational symmetry ◆ Determine congruency by measuring sides and/or angles
6.G.7	Identify types of symmetry, including line and rotational.	
6.G.8	Determine if two shapes are congruent by measuring sides or a combination of sides and angles, as necessary; or by motions or series of motions, e.g., translations, rotations, and reflections.	

Possible ENTRY POINTS to Learning Standard(s)

Less Complex

More Complex



The student will:

- ◆ Name the transformations performed on two-dimensional shapes
- ◆ Sort items that have line and rotational symmetry
- ◆ Demonstrate rotation of an object

See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials

The student will:

- ◆ Define, describe, and/or demonstrate symmetry of a two-dimensional shape
- ◆ Determine congruency of two similar shapes by measuring their sides and angles

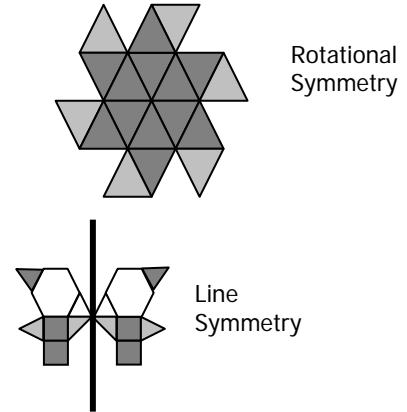
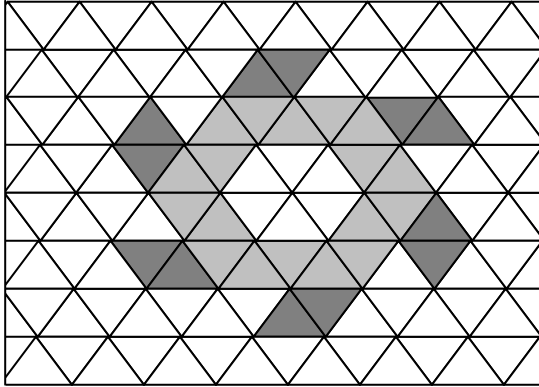
The student will:

- ◆ Perform simplified transformations, using two- and three-dimensional objects
- ◆ Using line and angle measurements for two different shapes, determine if they are congruent
- ◆ Use technology to create, rotate, skew, and transform shapes digitally

Continue to address skills and concepts that approach grade-level expectations in this subject

Instructional Idea based on Learning Standard 6.G.7

At grades 5 – 6, students determine the type(s) of symmetry (line and/or rotational) that exist(s) in the figure below. Students then draw their own patterns demonstrating line and rotational symmetry.



How all students can participate in this activity

Addressing Learning Standard(s) as written for this grade level	Possible Assessment Strategies and Portfolio Products
Rashida identifies the types of symmetry represented in different figures. Rashida then creates her own visual representations of both types of symmetry.	<ul style="list-style-type: none"> • Data chart showing Rashida's ability to identify line and rotational symmetry • Rashida's completed homework in which she identifies line and rotational symmetry in different figures • Rashida's work sample of symmetrical objects she has created
Addressing Learning Standard(s) at lower levels of complexity (Entry Points)	Possible Assessment Strategies and Portfolio Products
Carlos uses less complex shapes to determine line symmetry by folding each design in half. Carlos creates his own shapes using manipulatives, and demonstrates line symmetry using a mirror.	<ul style="list-style-type: none"> • Data chart showing Carlos's ability to identify lines of symmetry in various figures • Videotape of Carlos manipulating figures to determine lines of symmetry • Carlos's worksheet in which he identified line symmetry for objects in his classroom and home
Addressing Access Skills embedded in academic instruction	Possible Assessment Strategies and Portfolio Products
Kaya works on the computer using her touch screen, selecting and placing shapes of varying color and size. Her peers use these visuals to determine their symmetry.	<ul style="list-style-type: none"> • Data chart showing Kaya's progress using her touch screen to make choices during mathematics activities • Computer printout of Kaya's shape selections noting how she made each choice • Videotape showing Kaya using her computer program to make choices

Mathematics
Strand 3
Geometry
Transformation/
Symmetry

7 – 8

CONTENT Mathematics
STRAND Geometry
Learning Standards for Transformation/Symmetry

Grade Level: 7 – 8

Learning Standards as written **Essence of the Standard(s)**

8.G.6	Predict the results of transformations on unmarked or coordinate planes and draw the transformed figure, e.g., predict how tessellations transform under translations, reflections, and rotations.	♦ Predict, describe, and perform transformations, including tessellations for more complex shapes
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Possible ENTRY POINTS to Learning Standard(s)

<p style="font-size: 1.2em; font-weight: bold;">← Less Complex</p> <p style="text-align: center; font-weight: bold;">The student will:</p> <ul style="list-style-type: none"> ♦ Describe the outcome of transforming a figure ♦ Describe how an object is the same object after a transformation has occurred <p><i>See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials</i></p>	<p style="font-size: 1.2em; font-weight: bold;">← More Complex</p> <p style="text-align: center; font-weight: bold;">The student will:</p> <ul style="list-style-type: none"> ♦ Select the appropriate two- or three-dimensional figure when given the scale factor (e.g., find the box that is three times as big) ♦ Translate, reflect, and rotate figures on a plane 	<p style="font-size: 1.2em; font-weight: bold;">← More Complex</p> <p style="text-align: center; font-weight: bold;">The student will:</p> <ul style="list-style-type: none"> ♦ Identify and describe a tessellation ♦ Predict, perform, and describe transformations of two-dimensional shapes ♦ Translate, reflect, and rotate figures on a plane ♦ Perform and interpret translations, reflections, rotations, scale factors, or size changes on an object <p><i>Continue to address skills and concepts that approach grade-level expectations in this subject</i></p>
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CONTENT Mathematics
STRAND Geometry
Learning Standards for Transformation/Symmetry

Grade Level: 9 – 10

Learning Standards as written		Essence of the Standard(s)
10.G.1	Identify figures using properties of sides, angles, and diagonals. Identify the figures' type(s) of symmetry.	<ul style="list-style-type: none"> ◆ Identify figures given their properties of: <ul style="list-style-type: none"> • sides • angles • diagonals ◆ Identify types of symmetry of a figure ◆ Perform and interpret transformations on figures in the coordinate plane, and draw the results ◆ Use transformations to solve problems
10.G.9	Draw the results, and interpret transformations on figures in the coordinate plane, e.g., translations, reflections, rotations, scale factors, and the results of successive transformations. Apply transformations to the solutions of problems.	

Possible ENTRY POINTS to Learning Standard(s)

The student will:	The student will:	The student will:
<ul style="list-style-type: none"> ◆ Describe and identify shapes by attributes, including sizes, angles, and sides ◆ Rotate a figure and draw or identify the result ◆ Identify sides and angles on a figure ◆ Identify and draw diagonal lines through a figure ◆ Draw symmetrical figures <p><i>See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials</i></p>	<ul style="list-style-type: none"> ◆ Determine if two shapes are congruent by measuring sides and angles ◆ Identify and describe a variety of transformations on figures, including translation, reflection, rotation, and scale factors 	<ul style="list-style-type: none"> ◆ Predict the outcome and perform a variety of transformations on figures, including translation, reflection, rotation, and scale factors ◆ Identify the types of symmetry between two figures ◆ Identify and describe figures using properties of sides and angles <p><i>Continue to address skills and concepts that approach grade-level expectations in this subject</i></p>

Mathematics

Strand 3
Geometry

Visualization/
Spatial Reasoning/
Geometric Modeling

1 – 2

CONTENT Mathematics

STRAND Geometry

Learning Standards for Visualization/Spatial Reasoning/
Geometric Modeling

Grade Level: 1 – 2

Learning Standards as written

Essence of the Standard(s)

2.G.6	Predict the results of putting shapes together and taking them apart.	<ul style="list-style-type: none"> ◆ Put shapes together ◆ Take shapes apart ◆ Relate geometric ideas to numbers
2.G.7	Relate geometric ideas to numbers, e.g., seeing rows in an array as a model of repeated addition.	

