

# Mathematics Learning Community

## *Facilitator Materials Overview*

## *Preface*

The Regional Science Resource Center at the University of Massachusetts Medical School (RSRC), in partnership with the Massachusetts Department of Elementary and Secondary Education (ESE), has envisioned and developed the Mathematics Learning Community (MLC) curriculum materials as a means to support job-embedded professional development for teachers of mathematics in grades K-8. The initial concept development of the MLC was funded by ESE along with the Noyce Foundation and the Intel Foundation as an extension of activities that originated as part of the Massachusetts Partnership Advancing the Learning of Mathematics and Science (PALMS) program, funded by the National Science Foundation.

The materials support productive use of common teacher planning time through establishment and development of a professional learning community with a focus on student learning. Fundamental math concepts addressed in the sessions provide teachers the opportunity to extend mathematical and pedagogical understanding, first through direct engagement with the mathematics, and then through looking at student work and exploring how student understanding develops. The MLC model incorporates a gradual release of responsibility, beginning with modules that are highly structured and scripted to support early stages of facilitation. Later sessions are less prescriptive as the MLC facilitator gains experience with the format of the sessions. The early modules incorporate anonymous – though authentic – student work to promote objectivity in discussions of student understanding. As the sessions progress, teachers look at their own students' work using the descriptive, evidence-based MLC protocol.

Although it would be possible to use the MLC materials with no prior orientation or training, the developers recommend Facilitator Training to ensure a successful, robust implementation. The training addresses practical aspects of MLC implementation as well as promoting a metacognitive perspective that connects MLCs to mathematics curriculum, professional development and initiatives. For more information on MLC Facilitator Training seminars, please contact Sandra Mayrand at 508-856-5097 or [sandra.mayrand@umassmed.edu](mailto:sandra.mayrand@umassmed.edu).

Dona Apple and Wendy Cleaves, the MLC primary authors, wish to thank the mathematics coaches and teachers of Springfield, New Bedford and Worcester who were part of the MLC pilot program. The sharing of their MLC experiences, the thoughtfulness of their mathematics dialog, and their insights and suggestions were invaluable. The authors also thank the many Massachusetts teachers whose classes provided the thought-provoking student work; WestEd as the evaluator for providing crucial formative feedback; Barbara Libby, Life LeGeros, David Parker, and Emily Veader for coordination at the ESE; and Sandra Mayrand, the Director of RSRC, for overall project management.

It is the sincere hope of the MLC development team that these materials will promote student success in mathematics through supporting knowledgeable and reflective teaching and learning.

## *Introduction to the Mathematics Learning Community*

A growing consensus in the research points to the need for coordinated, classroom- and content-focused professional development that links challenging learning standards and curriculum to solve the specific problems hindering the advancement of students of mathematics (Bransford et.al., 2000; Olson, 2002; Smith, 2001; Hill and Cohen, 2000; Whitehurst, 2002; Wiley and Yoon, 1995; Kennedy, 1998; NRC Mathematics Learning Study Committee, 2000; DuFour, 1998). These professional development curriculum materials are intended to help teachers and schools make these critical connections.

These materials provide the structure and content to facilitate school-based discussions about how students understand, express and think about mathematics. Three modules allow teachers to probe student thinking about proportional reasoning, data analysis, and number sense & pre-algebra by learning how to examine student work with a diagnostic eye followed by meaningful collegial conversations. These online materials comprise the fifteen sessions within the number sense & pre-algebra MLC module.

The protocols, student work and guiding questions allow groups of teachers to uncover and explore collaboratively how students think and learn about specific mathematics concepts. With this deeper understanding of how students perceive and apply a concept, teachers are better able to further develop students' understanding in a classroom setting. To clarify goals related to the improvement of instructional practice, the MLC materials are directly linked (see cross-walk later in this document) to the Characteristics of an Effective Standards-Based Mathematics Classroom, available at [http://www.doe.mass.edu/omste/news07/mathclass\\_char.doc](http://www.doe.mass.edu/omste/news07/mathclass_char.doc).

It is recommended that the MLC is implemented as a complement to professional development courses that focus on increasing mathematical content knowledge, so that teachers can directly connect subject matter knowledge to the development of mathematical thinking in students. To this end, a version of the MLC that is aligned to the Intel Math course is available (see Sandra Mayrand's contact information below).

### *MLC Facilitator Training Seminars*

The MLC is meant to serve as a long-term, job-embedded, practical approach to professional growth centered around student thinking and learning. In order to make the MLC as effective as possible, skilled facilitation is required. Math coaches, specialists, and lead teachers often fill MLC facilitator roles. It is strongly recommended that an MLC be co-facilitated for many reasons, including sharing of the workload, continuity of content in case of sickness or emergency, greater depth of knowledge, and the ability to listen more attentively to the voices of the MLC members.

Facilitator Training Seminars are built on the premise that the training group acts as a model MLC. Through this approach, facilitators can experience first-hand the types of questions that will be raised, solutions that will be shared, student ideas that will be analyzed, and mathematics that will be discussed. Training participants practice their facilitation skills within this safe learning environment. Common facilitation dilemmas are addressed, along with background knowledge of professional learning communities. Video clips of MLCs in action, along with a case study of an MLC member and insights from current MLC Facilitators round out the training. Facilitator Training Seminars can be conducted in-district or at a more cost-effective setting with multiple districts in attendance in Shrewsbury, MA.

For more information on MLC Facilitator Training Seminars, contact Sandra Mayrand at 508-856-5097 or [sandra.mayrand@umassmed.edu](mailto:sandra.mayrand@umassmed.edu).

# Mathematics Learning Community

## MLC

The MLC uses student work as a means to connect professional development to the classroom and to stimulate authentic discussions about how students learn mathematics. The goal of the MLC is to improve students' understanding of mathematics.

### What is the MLC?

The purpose of the MLC is to provide teachers of mathematics with an opportunity to work together with colleagues to relate important content and pedagogy to their own classroom practice. Over the course of the school year, MLC members engage in up to 15 school- or district-based sessions facilitated by a math coach or lead teacher. Each of the MLC sessions consists of a math discussion, mental math tasks, problem solving and the examination of student work. Student work samples examined during MLC sessions represent various grade levels, yet each session's content is always connected to three grade bands: K – 2, 3 – 5, and 6 – 8. Three key mathematical ideas – counting, composition, and context – act as threads tying all content together within the MLC sessions.

### Goals of the MLC

- To form a collaborative learning community that focuses on student learning
- To understand mathematical content in a deeper, more conceptual way
- To closely examine student work and formative assessments in order to determine students' progress and instructional needs
- To reflect on classroom practice, share strategies, discuss best practices, and expand professional expertise

### MLC Protocol for Looking at Student Work

- ✓ Read the problem and discuss what it is assessing
- ✓ Solve the problem individually
- ✓ Share your thinking with a partner
- ✓ Discuss the mathematics of the problem as a whole group
- ✓ Look at how students solved the same problem
- ✓ Identify evidence of understanding by using guiding questions
- ✓ Discuss evidence of student understanding as a whole group

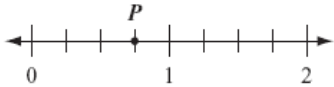
## MLC Number Sense Module

### Overview of Sessions 1 – 10

Session Title	Math Metacognition Problem(s)	Looking at Student Work Problem(s)
<b>1</b> Getting Started with the MLC	N/A	<b>Grade 6:</b> Lisa has three jobs: walking the family dog, washing the dishes, and vacuuming. She walks the family dog once every 3 days. She washes the dishes once every 4 days. She vacuums once every 6 days. On which day will Lisa again do all three jobs on the same day?
<b>2</b> Understanding Counting	Have participants figure out how long it is until their birthday. Then, give that answer to another participant and see if they can determine when the birthday is.	<b>Grade 3:</b> Carlos' birthday is April 30. How long is it until his birthday? Show how you figured it out.
<b>3</b> Working with Addition	$78 + 43 = ?$ $192 = 67 + ?$	<b>Grade 2:</b> Write a story problem for this number sentence: $18 + \underline{\quad} = 72$
<b>4</b> The Relationship between Addition and Subtraction	<i>Represent each of the following as a number sentence:</i> A. Sandy has 43 flowers in her garden. 25 are red, the rest are yellow. How many are yellow? B. I have some crayons. I gave 13 to Beth. Now I have 38. How many crayons did I start with?	<b>Grade 2:</b> Jackie and Sara collected shells at the beach. Jackie collected 48 shells in all. She collected 13 more shells than Sara. How many shells did Sara collect?
<b>5</b> Subtraction Strategies	$52 - 17 = ?$ $603 - 456 = ?$ $1001 - 439 = ?$	<b>Grade 7:</b> -16 is 9 less than a number. Find the number. Show your thinking using a picture or a number line.
<b>6</b> Multiplication Strategies	$29 \times 4 = ?$	<b>Grade 4:</b> Haley swam 22 laps each day for 18 days. Then she swam 25 laps each day for 10 days. What was the total number of laps she swam over the 28 days?
<b>7</b> The Distributive Property	$39 \times 22$ $325 \times 12$	<b>Grade 7:</b> $(20 + 4)(30 + 5)$ Write a word problem that represents this expression. Solve the problem and show your work.
<b>8</b> Dealing with Division	$182 \div 15 =$ Make note of the first step you think about to begin this problem	<b>Grade 4:</b> Solve each problem using a method other than the traditional algorithm for division. $56 \div 3$ $70 \div 9$ $128 \div 6$ $482 \div 5$ $972 \div 2$ $371 \div 8$
<b>9</b> Partitive and Quotative Division	$15 \div 5 = 3$ 1. Model the action in the problem above in two different ways. 2. Write a word problem that represents each of the actions modeled.	<b>Grade 5:</b> Twenty divided by 5 is 4. 1. Represent this statement as a division problem. 2. Write a word problem that models this division problem. Be sure to include labels for each of the three numbers. 3. Draw a picture that models your word problem. Be sure to include labels in your picture.
<b>10</b> Connecting Division to Fractions	Write a division problem (two-digit by a one-digit number) where you have a remainder. Write a word problem that models this division problem. Write a brief explanation interpreting your remainder.	<b>Grade 7:</b> Write a word problem for $44 \div 6$ where the answer to the problem is $7\frac{1}{3}$ . Write a word problem for $44 \div 6$ where the answer to the problem is 7 or 8.