Possible ENTRY POINTS to Learning Standard(s) (and ACCESS SKILLS embedded in standards-based activities)

Less Complex

More Complex

ACCESS SKILLS The student will:	ENTRY POINTS The student will:	The student will:	The student will:
<ul style="list-style-type: none"> ◆ Grasp, release, and manipulate geometric figures ◆ Activate an electronic device to access text, communicate with others, or participate in an instructional activity ◆ Establish joint attention to shapes ◆ Initiate or respond to request for joint attention ◆ Turn attention toward another person ◆ Use appropriate social greetings ◆ Follow simple directions ◆ Turn attention toward another person ◆ Apply rules for appropriate classroom behavior 	<ul style="list-style-type: none"> ◆ Manipulate and combine simple shapes to create new, complex shapes ◆ Manipulate and take apart shapes ◆ Create an array of objects ◆ Replicate an array of objects 	<ul style="list-style-type: none"> ◆ Describe/show the result of manipulating and combining/taking apart simple shapes to create new shapes ◆ Manipulate and combine/take apart simple shapes to create or replicate a model ◆ Describe a geometric array or model 	<ul style="list-style-type: none"> ◆ Manipulate simple and complex shapes to create new three-dimensional forms, and describe them ◆ Given a model of a shape, identify simple and more complex shapes that make up the model ◆ Explain why geometric ideas can be explained numerically ◆ Describe and compare geometric arrays and models <p><i>Continue to address skills and concepts that approach grade-level expectations in this subject</i></p>

CONTENT Mathematics

STRAND Geometry

Learning Standards for: Visualization/Spatial Reasoning/
Geometric Modeling

Grade Level: 5 – 6

Learning Standards as written

Essence of the Standard(s)

6.G.9 Match three-dimensional objects and their two-dimensional representations, e.g., nets, projections, and perspective drawings.

◆ Represent two-dimensional objects related to three-dimensional objects

Possible ENTRY POINTS to Learning Standard(s)

Less Complex

More Complex

The student will:

- ◆ Identify a variety of two- and three-dimensional shapes
- ◆ Match three-dimensional objects with their two-dimensional representations

See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials

The student will:

- ◆ Use manipulatives to match three-dimensional shapes to their two-dimensional representations (e.g., the projection or perspective drawing of a cube)

The student will:

- ◆ Show, describe, and/or identify the following representations of geometric figures:
 - nets
 - projections
 - perspective drawings
- ◆ Manipulate simple and complex shapes to create new three-dimensional forms, and describe them
- ◆ Draw or describe two-dimensional representations of three-dimensional objects

Continue to address skills and concepts that approach grade-level expectations in this subject

CONTENT Mathematics
STRAND Geometry

**Learning Standards for Visualization/Spatial Reasoning/
Geometric Modeling**

Grade Level: 7 – 8

Learning Standards as written

Essence of the Standard(s)

<p>8.G.5</p>	<p>Use a straight-edge, compass, or other tools to formulate and test conjectures, and to draw geometric figures.</p>	<ul style="list-style-type: none"> ◆ Use tools to formulate and test conjectures and draw geometric figures ◆ Recognize three-dimensional objects from two-dimensional representations, e.g., perspective drawings
<p>8.G.8</p>	<p>Recognize and draw two-dimensional representations of three-dimensional objects, e.g., nets, projections, and perspective drawings.</p>	

Possible ENTRY POINTS to Learning Standard(s)

Less Complex

More Complex



The student will:

- ◆ Use a straightedge, compass, computer software, and/or other tools to replicate two-dimensional geometric figures from a model
- ◆ Match two-dimensional objects

See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials

The student will:

- ◆ Manipulate simple and complex two- and three-dimensional shapes to create a complex visual model
- ◆ Draw and describe two-dimensional objects
- ◆ Match three-dimensional objects with their two-dimensional representations

The student will:

- ◆ Match three-dimensional objects and their two-dimensional representations, e.g., nets, projections, and perspective drawings
- ◆ Draw and describe nets, projections, perspective drawings, and other two-dimensional representations of three-dimensional objects

Continue to address skills and concepts that approach grade-level expectations in this subject

CONTENT Mathematics

STRAND Geometry

Learning Standards for Visualization/Spatial Reasoning/
Geometric Modeling

Grade Level: 9 – 10

Learning Standards as written

Essence of the Standard(s)

10.G.5	Solve simple triangle problems using the triangle angle sum property and/or the Pythagorean theorem.	<ul style="list-style-type: none"> ◆ Solve simple triangle problems using both the Pythagorean theorem and the triangle angle sum property ◆ Use properties of special triangles (i.e., isosceles, equilateral, 30°-60°-90°, 45°-45°-90°) to solve problems ◆ Visualize solid objects and their cross sections and projections ◆ Use vertex-edge graphs to solve problems
10.G.6	Use the properties of special triangles (e.g. isosceles, equilateral, 30°- 60°-90°, 45°-45°-90°) to solve problems.	
10.G.10	Demonstrate the ability to visualize solid objects and recognize their projections and cross sections.	
10.G.11	Use vertex-edge graphs to model and solve problems.	

Possible ENTRY POINTS to Learning Standard(s)

Less Complex

More Complex

The student will:

- ◆ Identify/describe cross section and projection
- ◆ Sort and match the following triangles by type: isosceles, equilateral, and right angle
- ◆ Recognize and match solid objects placed in different positions

See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials

The student will:

- ◆ Measure the angles of a triangle
- ◆ Measure angles of the following special triangles: isosceles, equilateral, and right angle
- ◆ Select the correct representation of a cross section of a three-dimensional solid object, given several choices

The student will:

- ◆ Represent a projection or cross section of a solid object
- ◆ Identify and describe a vertex-edge graph
- ◆ Measure, identify and classify triangles (e.g., isosceles, equilateral)
- ◆ Solve simple problems involving angles and sides of triangles

Continue to address skills and concepts that approach grade-level expectations in this subject

Measurement

PreK – K	Page 208
1 – 2	Pages 209 – 211
3 – 4	Pages 212 – 214
5 – 6	Pages 215 – 216
7 – 8	Page 217
9 – 10	Page 218

CONTENT Mathematics
STRAND Measurement

Grade Level: PreK – K

Learning Standards as written	Essence of the Standard(s)
K.M.1 Recognize and compare the attributes of length, volume/capacity, weight, area, and time using appropriate language, e.g., longer, taller, shorter, same length; heavier, lighter, same weight; holds more, holds less, holds the same amount.	<ul style="list-style-type: none"> ◆ Use appropriate language to discuss, compare, and contrast measurement attributes ◆ Understand the notion of measurement ◆ Estimate measurements ◆ Measure objects using informal units of measurement
K.M.2 Make and use estimates of measurements from everyday experiences.	
K.M.3 Use nonstandard units to measure length, area, weight, and capacity.	