## MLC Number Sense Module

### Overview of Sessions 11 – 15

Session Title	Math Metacognition Problem(s)	Looking at Student Work Problem(s)
<b>11</b> <b>Representing and Interpreting Fractions</b>	Point $P$ is located on the number line shown below.  Which of the following fractions best represents the location of point $P$ ? A) $\frac{1}{4}$ B) $\frac{3}{8}$ C) $\frac{3}{4}$ D) $\frac{4}{5}$	<b>Grade 5:</b> The table below shows the amount of time each of four students spent on a mathematics test yesterday. Time Spent on Mathematics Test (Time in hours): Joe: $\frac{1}{2}$ , Keith: $\frac{2}{3}$ , Lena: $\frac{1}{4}$ , Mia: $\frac{2}{5}$ Which student spent the <b>greatest</b> amount of time on the test? Show how you figured out your answer.
<b>12</b> <b>Adding and Subtracting Fractions</b>	Examine the following pairs of problems. Keep track of your thinking as you determine which sum or difference is greater. $\frac{8}{7} - \frac{3}{4}$ or $\frac{4}{5} - \frac{6}{11}$ $\frac{1}{15} + \frac{1}{11}$ or $\frac{4}{3} - \frac{3}{4}$ $\frac{3}{4} + \frac{6}{7}$ or $\frac{4}{5} + \frac{5}{6}$	<b>Grades 6 – 8:</b> A new fitness center is going to be built soon in your city. 1. Label each section with its fraction of the total area of the fitness center. 2. The owners of the fitness center have now decided to use $\frac{1}{8}$ of the Pool's area to install showers. What fraction of the total area of the fitness center will the Pool now cover? Explain your thinking. 3. The Weights, Aerobics, and Treadmills sections will be renamed as one section called the Workout Zone. What fraction of the total area of the fitness center will the Workout Zone now cover?
<b>13</b> <b>Multiplying Fractions</b>	Estimate the following products. $2\frac{1}{3} * \frac{5}{8}$ $4\frac{1}{2} * 3\frac{12}{13}$	<b>Grade 6 – 8:</b> I baked a rectangular sheet cake for a party. $\frac{3}{8}$ of the cake was left after the party. Draw and label how much of the cake was left. My son came to visit and ate $\frac{1}{3}$ of the remaining cake. Draw and label how much of the remaining cake he ate. What portion of the whole cake did he eat?
<b>14</b> <b>Dividing Fractions</b>	Consider the following problem: How many $\frac{2}{3}$ 's are in 3? Be prepared to explain how you solved the problem.	<b>Grades 6 – 8:</b> Solve each of the following problems. Draw a model to show your thinking. A. Juan has 3 gallons of yellow paint that needs to be put into new containers for an art project. Each container holds $\frac{2}{7}$ of a gallon of paint. How many containers can Juan fill if he uses up all of the paint? B. The art class has $\frac{5}{6}$ of a gallon of paint that needs to be shared equally among 3 students. How much paint will each student get? C. The art class wants to make banners out of construction paper. Each banner takes $\frac{3}{5}$ yard of paper. How many banners can they make if they have $2\frac{1}{2}$ yards of paper to use?
<b>15</b> <b>Working with Fractions, Decimals &amp; Percents</b>	Mentally determine each of the following: $12\frac{1}{2}\% + 0.50 + \frac{5}{8} = ?$ $0.15 + \frac{4}{9} + 67\% = ?$ $\frac{2}{8} + 0.435 - 25\% = ?$	<b>Grade 7:</b> The circle graph below shows the student attendance at the Central Middle School Fall Festival. a. What percent of the students who attended the Fall Festival were grade 7 girls? Show or explain how you got your answer. b. What part of the students attending the Fall Festival were girls? Write your answer as a fraction. Show or explain how you got your answer. c. There were 32 grade 7 girls who attended the Fall Festival. What was the total number of students who attended the Fall Festival? Show or explain how you got your answer.