Possible ENTRY POINTS to Learning Standard(s) (and ACCESS SKILLS embedded in standards-based activities)

Less Complex		More Complex
<p>ACCESS SKILLS The student will:</p> <ul style="list-style-type: none"> ◆ Grasp objects to be measured and tools for measurement ◆ Manipulate objects of various weights, sizes, lengths, heights, etc. ◆ Initiate or respond to request for joint attention ◆ Follow simple directions ◆ Use appropriate social greetings ◆ Respond to/initiate contacts with others ◆ Activate an electronic device to access text, communicate with others, or participate in an instructional activity ◆ Turn attention toward another person ◆ Apply rules for appropriate classroom behavior 	<p>ENTRY POINTS The student will:</p> <ul style="list-style-type: none"> ◆ Match two pictures, objects, and/or manipulatives of the same length or weight ◆ Match objects using appropriate measurement tools 	<p>The student will:</p> <ul style="list-style-type: none"> ◆ Demonstrate understanding of concepts of length, volume/capacity, weight, area, and time ◆ Describe the similarity(ies) between two pictures, objects, and/or manipulatives using measurement concepts ◆ Match two objects of the same approximate area or capacity
		<p>The student will:</p> <ul style="list-style-type: none"> ◆ Categorize and sort items of similar length, volume/capacity, weight, area, and/or duration ◆ Measure length, area, weight, and/or capacity of familiar items using standard and nonstandard units of measurement ◆ Estimate measurement based on observation of similar objects <p style="text-align: right;"><i>Continue to address skills and concepts that approach grade-level expectations in this subject</i></p>

CONTENT Mathematics
STRAND Measurement

Grade Level: 1 – 2

Learning Standards as written

Essence of the Standard(s)

2.M.1	Identify parts of the day (e.g., morning, afternoon, evening), days of the week, and months of the year. Identify dates using a calendar.	<ul style="list-style-type: none"> ◆ Identify parts of day, days of the week, and months of the year ◆ Measure length, weight, volume, capacity, and time, using standard units ◆ Tell time/read and use a calendar ◆ Use simple measurement tools correctly ◆ Compare objects based on measurement attributes ◆ Estimate measurements
2.M.2	Tell time at quarter-hour intervals on analog and digital clocks using a.m. and p.m.	
2.M.3	Compare the length, weight, area, and volume of two or more objects by using direct comparison.	
2.M.4	Measure and compare common objects using metric and English units of length measurement, e.g., centimeter, inch.	
2.M.5	Select and correctly use the appropriate measurement tools, e.g., ruler, balance scale, thermometer.	
2.M.6	Make and use estimates of measurement, including time, volume, weight, and area.	

Possible ENTRY POINTS to Learning Standard(s)

Less Complex

More Complex

The student will:

- ◆ Use picture-based schedules and calendars to manage time throughout the day/week/month
- ◆ Identify the date using a calendar
- ◆ Identify and use environmental cues that relate to activities from daily schedule
- ◆ Compare familiar objects by size, weight, or other measurement attributes

See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials

The student will:

- ◆ Demonstrate understanding of the concepts today/tomorrow/yesterday
- ◆ Identify clock as a tool to measure time
- ◆ Identify intervals of time
- ◆ Identify familiar tools that are used for measurement
- ◆ Use descriptive language of measurement to compare two objects
- ◆ Compare two objects of different lengths, weights, sizes, or other measurement attribute

The student will:

- ◆ Identify today/tomorrow/yesterday on the calendar
- ◆ Describe the schedule for a given school day using pictures or other mode of communication
- ◆ Tell time to the hour on analog and digital clocks using a.m. and p.m.
- ◆ Match digital time to analog time
- ◆ Given a situation requiring measurement, identify the correct tool to use
- ◆ Use appropriate tools to measure common objects using English units of length
- ◆ Compare two or more objects of different lengths, weights, sizes or other measurement attribute
- ◆ Make estimates of weight, height, volume, and length for a given object

Continue to address skills and concepts that approach grade-level expectations in this subject

Instructional Idea based on Learning Standards 2.M.1 and 2.M.2

At grades 1 – 2, students make a schedule of school activities, recording times to the hour and half-hour. An alarm is set on both a digital clock and an analog clock to ring at the start of each new activity. Students read the clocks to verify that they match the times on their schedules.

How all students can participate in this activity

Addressing Learning Standard(s) as written for this grade level	Possible Assessment Strategies and Portfolio Products
Heidi records the time to the hour or half-hour for the start of each daily activity. She matches clocks at the start of each activity with her schedule and identifies the time.	<ul style="list-style-type: none"> • Data chart showing Heidi's ability to tell time to the hour and half hour • Homework assignment for which Heidi created her schedule to the hour and half hour showing beginning times of activities • Heidi's quiz on telling time to the hour and half hour
Addressing Learning Standard(s) at lower levels of complexity (Entry Points)	Possible Assessment Strategies and Portfolio Products
When the alarm rings at the start of each new activity, Aidan uses a stamp to indicate on his picture schedule the activity that is finished. He then points to the next activity on his schedule.	<ul style="list-style-type: none"> • Data chart showing Aidan's ability to identify a finished activity and the next activity on his schedule • Videotape of Aidan responding after hearing the alarm ring • Aidan's selection of one day's schedule for his portfolio showing his stamps for each activity
Addressing Access Skills embedded in academic instruction	Possible Assessment Strategies and Portfolio Products
When the alarm rings, a peer guides Evan to his schedule box. The peer assists him in transitioning to the new activity by putting away the objects from the finished activity and selecting the objects representing the next scheduled activity.	<ul style="list-style-type: none"> • Field data chart showing Evan's ability to put items into his schedule box without dropping or throwing them • Line or bar graph summarizing the field data chart • Teacher-scribed work sample charting Evan's ability to place each item into the schedule box

CONTENT Mathematics
STRAND Measurement

Grade Level: 3 – 4

Learning Standards as written

Essence of the Standard(s)

4.M.1	Demonstrate an understanding of such attributes as length, area, weight, and volume, and select the appropriate type of unit for measuring each attribute.	<ul style="list-style-type: none"> ◆ Demonstrate understanding of attributes of measurement such as length, weight, volume, capacity, time ◆ Covert measurement units (inches to feet, minutes to hours) ◆ Tell time/read the calendar ◆ Estimate and measure perimeter and area ◆ Use appropriate measurement tools ◆ Measure and estimate using standard and metric units
4.M.2	Carry out simple unit conversions within a system of measurement, e.g., hours to minutes, cents to dollars, yards to feet or inches, etc.	
4.M.3	Identify time to the minute on analog and digital clocks using a.m. and p.m. Compute elapsed time using clock (e.g., hours and minutes since...) and using a calendar (e.g., days since...).	
4.M.4	Estimate and find area and perimeter of a rectangle, triangle, or irregular shape using diagrams, models, and grids or by measuring.	
4.M.5	Identify and use appropriate metric and English units and tools (e.g., ruler, angle ruler, graduated cylinder, thermometer) to estimate, measure, and solve problems involving length, area, volume, weight, time, angle size, and temperature.	

Less Complex

More Complex



The student will:

- ◆ Identify the appropriate tool for measuring, for example:
 - clock for telling time
 - ruler for measuring length, width, and height
 - scale for measuring weight
 - thermometer for measuring temperature
- ◆ Identify the difference between hot/warm/cool/cold as designated by pictures, measurements, numbers on a thermometer, and/or personal experience
- ◆ Use environmental cues to understand concept of time and its association with certain activities
- ◆ Recognize money as the means for making purchases (Note: actual use of money is addressed in Number Sense and Operations)

See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials

The student will:

- ◆ Measure elapsed time to half-hour and quarter-hour intervals
- ◆ Match/read a thermometer to 10 degree intervals and describe temperature with appropriate vocabulary
- ◆ Make estimates of measurement, and use measurement tools to verify estimates
- ◆ Correlate linear measurement to numerical representation (5 feet = 5')
- ◆ State equivalents within a system of measurement (e.g., minutes/hour, inches/feet)
- ◆ Use appropriate tools to measure lines and angles in a figure

The student will:

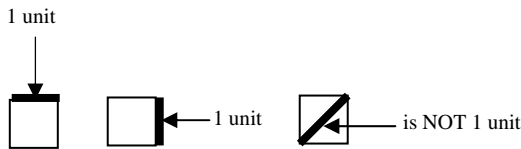
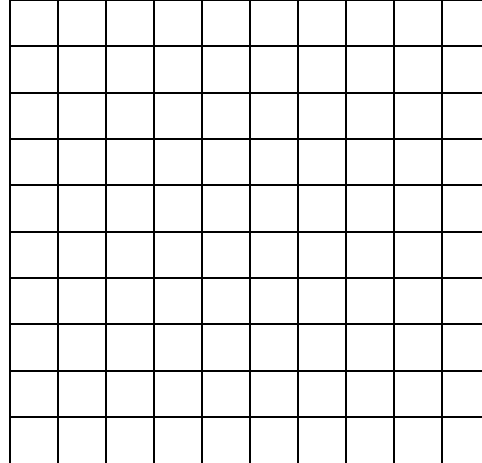
- ◆ Measure elapsed time to ten- and five-minute intervals
- ◆ Read a thermometer and describe temperature using appropriate vocabulary
- ◆ Use appropriate units and tools to measure perimeter, make measurements, estimates, and compare areas
- ◆ Use a ruler to measure objects using standard units (e.g., centimeters and inches)
- ◆ Apply measurement concepts and tools (English or metric) to solve problems involving length, area, volume, weight, time, angle size, and temperature
- ◆ Label the result of a measurement with the correct unit
- ◆ Perform simple conversions of units (inches to feet; ounces to pounds)

Continue to address skills and concepts that approach grade-level expectations in this subject

Instructional Idea based on Learning Standard 4.M.4

At grades 3 – 4, students are told that the City Park Committee wants to fence in an area of the neighborhood park for young children to use as a playground. The playground must use twelve sections of fence. Each section of fence is one unit long. Students are then asked to follow the directions below.

1. On the grid at the right, draw 4 different **closed** shapes using 12 sections of fence.
2. What is the area of each of your shapes? Write the area inside each shape.
3. Decide which shape would be the best one for the playground. Circle your choice. Explain why this is the best shape for a playground.



How all students can participate in this activity

Addressing Learning Standard(s) as written for this grade level	Possible Assessment Strategies and Portfolio Products
<p>Chyou plots four possible closed shapes for the park on the grid provided. She determines the area of each shape and makes recommendations for the best possible design. Chyou makes a three-dimensional model of the design for her final project.</p>	<ul style="list-style-type: none"> • Data chart showing Chyou's ability to determine the area of shapes • Chyou's work sample determining the area of each design • Photograph of Chyou's three-dimensional design and calculation of the area for that design
Addressing Learning Standard(s) at lower levels of complexity (Entry Points)	Possible Assessment Strategies and Portfolio Products
<p>Rebekka works with Chyou. She counts and records the number of squares contained in each design. Rebekka compares each number and decides which playground will have the greatest area.</p>	<ul style="list-style-type: none"> • Data chart showing Rebekka's ability to determine "large" and "small" in relation to area • Rebekka's work sample recording the number of squares per design • Photographs of Rebekka counting squares using the design from Chyou's three-dimensional model
Addressing Access Skills embedded in academic instruction	Possible Assessment Strategies and Portfolio Products
<p>Sabrina works on her fine motor skills by helping Chyou construct her model of the playground design. Sabrina hands the tools to Chyou.</p>	<ul style="list-style-type: none"> • Field data chart showing Sabrina's ability to group tools and materials as instructed by Chyou • Line or bar graph summarizing the field data chart • Videotape of Sabrina helping Chyou

CONTENT Mathematics
STRAND Measurement

Grade Level: 5 – 6

Learning Standards as written

Essence of the Standard(s)

6.M.1	Apply the concepts of perimeter and area to the solution of problems. Apply formulas where appropriate.	<ul style="list-style-type: none"> ◆ Use and apply concepts and formulas for measuring and describing <ul style="list-style-type: none"> • perimeter • area • proportions • angles • triangles • quadrilaterals ◆ Identify, measure, and describe circles and their properties, and the relationships among parts of a circle ◆ Calculate volume and surface area of a rectangular prism ◆ Calculate the sum of angles in a polygon
6.M.2	Identify, measure, describe, classify, and construct various angles, triangles, and quadrilaterals.	
6.M.3	Solve problems involving proportional relationships and units of measurement, e.g., same system unit conversions, scale models, maps, and speed.	
6.M.4	Find areas of triangles and parallelograms. Recognize that shapes with the same number of sides but different appearances can have the same area. Develop strategies to find the area of more complex shapes.	
6.M.5	Identify, measure, and describe circles and the relationships of the radius, diameter, circumference, and area (e.g., $d = 2r$, $p = C/d$), and use the concepts to solve problems.	
6.M.6	Find volumes and surface areas of rectangular prisms.	
6.M.7	Find the sum of the angles in simple polygons (up to eight sides) with and without measuring the angles.	

Possible ENTRY POINTS to Learning Standard(s)

Less Complex

More Complex

The student will:

- ◆ Measure the lengths of sides/angles of various shapes
- ◆ Measure area for simple two-dimensional shapes
- ◆ State the correct measurement to use in solving a given mathematical problem
- ◆ Match measurements to corresponding terms and abbreviations (d=diameter, p=perimeter)
- ◆ Compare similar shapes and identify which is heavier, longer, wider, etc.

See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials

The student will:

- ◆ Find the areas of parallelograms and triangles when given their formulas for area
- ◆ Find the distance between two locations on a map using the scale provided
- ◆ Calculate circumference/perimeter for a circle, square, rectangle, or polygon

The student will:

- ◆ Understand and apply increasingly complex measurement concepts
- ◆ Perform increasingly complex conversions of units
- ◆ Compute area and perimeter using square unit measurements
- ◆ Find the sum of angles for triangles, squares, and rectangles
- ◆ Identify, describe, and apply concepts of radius, diameter, circumference, and area for circles

Continue to address skills and concepts that approach grade-level expectations in this subject

CONTENT Mathematics
STRAND Measurement

Grade Level: 7 – 8

Learning Standards as written		Essence of the Standard(s)
8.M.1	Select, convert (within the same system of measurement), and use appropriate units of measurement or scale.	<ul style="list-style-type: none"> ◆ Select and use appropriate units of measurement ◆ Determine area and perimeter/circumference for parallelograms, trapezoids, and circles ◆ Determine surface area, perimeter, circumference, and volume for simple three-dimensional shapes ◆ Convert units of length, capacity, weight, and time from one system to another (metric and customary/English) ◆ Solve problems involving ratio/proportion ◆ Solve problems involving rate (e.g., velocity, density)
8.M.2	Given the formulas, convert from one system of measurement to another. Use technology as appropriate.	
8.M.3	Demonstrate an understanding of the concepts and apply formulas and procedures for determining measures, including those of area and perimeter/circumference of parallelograms, trapezoids, and circles. Given the formulas, determine the surface area and volume of rectangular prisms, cylinders, and spheres. Use technology as appropriate.	
8.M.4	Use ratio and proportion (including scale factors) in the solution of problems, including problems involving similar plane figures and indirect measurement.	
8.M.5	Use models, graphs, and formulas to solve simple problems involving rates, e.g., velocity and density.	