## MLC Number Sense Module

### Crosswalk of Sessions 1 – 4

Session Title & Student Work Problem	MA Learning Standards Addressed in Student Work	NCTM Process Standards Addressed in Student Work	MA Learning Standards: Vertical Connections	Standards-Based Teaching/Learning Characteristics Addressed
<p><b><i>1: Getting Started with the MLC</i></b></p> <p>Lisa has three jobs: walking the family dog, washing the dishes, and vacuuming.</p> <ul style="list-style-type: none"> <li>▪ She walks the family dog once every 3 days.</li> <li>▪ She washes the dishes once every 4 days.</li> <li>▪ She vacuums once every 6 days.</li> </ul> <p>The calendar below shows that Lisa did all three jobs on Monday the 2<sup>nd</sup>. On which day will Lisa again do all three jobs on the same day?</p>	<p>6.N.8: Apply number theory concepts—including prime and composite numbers, prime factorization, greatest common factor, least common multiple, and divisibility rules for 2, 3, 4, 5, 6, 9, and 10—to the solution of problems.</p>	<p>Problem Solving – solve problems that arise in mathematics and in other contexts</p>	<p>K.N.1 2.P.4 2.M.1 3.N.5 4.N.7 5.N.8</p>	<ul style="list-style-type: none"> <li>• 5.1</li> <li>• 5.2</li> <li>• 5.3</li> <li>• 5.4</li> </ul>
<p><b><i>2: Understanding the Conventions of Counting</i></b></p> <p>Carlos’ birthday is April 30. How long is it until his birthday? Show how you figured it out.</p>	<p>3.M.3 Identify time to the minute on analog and digital clocks using am and pm. Compute elapsed time less than one hour using a clock and using a calendar.</p>	<p>Problem Solving – apply and adapt a variety of appropriate strategies to solve problems</p> <p>Representation – create and use representations to organize, record, and communicate mathematical ideas</p>	<p>K.M.1 2.M.1 3.M.3 4.M.5</p>	<ul style="list-style-type: none"> <li>• 5.1</li> <li>• 5.2</li> <li>• 5.3</li> <li>• 5.4</li> <li>• 5.5</li> </ul>
<p><b><i>3: Working with Addition</i></b></p> <p>Write a story problem for this number sentence: <math>18 + \underline{\quad} = 72</math></p>	<p>2.N.7 Demonstrate an understanding of various meanings of addition and subtraction, e.g. addition as combination, subtraction as comparison, equalizing, and separation.</p> <p>2.N.12 Estimate, calculate, and solve problems involving addition and subtraction of two-digit numbers. Describe differences between estimates and actual calculations.</p>	<p>Problem Solving – apply and adapt a variety of appropriate strategies to solve problems</p> <p>Communication – use the language of mathematics to express mathematical ideas precisely</p> <p>Representation – Select, apply, and translate among mathematical representations to solve problems</p>	<p>2.N.7 2.N.12 3.P.4 4.P.4 5.P.4 6.P.3 8.N.8 7.P.4 8.P.7</p>	<ul style="list-style-type: none"> <li>• 5.1</li> <li>• 5.2</li> <li>• 5.3</li> <li>• 5.4</li> <li>• 5.5</li> </ul>
<p><b><i>4: The Relationship Between Addition and Subtraction</i></b></p> <p>Jackie and Sara collected shells at the beach. Jackie collected 48 shells in all. She collected 13 more shells than Sara. How many shells did Sara collect?</p>	<p>2.N.7 Demonstrate an understanding of various meanings of addition and subtraction, e.g. addition as combination, subtraction as comparison, equalizing, and separation.</p> <p>2.N.8 Understand and use the inverse relationship between addition and subtraction to solve problems and check solutions.</p>	<p>Problem Solving – solve problems that arise in mathematics and in other contexts; apply and adapt a variety of appropriate strategies to solve problems.</p> <p>Connections – recognize and use connections among mathematical ideas; understand how mathematical ideas interconnect and build on one another to produce a coherent whole.</p>	<p>2.N.7 2.N.8 3.N.6 4.N.8 5.N.11 6.N.10 6.N.12 6.N.15 7.N.6 7.N.9 8.N.9 8.N.12</p>	<ul style="list-style-type: none"> <li>• 5.1</li> <li>• 5.2</li> <li>• 5.3</li> <li>• 5.4</li> <li>• 5.5</li> </ul>

## MLC Number Sense Module

### Crosswalk of Sessions 5 – 8

Session Title & Student Work Problem	MA Learning Standards Addressed in Student Work	NCTM Process Standards Addressed in Student Work	MA Learning Standards: Vertical Connections	Standards-Based Teaching/Learning Characteristics Addressed
<p><b>5: Subtraction Strategies</b></p> <p>-16 is 9 less than a number. Find the number. Show your thinking using a picture or a number line.</p>	<p>7.N.7 Estimate and compute with fractions, integers, decimals, and percents.</p> <p>7.N.9 Select and use appropriate operations – addition, subtraction, multiplication, division, and positive integer exponents – to solve problems with rational numbers (including negatives).</p>	<p>Problem Solving – apply and adapt a variety of appropriate strategies to solve problems.</p> <p>Connections - understand how mathematical ideas interconnect and build on one another to produce a coherent whole.</p> <p>Representation – use representations to model and interpret physical, social and mathematical phenomena.</p>	<p>2.N.7 2.N.8 5.N.6 6.N.10 6.N.15 7.N.7 7.N.9 8.N.12</p>	<ul style="list-style-type: none"> <li>• 5.1</li> <li>• 5.2</li> <li>• 5.3</li> <li>• 5.4</li> <li>• 5.5</li> </ul>
<p><b>6: Multiplication Strategies</b></p> <p>Haley swam 22 laps each day for 18 days. Then she swam 25 laps each day for 10 days. What was the total number of laps she swam over the 28 days?</p>	<p>4.N.12 Add and subtract (up to five-digit numbers) and multiply (up to three digits by two digits) accurately and efficiently.</p>	<p>Problem Solving – solve problems that arise in mathematics and in other contexts</p> <p>Representation – create and use representations to organize, record, and communicate mathematical ideas</p>	<p>K.N.1 2.N.7 3.N.6 3.N.8 3.N.9 3.N.12 4.N.8 4.N.11 4.N.14 5.N.9 5.N.12 6.N.9 6.N.14 7.N.7 7.N.9 8.N.10 8.N.12</p>	<ul style="list-style-type: none"> <li>• 5.1</li> <li>• 5.2</li> <li>• 5.3</li> <li>• 5.4</li> <li>• 5.5</li> </ul>
<p><b>7: The Distributive Property</b></p> <p><math>(20 + 4)(30 + 5)</math> Write a word problem that represents this expression. Solve the problem and show your work.</p>	<p>7.N.9 Select and use appropriate operations – addition, subtraction, multiplication, division, and positive integer exponents – to solve problems with rational numbers (including negatives).</p>	<p>Problem Solving – apply and adapt a variety of appropriate strategies to solve problems.</p> <p>Reasoning and Proof – select and use various types of reasoning and methods of proof.</p> <p>Communication – analyze and evaluate the mathematical thinking and strategies of others; use the language of mathematics to express mathematical ideas precisely.</p> <p>Connections – understand how mathematical ideas interconnect and build on one another to produce a coherent whole.</p> <p>Representation – use representations to model and interpret physical, social, and mathematical phenomena.</p>	<p>K.N.1 2.N.7 3.N.6 3.N.8 3.N.9 3.N.12 4.N.8 4.N.11 4.N.14 5.N.9 5.N.12 6.N.13 7.N.9 8.N.8 8.N.12</p>	<ul style="list-style-type: none"> <li>• 5.1</li> <li>• 5.2</li> <li>• 5.3</li> <li>• 5.4</li> <li>• 5.5</li> </ul>
<p><b>8: Dealing with Division</b></p> <ul style="list-style-type: none"> <li>▪ <math>56 \div 3</math></li> <li>▪ <math>70 \div 9</math></li> <li>▪ <math>128 \div 6</math></li> <li>▪ <math>482 \div 5</math></li> <li>▪ <math>972 \div 2</math></li> <li>▪ <math>371 \div 8</math></li> </ul>	<p>4.N.8 Select, use, and explain various meanings and models of multiplication and division of whole numbers. Understand and use the inverse relationship between the two operations.</p> <p>4.N.13 Divide up to a three-digit whole number with a single-digit divisor (with or without remainders) accurately and efficiently. Interpret any remainders.</p>	<p>Problem Solving – apply and adapt a variety of appropriate strategies to solve problems.</p> <p>Communication –use the language of mathematics to express mathematical ideas precisely.</p> <p>Connections – understand how mathematical ideas interconnect and build on one another to produce a coherent whole.</p> <p>Representation – use representations to model and interpret physical, social, and mathematical phenomena.</p>	<p>K.N.1 2.N.7 4.N.8 4.N.13 4.N.15 5.N.9 5.N.12 6.N.9 6.N.13 7.N.9 8.N.12</p>	<ul style="list-style-type: none"> <li>• 5.1</li> <li>• 5.2</li> <li>• 5.3</li> <li>• 5.4</li> <li>• 5.5</li> </ul>