Possible ENTRY POINTS to Learning Standard(s)

Less Complex		More Complex
<p><u>The student will:</u></p> <ul style="list-style-type: none"> ◆ Given everyday situations, differentiate between number vs. rate (25 pages vs. 25 pages/hour) ◆ Measure perimeter for simple two-dimensional shapes <p><i>See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials</i></p>	<p><u>The student will:</u></p> <ul style="list-style-type: none"> ◆ Convert from one system of measurement to another using a conversion table ◆ Indicate whether a measurement applies to surface area or volume (square inches vs. cubic inches); identify for which shape (two- or three- dimensional) the measurement is used ◆ Calculate area for simple two-dimensional shapes 	<p><u>The student will:</u></p> <ul style="list-style-type: none"> ◆ Apply increasingly complex measurement concepts for solving problems involving angles, proportional relationships, measurements of geometric shapes (including circles, parallelograms, trapezoids), and surface areas/volumes (including prisms, cylinders, spheres) ◆ Perform conversions of units between systems or scales of measurement ◆ Solve problems using ratio, proportions, and/or scale factors ◆ Calculate area for two-dimensional shapes, including circles ◆ Solve problems involving rate <p><i>Continue to address skills and concepts that approach grade-level expectations in this subject</i></p>

CONTENT Mathematics
STRAND Measurement

Grade Level: 9 – 10

Learning Standards as written		Essence of the Standard(s)
10.M.1	Calculate perimeter, circumference, and area of common geometric figures such as parallelograms, trapezoids, circles, and triangles.	<ul style="list-style-type: none"> ◆ Determine area, perimeter, and circumference for more complex two-dimensional shapes ◆ Describe how a change in one attribute causes changes in other attributes of an object ◆ Estimate measurements and determine the effects of rounding ◆ Determine surface area, lateral area, and volume for complex three-dimensional shapes
10.M.2	Given the formula, find the lateral area, surface area, and volume of prisms, pyramids, spheres, cylinders, and cones, e.g., find the volume of a sphere with a specified surface area.	
10.M.3	Relate changes in the measurement of one attribute of an object to changes in other attributes, e.g., how changing the radius or height of a cylinder affects its surface area or volume.	
10.M.4	Describe the effects of approximate error in measurement and rounding on measurements and on computed values from measurements.	

Possible ENTRY POINTS to Learning Standard(s)

Less Complex	More Complex
<p style="text-align: center;"><u>The student will:</u></p> <ul style="list-style-type: none"> ◆ Identify prisms, pyramids, spheres, cylinders, and cones ◆ Identify parallelograms, trapezoids, circles, and triangles ◆ Compare height, width, perimeter, circumference, and radius of similar shapes <p><i>See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials</i></p>	<p style="text-align: center;"><u>The student will:</u></p> <ul style="list-style-type: none"> ◆ Match a formula for area and volume to its appropriate shape ◆ Indicate, or estimate and verify, which shape has the larger surface area or volume, given two similar shapes ◆ Compare surface area and volume of two shapes for which one attribute (e.g., height or width) is dissimilar
	<p style="text-align: center;"><u>The student will:</u></p> <ul style="list-style-type: none"> ◆ Apply increasingly complex measurement concepts for solving problems involving angles, proportional relationships, measurements of geometric shapes (including circles, parallelograms, trapezoids, triangles), and surface areas/volumes (including prisms, cylinders, spheres, pyramids, cones) ◆ Explore changes in the attributes of shapes and how these affect surface area and/or volume ◆ Understand and apply formulas to solve problems involving measurement <p><i>Continue to address skills and concepts that approach grade-level expectations in this subject</i></p>

Data Analysis, Statistics, and Probability

	Data and Statistics	Probability
PreK – K	Page 220	No learning standards for this grade span
1 – 2	Page 221	Page 230
3 – 4	Pages 222 – 223	Page 231
5 – 6	Pages 224 – 225	Page 232
7 – 8	Pages 226 – 228	Page 233
9 – 10	Page 229	Page 234

Mathematics

Strand 5
***Data Analysis,
Statistics, and
Probability***

Data and Statistics

PreK – K

CONTENT Mathematics

STRAND Data Analysis, Statistics, and Probability
Learning Standards for Data and Statistics

Grade Level: PreK – K

Learning Standards as written

Essence of the Standard(s)

K.D.1	Collect, sort, organize, and draw conclusions about data using concrete objects, pictures, numbers, and graphs.	◆ Represent data concretely and draw conclusions
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Possible ENTRY POINTS to Learning Standard(s) (and ACCESS SKILLS embedded in standards-based activities)

Less Complex

More Complex

ACCESS SKILLS

The student will:

- ◆ Follow one-step directions
- ◆ Grasp, manipulate, and release objects and materials used in the instructional activity
- ◆ Initiate or respond to request for joint attention
- ◆ Follow simple directions
- ◆ Use appropriate social greetings
- ◆ Respond to/initiate contacts with others
- ◆ Activate an electronic device to access text, communicate with others, or participate in an instructional activity
- ◆ Turn attention toward another person
- ◆ Apply rules for appropriate classroom behavior

ENTRY POINTS

The student will:

- ◆ Collect, match, and/or sort objects with similar characteristics
- ◆ Describe characteristics of an object, picture, or manipulative

The student will:

- ◆ Given two attributes sort and organize objects, pictures, or manipulatives

The student will:

- ◆ Describe/draw conclusions about data using concrete objects, pictures, and/or manipulatives
 - ◆ Match representations of data (objects, pictures, numbers, graphs) to descriptions of data collection activities
- Continue to address skills and concepts that approach grade-level expectations in this subject*

CONTENT Mathematics

STRAND Data Analysis, Statistics, and Probability

Learning Standards for Data and Statistics

Grade Level: 1 – 2

Learning Standards as written

Essence of the Standard(s)

2.D.1	Use interviews, surveys, and observations to gather data about themselves and their surroundings.	<ul style="list-style-type: none"> ◆ Represent simple data in different formats ◆ Interpret and draw conclusions about simple data
2.D.2	Organize, classify, represent, and interpret data using tallies, charts, tables, bar graphs, pictographs, and Venn diagrams; interpret the representations.	
2.D.3	Formulate inferences (draw conclusions) and make educated guesses (conjectures) about a situation based on information gained from data.	