## MLC Number Sense Module

### Crosswalk of Sessions 9 – 11

Session Title & Student Work Problem	MA Learning Standards Addressed in Student Work	NCTM Process Standards Addressed in Student Work	MA Learning Standards: Vertical Connections	Standards-Based Teaching/Learning Characteristics Addressed
<p><b><i>9: Partitive and Quotative Division</i></b></p> <p>Twenty divided by 5 is 4.</p> <ul style="list-style-type: none"> <li>▪ Represent this statement as a division problem.</li> <li>▪ Write a word problem that models this division problem. Be sure to include labels for each of the three numbers.</li> <li>▪ Draw a picture that models your word problem. Be sure to include labels in your picture.</li> </ul>	5.N.9 Solve problems involving multiplication and division of whole numbers, and multiplication of positive fractions with whole numbers.	<p>Problem Solving – build new mathematical knowledge through problem solving.</p> <p>Communication – use the language of mathematics to express mathematical ideas precisely.</p> <p>Connections – understand how mathematical ideas interconnect and build on one another to produce a coherent whole.</p> <p>Representation – create and use representations to organize, record, and communicate mathematical ideas; select, apply, and translate among mathematical representations to solve problems</p>	K.N.1 K.N.7 2.N.7 4.N.8 4.N.10 4.N.13 4.N.15 4.P.4 5.N.12 5.P.4 6.N.9 6.N.13 6.P.4 7.N.9 8.N.12	<ul style="list-style-type: none"> <li>• 5.1</li> <li>• 5.2</li> <li>• 5.3</li> <li>• 5.4</li> <li>• 5.5</li> </ul>
<p><b><i>10: Connecting Division to Fractions</i></b></p> <p>Write a word problem for <math>44 \div 6</math> where the answer to the problem is <math>7\frac{1}{3}</math>.</p> <p>Write a word problem for <math>44 \div 6</math> where the answer to the problem is 7 or 8.</p>	8.N.12 Select and use appropriate operations – addition, subtraction, multiplication, division, and positive integer exponents – to solve problems with rational numbers (including negatives).	<p>Problem Solving – build new mathematical knowledge through problem solving.</p> <p>Communication – use the language of mathematics to express mathematical ideas precisely.</p> <p>Connections – understand how mathematical ideas interconnect and build on one another to produce a coherent whole.</p> <p>Representation – select, apply, and translate among mathematical representations to solve problems.</p>	K.N.5 2.N.3 3.N.3 4.N.3 4.N.4 4.N.8 4.N.13 4.N.15 5.N.4 5.N.9 5.N.12 6.N.9 6.N.13 7.N.9 8.N.12	<ul style="list-style-type: none"> <li>• 5.1</li> <li>• 5.2</li> <li>• 5.3</li> <li>• 5.4</li> <li>• 5.5</li> </ul>
<p><b><i>11: Representing and Interpreting Fractions</i></b></p> <p>The table below shows the amount of time each of four students spent on a mathematics test yesterday. Which student spent the <b>greatest</b> amount of time on the test? Show how you figured out your answer.</p>	5.N.4 Demonstrate an understanding of fractions as a ratio of whole numbers, as parts of unit wholes, as parts of a collection, and as locations on the number line.	<p>Reasoning and Proof – select and use various types of reasoning and methods of proof.</p> <p>Representation – select, apply, and translate, among mathematical representations to solve problems; use representations to model and interpret physical, social and mathematical phenomena.</p>	K.N.5 2.N.3 3.N.3 3.N.4 4.N.3 4.N.4 4.N.5 5.N.4 5.N.5 5.N.6 5.N.7 6.N.4 6.N.5 6.N.6 6.N.7 6.N.9 6.N.14 6.N.16 7.N.1 7.N.7 7.N.9 8.N.1 8.N.10 8.N.12	<ul style="list-style-type: none"> <li>• 5.1</li> <li>• 5.2</li> <li>• 5.3</li> <li>• 5.4</li> <li>• 5.5</li> </ul>

## MLC Number Sense Module

### Crosswalk of Sessions 12 – 13

Session Title & Student Work Problem	MA Learning Standards Addressed in Student Work	NCTM Process Standards Addressed in Student Work	MA Learning Standards: Vertical Connections	Standards-Based Teaching/Learning Characteristics Addressed
<p><b><i>12: Adding and Subtracting Fractions</i></b></p> <p>A new fitness center is going to be built soon in your city.</p> <ol style="list-style-type: none"> <li>Label each section with its fraction of the total area of the fitness center.</li> <li>The owners of the fitness center have now decided to use <math>\frac{1}{8}</math> of the Pool's area to install showers. What fraction of the total area of the fitness center will the Pool now cover? Explain your thinking.</li> <li>The Weights, Aerobics, and Treadmills sections will be renamed as one section called the Workout Zone. What fraction of the total area of the fitness center will the Workout Zone now cover?</li> </ol>	<p>6.N.9 Select and use appropriate operations to solve problems involving addition, subtraction, multiplication, division, and positive integer exponents with whole numbers, and with positive fractions, mixed numbers, decimals, and percents.</p> <p>6.N.14 Accurately and efficiently add, subtract, multiply, and divide positive fractions and mixed numbers. Simplify fractions.</p>	<p>Problem Solving – solve problems that arise in mathematics and in other contexts</p> <p>Reasoning and Proof – select and use various types of reasoning and methods of proof.</p>	<p>K.N.4 K.N.5 2.N.3 2.N.4 2.N.7 2.N.12 3.N.4 3.N.13 4.N.3 4.N.4 4.N.18 5.N.4 5.N.5 5.N.7 5.N.13 6.N.4 6.N.5 6.N.7 6.N.9 6.N.14 7.N.1 7.N.7 7.N.9 8.N.1 8.N.10 8.N.12</p>	<ul style="list-style-type: none"> <li>• 5.1</li> <li>• 5.3</li> <li>• 5.4</li> </ul>
<p><b><i>13: Multiplying Fractions</i></b></p> <p>I baked a rectangular sheet cake for a party. <math>\frac{3}{8}</math> of the cake was left after the party. Draw and label how much of the cake was left. My son came to visit and ate <math>\frac{1}{3}</math> of the remaining cake. Draw and label how much of the remaining cake he ate. What portion of the whole cake did he eat?</p>	<p>6.N.9 Select and use appropriate operations to solve problems involving addition, subtraction, multiplication, division, and positive integer exponents with whole numbers, and with positive fractions, mixed numbers, decimals, and percents.</p> <p>6.N.14 Accurately and efficiently add, subtract, multiply, and divide positive fractions and mixed numbers. Simplify fractions.</p>	<p>Connections – understand how mathematical ideas interconnect and build on one another to produce a coherent whole</p> <p>Representation – use representations to model and interpret physical, social, and mathematical phenomena</p>	<p>K.N.5 K.N.8 2.N.3 2.N.12 3.N.6 3.N.7 3.N.8 3.N.9 3.N.10 3.N.11 4.N.3 4.N.4 4.N.5 4.N.8 4.N.10 4.N.11 4.N.12 4.N.16 4.N.17 4.N.18 5.N.4 5.N.5 5.N.7 5.N.13 6.N.4 6.N.5 6.N.9 6.N.13 6.N.14 6.N.16 7.N.1 7.N.7 7.N.8 7.N.9 8.N.1 8.N.8 8.N.9 8.N.10 8.N.11 8.N.12</p>	<ul style="list-style-type: none"> <li>• 5.1</li> <li>• 5.3</li> <li>• 5.4</li> <li>• 5.5</li> </ul>