Possible ENTRY POINTS to Learning Standard(s)

Less Complex

More Complex



The student will:

- ◆ Collect, sort, and organize objects by characteristics

See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials

The student will:

- ◆ Represent raw data on a template of a graph, table, or chart

The student will:

- ◆ Gather data about things that are familiar and represent using an organizational tool (e.g., chart, list, tally, table)
- ◆ Identify and describe the information on graphs, tables, charts used to display data
- ◆ Label data accurately
- ◆ Make observations on data collected

Continue to address skills and concepts that approach grade-level expectations in this subject

CONTENT Mathematics
STRAND Data Analysis, Statistics, and Probability
Learning Standards for Data and Statistics

Grade Level: 3 – 4

Learning Standards as written		Essence of the Standard(s)
4.D.1	Collect and organize data using observations, measurements, surveys, or experiments, and identify appropriate ways to display the data.	<ul style="list-style-type: none"> ◆ Collect, organize, and describe data in a variety of formats ◆ Match different formats that represent similar data ◆ Draw conclusions and make predictions about data represented in different formats
4.D.2	Match representations of a data set such as lists, tables, or graphs (including circle graphs) with actual set of data.	
4.D.3	Construct, draw conclusions, and make predictions from various representations of data sets, including tables, bar graphs, pictographs, line graphs, line plots, and tallies.	

Possible ENTRY POINTS to Learning Standard(s)

Less Complex

More Complex



The student will:

- ◆ Make observations about data and use observations to predict patterns in outcomes

See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials

The student will:

- ◆ Identify which data is important to collect in a given situation
- ◆ Match data with a corresponding graph
- ◆ Identify data on a graph, table, or other graphical representation

The student will:

- ◆ Given a set of data, construct a graph, table, or chart
- ◆ Identify data on a graph, table, or chart
- ◆ Organize data efficiently and label data accurately on a graph, table, or list
- ◆ Draw conclusions and make predictions based on data gathered (or to be gathered)

Continue to address skills and concepts that approach grade-level expectations in this subject

Instructional Idea based on Learning Standard 4.D.1

At grades 3 – 4, during a presidential campaign, students read and discuss polls that attempt to predict the outcome of the election. Students talk about whether it is practical to survey every voter in the United States as to how they voted. The class discusses how they might go about finding an appropriate sample of voters.

How all students can participate in this activity

Addressing Learning Standard(s) as written for this grade level	Possible Assessment Strategies and Portfolio Products
Gayla discusses with her classmates how to take a sample of data and use it to draw conclusions about the presidential election with her classmates. She outlines strategies for sampling data and organizes the information in a visual presentation.	<ul style="list-style-type: none"> • Data chart showing Gayla's ability to collect data • List of strategies Gayla developed to collect data • Gayla's data from a completed survey of classmates
Addressing Learning Standard(s) at lower levels of complexity (Entry Points)	Possible Assessment Strategies and Portfolio Products
Sam conducts a poll of his classmates during the presidential election. He surveys ten peers and records their votes on a list. Sam then graphs the data on a pie chart.	<ul style="list-style-type: none"> • Data chart showing Sam's ability to record data • Sam's completed pie chart • Sam's completed homework assignment on calculating and organizing data
Addressing Access Skills embedded in academic instruction	Possible Assessment Strategies and Portfolio Products
Lian assists Sam in polling classmates by using his communication device to ask questions about their vote.	<ul style="list-style-type: none"> • Field data chart showing Lian's ability to use his communication device to ask classmates questions • Line or bar graph summarizing the field data chart • Videotape of Lian participating in the interview using his communication device

CONTENT Mathematics
STRAND Data Analysis, Statistics, and Probability
Learning Standards for Data and Statistics

Grade Level: 5 – 6

Learning Standards as written		Essence of the Standard(s)
6.D.1	Describe and compare data sets using the concepts of median, mean, mode, maximum and minimum, and range.	<ul style="list-style-type: none"> ◆ Understand the concepts of mean, median, mode, maximum, minimum, and range ◆ Construct and interpret the following: <ul style="list-style-type: none"> • stem-and-leaf plots • line plots • circle graphs
6.D.2	Construct and interpret stem-and-leaf plots, line plots, and circle graphs.	

Possible ENTRY POINTS to Learning Standard(s)

Less Complex

More Complex



The student will:

- ◆ Accurately construct graphs, charts, and/or tables for a set of data
- ◆ Demonstrate understanding and answer questions about organized data on a graph, table, or chart
- ◆ Make inferences and draw conclusions about data on a graph, table, or chart
- ◆ Compare and contrast data represented in different graphical formats (e.g., a pie chart vs. a line graph)

See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials

The student will:

- ◆ Demonstrate understanding of the concepts maximum, minimum, and range
- ◆ Identify different types of graphical representations, including stem-and-leaf plots, line plots, and circle graphs
- ◆ Select the most effective graphical representations for displaying different sets of data

The student will:

- ◆ Demonstrate understanding of the meanings and use of the terms median, mean, mode, maximum, minimum, and range to describe data
- ◆ Match sets of data to their graphical representations, including stem-and-leaf plots, line plots, and circle graphs
- ◆ Construct a stem-and-leaf plot, line plot, and/or circle graphs for a simple set of data

Continue to address skills and concepts that approach grade-level expectations in this subject

Instructional Idea based on Learning Standard 6.D.1

At grades 5 – 6, students review their test scores at the end of a semester before taking their final exam. They determine the lowest possible score they can get on the final exam to maintain their current letter grade, and the score they need in order to progress to the next letter grade.

How all students can participate in this activity

Addressing Learning Standard(s) as written for this grade level	Possible Assessment Strategies and Portfolio Products
<p>Olivia has the following scores on her tests: 87, 86, 96, and 87. She calculates that she must receive at least a 94 to end the semester with an A-. Her additional calculations prove that an A- is the highest grade she can get, even if she receives 100 on the final exam. Olivia graphs her test results on a bar graph.</p>	<ul style="list-style-type: none"> • Data chart showing Olivia's progress in demonstrating her knowledge of maximum, minimum, mean, median, range, and mode • Work sample showing Olivia's calculations of mean, median, range and mode • Olivia's calculation of her mean test scores given a variety of final test scores
Addressing Learning Standard(s) at lower levels of complexity (Entry Points)	Possible Assessment Strategies and Portfolio Products
<p>Parker reviews his progress on meeting his math goals for the year, using his homework and class assignments as guides. He graphs his progress in terms of (1) not meeting, (2) meeting, and (3) exceeding his goals during the year. Parker reviews the results and discovers that he has made progress toward meeting all three of his math goals.</p>	<ul style="list-style-type: none"> • Data chart showing Parker's progress in organizing data on a graph and drawing conclusions based on that information • Parker's graph of his progress toward his math goals • Videotape of Parker interpreting his graph with teacher assistance
Addressing Access Skills embedded in academic instruction	Possible Assessment Strategies and Portfolio Products
<p>Fred helps his peers graph their data by using a rubber stamp to represent blocks on a graph. He marks the correct block on the graph paper with his stamp, following the directions of peers.</p>	<ul style="list-style-type: none"> • Field data chart showing Fred's accuracy using his visual-motor skills to produce a graph • Line or bar graph summarizing the field data chart • Videotape of Fred using his stamp

CONTENT Mathematics

STRAND Data Analysis, Statistics, and Probability

Learning Standards for Data and Statistics

Grade Level: 7 – 8

Learning Standards as written

Essence of the Standard(s)

<p>8.D.1</p>	<p>Describe the characteristics and limitations of a data sample. Identify different ways of selecting a sample, e.g., convenience sampling, responses to a survey, random sampling.</p>	<ul style="list-style-type: none"> ◆ Describe the characteristics and limitations of a data sample and identify different ways to select a data sample ◆ Represent and interpret data using a variety of graphical representations ◆ Differentiate between continuous and discrete data ◆ Determine, describe and interpret measures of central tendency and range
<p>8.D.2</p>	<p>Select, create, interpret, and utilize various tabular and graphical representations of data, e.g., circle graphs, Venn diagrams, scatterplots, stem-and-leaf plots, box-and-whisker plots, histograms, tables, and charts. Differentiate between continuous and discrete data and ways to represent them.</p>	
<p>8.D.3</p>	<p>Find, describe, and interpret appropriate measures of central tendency (mean, median, and mode) and spread (range) that represent a set of data. Use these notions to compare different sets of data.</p>	

Possible ENTRY POINTS to Learning Standard(s)

Less Complex

More Complex



The student will:

- ◆ Identify different ways to collect data
- ◆ For an ordered set of data, identify the mean, median, mode, range, minimum, and maximum

See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials

The student will:

- ◆ Consider different ways and formats to represent data
- ◆ Draw conclusions given a data sample
- ◆ Match and label a variety of tabular and graphical representations
- ◆ Arrange data in order and identify the mean, median, mode, range, minimum, and maximum of a set of data

The student will:

- ◆ Collect a data sample and make observations about data
- ◆ Identify the most appropriate way to collect a sample of data
- ◆ Design and conduct statistical experiments and interpret findings
- ◆ Use technology to represent data in a variety of formats
- ◆ Use a representative data sample to draw conclusions
- ◆ Identify and create the most appropriate representations of data
- ◆ Determine the mean, median, mode, minimum, maximum, and range of a set of data
- ◆ Identify and create various representations of data (e.g., box and whisker plot)

Continue to address skills and concepts that approach grade-level expectations in this subject

Instructional Idea based on Learning Standard 8.D.2

At grades 7 – 8, students use different graphical representations to display and interpret a variety of weather patterns over a period of two months. Based on their data and calculations, students draw conclusions about the incidence of each weather pattern, and make predictions and inferences about the statistical probability of each weather pattern.

How all students can participate in this activity

Addressing Learning Standard(s) as written for this grade level	Possible Assessment Strategies and Portfolio Products
Caleb graphs weather patterns using a variety of graphical formats. He draws conclusions and makes predictions from the data involving statistical probability.	<ul style="list-style-type: none"> • Data chart showing Caleb's progress creating and interpreting a variety of data charts • Caleb's graphs of weather data, including his interpretations • Caleb's calculations, observations, and predictions recorded in his journal
Addressing Learning Standard(s) at lower levels of complexity (Entry Points)	Possible Assessment Strategies and Portfolio Products
During a two-month period, Felix graphs the sunrise and sunset. He discovers a pattern of change of one to two minutes daily. He answers yes and no questions and estimates the probability of different times for sunrise and sunset the following week.	<ul style="list-style-type: none"> • Data chart showing Felix's increasing ability to create graphs • Felix's sunrise/sunset graph • Sample of Felix's graphic organizer on which he records daily observations
Addressing Access Skills embedded in academic instruction	Possible Assessment Strategies and Portfolio Products
Faye helps her peers record daily weather features (sun, clouds, rain). She selects, grasps, and places picture communication symbols on the graph.	<ul style="list-style-type: none"> • Data chart showing Faye's ability to grasp and manipulate objects with minimal physical assistance • Videotape of Faye using her pincer grasp • Teacher-scribed work sample indicating Faye's ability to select, grasp, and place pictures of each symbol on her graph

CONTENT Mathematics

STRAND Data Analysis, Statistics, and Probability

Learning Standards for Data and Statistics

Grade Level: 9 – 10

Learning Standards as written

Essence of the Standard(s)

<p>10.D.1</p>	<p>Select, create, and interpret an appropriate graphical representation (e.g., scatterplot, table, stem-and-leaf plots, box-and-whisker plots, circle graph, line graph, and line plot) for a set of data and use appropriate statistics (e.g., mean, median, range, and mode) to communicate information about the data. Use these notions to compare different sets of data.</p>	<ul style="list-style-type: none"> ◆ Select, create, and interpret the appropriate graphical representation for a set of data ◆ Compare sets of data using different graphical representations ◆ Approximate the line of best fit (trend line) for a set of data
<p>10.D.2</p>	<p>Approximate a line of best fit (trend line) given a set of data (e.g., scatterplot). Use technology when appropriate.</p>	

Possible ENTRY POINTS to Learning Standard(s)

Less Complex

More Complex



The student will:

- ◆ Describe a data sample
- ◆ Identify what a data sample might be useful for
- ◆ Arrange data in order and identify the minimum, maximum, mean, median, mode, and range

See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials

The student will:

- ◆ Identify different types of data representations
- ◆ Identify ways to select a representative data sample
- ◆ Use various graphical representations of data to communicate information
- ◆ Given a set of data represented in different graphical formats, make observations and interpretations about the path of that data

The student will:

- ◆ Given a set of data, select the best graphical representation to communicate information
- ◆ Given a set of data and its graphical representation, evaluate the effectiveness of the way in which the data was portrayed
- ◆ Explain the purpose of a trend line and demonstrate its use
- ◆ Interpret and represent data in a variety of graphical representations
- ◆ Determine the mean, median, mode, range, maximum, and minimum for a set of data

Continue to address skills and concepts that approach grade-level expectations in this subject

CONTENT Mathematics

STRAND Data Analysis, Statistics, and Probability
Learning Standards for Probability

Grade Level: 1 – 2

Learning Standards as written

Essence of the Standard(s)

2.D.4 Decide which outcomes of experiments are most likely.

- ◆ Classify events as likely or unlikely
- ◆ Understand cause/effect

Possible ENTRY POINTS to Learning Standard(s) (and ACCESS SKILLS embedded in standards-based activities)

Less Complex

More Complex

ACCESS SKILLS
The student will:

- ◆ Grasp, release, and manipulate objects and materials relevant to the instructional activity
- ◆ Initiate or respond to request for joint attention
- ◆ Follow simple directions
- ◆ Use appropriate social greetings
- ◆ Respond to/initiate contacts with others
- ◆ Activate an electronic device to access text, communicate with others, or participate in an instructional activity
- ◆ Turn attention toward another person
- ◆ Apply rules for appropriate classroom behavior

ENTRY POINTS
The student will:

- ◆ Make predictions as to the likeliest outcome during simple cause and effect experiments (e.g., using electronic switch, simple machine, or other objects, pictures, and/or manipulatives)

The student will:

- ◆ Given a simple cause and effect relationship, decide which outcome is most likely
- ◆ In a given cause and effect relationship, indicate possible actions and the likeliest outcome of each

The student will:

- ◆ Identify the variables in a situation or experiment
- ◆ Describe how a variable controls the outcome
- ◆ Given a cause, predict the likeliest effect
- ◆ Given an effect, speculate on the cause

Continue to address skills and concepts that approach grade-level expectations in this subject

CONTENT Mathematics

STRAND Data Analysis, Statistics, and Probability

Learning Standards for Probability

Grade Level: 3 – 4

Learning Standards as written		Essence of the Standard(s)
4.D.4	Represent the possible outcomes for a simple probability situation, e.g., the probability of drawing a red marble from a bag containing three red marbles and four green marbles.	<ul style="list-style-type: none"> ◆ Experiment with probability (e.g., using manipulatives) ◆ List and count numbers of possible combinations
4.D.5	List and count the number of possible combinations of objects from three sets, e.g., how many different outfits can one make from a set of three shirts, a set of two skirts, and a set of two hats?	
4.D.6	Classify outcomes as certain, likely, unlikely, or impossible by designing and conducting experiments using concrete objects such as counters, number cubes, spinners, or coins.	