## MLC Number Sense Module

### Crosswalk of Sessions 14 – 15

Session Title & Student Work Problem	MA Learning Standards Addressed in Student Work	NCTM Process Standards Addressed in Student Work	MA Learning Standards: Vertical Connections	Standards-Based Teaching/Learning Characteristics Addressed
<p><b>14: Dividing Fractions</b></p> <p><i>Solve each of the following problems. Draw a model to show your thinking.</i></p> <p>A. Juan has 3 gallons of yellow paint that needs to be put into new containers for an art project. Each container holds <math>\frac{2}{7}</math> of a gallon of paint. How many containers can Juan fill if he uses up all of the paint?</p> <p>B. The art class has <math>\frac{5}{6}</math> of a gallon of paint that needs to be shared equally among 3 students. How much paint will each student get?</p> <p>C. The art class wants to make banners out of construction paper. Each banner takes <math>\frac{3}{5}</math> yard of paper. How many banners can they make if they have <math>2\frac{1}{2}</math> yards of paper to use?</p>	<p>6.N.9 Select and use appropriate operations to solve problems involving addition, subtraction, multiplication, division, and positive integer exponents with whole numbers, and with positive fractions, mixed numbers, decimals, and percents.</p> <p>6.N.14 Accurately and efficiently add, subtract, multiply, and divide positive fractions and mixed numbers. Simplify fractions.</p>	<p>Problem Solving – build new mathematical knowledge through problem solving</p> <p>Communication – use the language of mathematics to express mathematical ideas precisely</p>	<p>K.N.5 K.N.8 2.N.3 3.N.6 3.N.8 3.N.9 4.N.3 4.N.4 4.N.5 4.N.8 4.N.10 4.N.11 4.N.13 4.N.15 5.N.4 5.N.5 5.N.7 5.N.9 5.N.13 6.N.4 6.N.5 6.N.9 6.N.13 6.N.14 7.N.1 7.N.6 7.N.7 7.N.9 8.N.1 8.N.8 8.N.9 8.N.10 8.N.12</p>	<ul style="list-style-type: none"> <li>• 5.1</li> <li>• 5.3</li> <li>• 5.4</li> <li>• 5.5</li> </ul>
<p><b>15: Working with Fractions, Decimals &amp; Percents</b></p> <p>The circle graph below shows the student attendance at the Central Middle School Fall Festival.</p> <p>a) What percent of the students who attended the Fall Festival were grade 7 girls? Show or explain how you got your answer.</p> <p>b) What part of the students attending the Fall Festival were girls? Write your answer as a fraction. Show or explain how you got your answer.</p> <p>c) There were 32 grade 7 girls who attended the Fall Festival. What was the total number of students who attended the Fall Festival? Show or explain how you got your answer.</p>	<p>7.D.1 Select, create, interpret, and utilize the following tabular and graphical representations of data: circle graphs, Venn Diagrams, stem-and-leaf plots, tables and charts</p>	<p>Problem Solving – apply and adapt a variety of appropriate strategies to solve problems.</p> <p>Communication –use the language of mathematics to express mathematical ideas precisely.</p> <p>Connections – recognize and use connections among mathematical ideas</p> <p>Representation – select, apply, and translate, among mathematical representations to solve problems</p>	<p>K.N.5 K.N.6 K.D.1 2.N.1 2.N.3 2.N.6 2.D.2 3.N.8 3.D.3 4.N.4 4.N.5 4.N.6 4.N.10 4.D.3 5.N.2 5.N.3 5.N.5 5.N.7 5.N.12 5.N.14 5.D.2 6.N.2 6.N.3 6.N.5 6.N.7 6.N.9 6.N.16 6.D.2 7.N.1 7.N.2 7.N.7 7.N.9 7.D.1 8.N.1 8.N.3 8.N.10 8.N.12 8.D.2</p>	<ul style="list-style-type: none"> <li>• 5.1</li> <li>• 5.3</li> <li>• 5.4</li> <li>• 5.5</li> </ul>

## Student Discourse Summary of Available Video and Print Resources

*The following is a list of video and print resources that focus on different aspects of student discourse. These resources informed the development of the MLC and are all aligned with the content found within one or more MLC sessions.*

### VIDEO

#### When viewing video clips, consider the following....

1. Take notes on the student's strategy.
2. What question(s) would you ask to engage other students in problem solving using this strategy?
3. What dialogue would you hope to hear, and what would you want to capture as it relates to student thinking?
4. What moves would you consider making in order to work productively with students' incomplete ideas?

#### ▪ **Annenberg Media**

- Note: videos are available for free online at: [www.learner.org](http://www.learner.org)
- Learning Math: Number and Operations
  - Meanings and Models for Operations
  - Fractions and Decimals
  - Case Studies: K – 2, 3 – 5, 6 – 8

#### ▪ ***Bridges to Classroom Mathematics, TERC/COMAP, 1998***

- Segment #1: Grade 2 & Grade 4 Subtraction
  - Gr. 2 Subtraction ( $63 - ? = 37$ ). Three student strategies are shared.
  - Gr. 4. Subtraction ( $9003 - 410 = ?$ ), Two students highlighted in this segment. The first student doesn't do much talking, but she explains her thinking later, as the teacher asks a good summary question. The second student uses a "friendly number" strategy, and the teacher asks the group to help out to explain to him why he's right or not. The teacher asks: "Is there some way you could check it?"
- Segment #3: Assessment Grade 4
  - Assessment –  $27 \times 4$ . Students are asked to solve this problem in two ways.
    - What kinds of questions is the teacher asking students as she circulates?
    - What kinds of things does the teacher learn as she observes and talks to the students?
    - This segment presents lots of teacher/student dialogue.
    - The teacher's goal is to find out what students know.
    - What do you see as the strengths of this way of assessing students, and what are the challenges?
    - The above-mentioned problem is a written assessment, but the same can be done orally as a student dialogue.

- Consider the following: what’s the level of sophistication of the students’ strategies? Of their understanding?
  
- ***Children’s Mathematics: Cognitively Guided Instruction* by T. Carpenter, et. al., Heinemann, 1999**
  - Disc 2
    - Kindergarten (A bee has six legs. How many legs do five bees have?) Students are working individually on multiplication problems. The teacher asks them questions about their solution strategies.
    - Grade 2 (There are 47 trees in Logan Grove. 31 of them are oak trees. The rest are hickory. How many are hickory?) The problem is read aloud to the class, and then students work individually to solve it. The teacher interacts with different students as each explains his/her strategy. Three children share with the whole class, and the teacher encourages other students to think about these presented strategies.
  
- ***Connecting Mathematical Ideas: Middle School Video Cases to Support Teaching and Learning* by J. Boaler and C. Humphreys, Heinemann, 2005**
  - Disc 1; Defending Reasonableness: Division of fractions
    - $1 \div \frac{2}{3}$ ; The teacher asks the students to solve this without using a known rule.
    - The teacher pushes for sense-making.
    - She uses the strategy: *Convince yourself – Convince a friend – Convince a skeptic*. The use of this strategy could be a good way to build classroom discourse.
  