Possible ENTRY POINTS Learning Standard(s)

Less Complex

More Complex



The student will:

- ◆ Predict the next event in a given routine
- ◆ Identify events that are likely (the sun rises every morning)
- ◆ Identify events that are not likely (snowfall in August)

See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials

The student will:

- ◆ Predict the next event in a given routine and explain why that prediction is likely
- ◆ Make predictions about events/situations that are not likely
- ◆ Use manipulatives to determine the number of possible combinations of objects for two or three sets
- ◆ Identify possible combinations, given three objects

The student will:

- ◆ Identify and calculate possible combinations of objects or manipulatives, given two sets (e.g., three colored marbles and three different triangles)
- ◆ Determine whether an outcome is likely, unlikely, or impossible, then perform an experiment to determine outcome
- ◆ Make several predictions of real-world events, then observe, gather data, and discuss possible/likely outcome(s)

Continue to address skills and concepts that approach grade-level expectations in this subject

CONTENT Mathematics

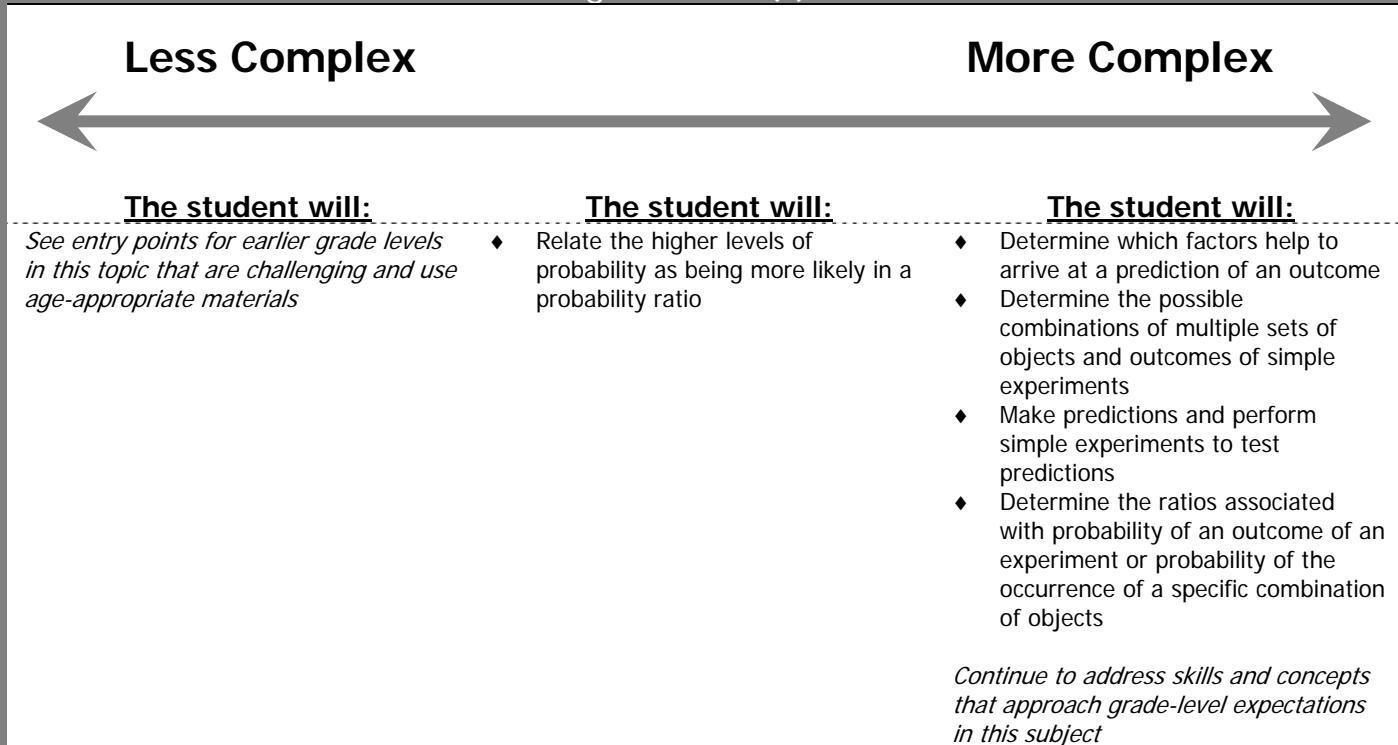
STRAND Data Analysis, Statistics, and Probability

Learning Standards for Probability

Grade Level: 5 – 6

Learning Standards as written		Essence of the Standard(s)
6.D.3	Use tree diagrams and other models (e.g., lists and tables) to represent possible or actual outcomes of trials. Analyze the outcomes.	<ul style="list-style-type: none"> ◆ Predict the probability of outcomes of simple experiments and test the prediction(s) ◆ Use a variety of methods to compute probabilities
6.D.4	Predict the probability of outcomes of simple experiments (e.g., tossing a coin, rolling a die) and test the predictions. Use appropriate ratios between 0 and 1 to represent the probability of the outcome and associate the probability with the likelihood of the event.	

Possible ENTRY POINTS to Learning Standard(s)



CONTENT Mathematics

STRAND Data Analysis, Statistics, and Probability
Learning Standards for Probability

Grade Level: 7 – 8

Learning Standards as written Essence of the Standard(s)

8.D.4	Use tree diagrams, tables, organized lists, basic combinatorics (“fundamental counting principle”), and area models to compute probabilities for simple compound events, e.g., multiple coin tosses or rolls of dice.	♦ Use graphical representations to compute probabilities for simple compound (multiple) events
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Possible ENTRY POINTS to Learning Standard(s)

Less Complex

More Complex



The student will:

- ♦ Determine which factors help to arrive at a likely prediction of an outcome
- ♦ Determine the possible outcomes of an event or experiment

See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials

The student will:

- ♦ Design and/or conduct an experiment in which data is used to make a prediction
- ♦ Use organized lists, graphs, models, tables, and diagrams to portray data and/or predict outcomes
- ♦ Record data and make predictions of outcomes for repeated trials

The student will:

- ♦ Use tree diagrams, tables, and lists to represent possible or actual outcomes of trials
- ♦ Use ratios to represent the probability of an outcome of an experiment

Continue to address skills and concepts that approach grade-level expectations in this subject

CONTENT Mathematics

STRAND Data Analysis, Statistics, and Probability

Learning Standards for Probability

Grade Level: 9 – 10

Learning Standards as written

Essence of the Standard(s)

10.D.3	Describe and explain how the relative sizes of a sample and the population affect the validity of predictions from a set of data.	<ul style="list-style-type: none"> ◆ Explain how relative sample size taken from a population affects the validity of a prediction
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Possible ENTRY POINTS to Learning Standard(s)

Less Complex

More Complex



The student will:

- ◆ Define and apply the terms *population, data sample, validity,* and *data set* to demonstrate understanding of probability

See entry points for earlier grade levels in this topic that are challenging and use age-appropriate materials

The student will:

- ◆ Identify a sample of a given population
- ◆ Recognize that data samples are taken for a population that is too large to analyze in its entirety

The student will:

- ◆ Given a population, select different sample sizes and make predictions about validity of results for each sample
- ◆ Recognize that the data taken for a sample is not exact, and will result in a prediction about the population
- ◆ Identify the variables that can affect the analysis/outcome of sampling data from a given population
- ◆ Identify the benefits and drawbacks to sampling data from a population to make analyses and observations

Continue to address skills and concepts that approach grade-level expectations in this subject