- ***Content-Focused Coaching: Transforming Mathematics Lessons*, by L. West and F. Staub, Heinemann/University of Pittsburgh, 2003.**
  - Disk 1 – (K/1 Bridge class) (counting, addition, commutativity) (fruit bowl problem, 11 pieces of fruit in all). The teacher discusses the task with the whole group, answers student questions and gets input from students on solution methods. Next, small group discussion during the completion of the task is presented. There is a lot of good teacher-student dialogue.
  - Disk 2 – (Grade 4) (fractions on a geoboard – fourths and eighths). The teacher discusses the point about trying to speak up during group discussion at the beginning. Students share insights about  $\frac{1}{4}$  and  $\frac{1}{8}$ . Teacher asks the students: “Can you restate that? (what someone else said). The teacher pushes the student into actually restating it in her own words, rather than just reading off the board. Teacher also uses other prompts to promote discourse, including: “Who can restate what our job is when we go off to work?”, “Need to prove it to a partner – explain it to them; ask questions – don’t prove it to me (the teacher).”
  - Disk 3 – (Grade 4) (4 people sharing 7 brownies). The whole group discusses the task. Then, students share their initial ideas, including the fact that there is more than one way to solve the problem. Students are asked to represent the answer using fraction numbers. This segment highlights students’ difficulty with combining fractions. In whole group discussion, students explain one another’s thinking.
  
- ***Developing Mathematical Ideas: “Number and Operations, Part 1: Building a System of Tens,”* by D. Schifter, V. Bastable, and S. Russell, Dale Seymour Publications, 1999.**
  - Session 2 Video on Subtraction (see Facilitator’s Guide p. 48 – 49)

- Naillil – From the teacher questioning and the student dialogue, the student clearly explains how what most adults would see as a subtraction problem, she sees as an addition problem of  $26 + x = 40$ .
    - Becky – This student is not seeing the numbers as a whole; instead she is seeing individual digits. The teacher allows her to discuss this, all the while pressing for incorrect thinking/misconceptions to surface in order to move her thinking forward.
  - Session 5 Video on Multiplication (see Facilitator’s Guide p. 76 – 78)
    - Thomas B – In this segment, students ask another student about their mathematical thinking. This is an example of an excellent way for a teacher to get a sense of the kind of intellectual community that exists in the classroom.
    - Thomas H. – Consider Thomas’ thinking; what was missing in his strategy? How could you use his thinking to come up with a correct answer? Have manipulatives available, including: base 10 blocks, linking cubes, graph paper
    - Jemea – The teacher asks: “Do you understand what Jemea did?” and then the student explains. This process becomes part of the culture; that each student who is part of the group needs to explain one another’s thinking process. This segment presents more of the teacher developing that community rather than presenting any particular student’s thinking.
- ***Developing Mathematical Ideas: “Number and Operations, Part 2: Making Meaning for Operations,”*** by D. Schifter, V. Bastable, and S. Russell, Dale Seymour Publications, 1999.
  - Session 3 Video on Multiplication and Division (Facilitator’s Guide p. 49)
    - How many legs do three elephants have? The teacher asks: “So what do the circles represent?” The teacher is trying, through questioning, to get at 4 legs x 3 elephants, and that the answer doesn’t represent the elephants, but the total number of legs. The teacher is trying to get the student to make sense of his/her representation, in particular, the difference between 4 legs on 3 elephants and 3 legs on 4 elephants.
- ***Lenses on Learning: “Course 1: Instructional Leadership in Mathematics,”*** by C. Grant, et. al., Dale Seymour Publications, 2003
  - Video – Course 1:
    - Interview w/ Sasha, (14 brownies to be shared by 6 kids). This is a long diagnostic interview. The teacher allows her to work through it. Sasha states that the answer would be  $2 \frac{1}{3}$  for each kid. She represents the 2 and then tries to draw the remaining two in fourths, and then she says: “Oh, wait a minute, it’s thirds, because  $2 \times 3$  is 6 and I need 6 pieces.” The teacher then asks her about a similar problem (1400 brownies and 600 kids), and Sasha states that: “I know that because it’s the same numbers.” Again, another similar problem is presented (14,000 and 600 kids). That is where her thinking begins to become fragile; she gets to  $20 \frac{10}{3}$ . This interview gave the teacher a lot of information. There were a lot of things about fractions that Sasha knows solidly, yet some things are still fragile. This segment is an example of how a series of well-scaffolded problems can serve as a good diagnostic tool, especially as a pre-assessment.
    - Video also includes a subtraction problem (2003 – 359) and a long division problem ( $7897 \div 6$ ). This segment shows a lot of misconceptions about the traditional algorithm. This segment is a compelling piece of video; it talks about what is the evidence of the student’s understanding. She understands a process and does well with that process; but there is no evidence of her understanding anything beyond

that (except for maybe working to extend her thinking on the 14/6 to 1400/600 problem).

- ***Relearning to Teach Arithmetic: Addition and Subtraction Guide / Multiplication and Division Package*, Dale Seymour Publications, 1999**
  - Session 3, Tape 1 (Subtraction)
    - Naillil – From the teacher questioning and the student dialogue, the student clearly explains how what most adults would see as a subtraction problem, she sees as an addition problem of  $26 + x = 40$ .
  - Session 4, Tape 1 (Multiplication)
    - Thomas B – In this segment, students ask another student about their mathematical thinking. This is an example of an excellent way for a teacher to get a sense of the kind of intellectual community that exists in the classroom.
    - Thomas H. – Consider Thomas’ thinking; what was missing in his strategy? How could you use his thinking to come up with a correct answer? Have manipulatives available, including: base 10 blocks, linking cubes, graph paper

## PRINT

- **“Assessing the Quality and Quantity of Student Discourse in Mathematics Classrooms,”** by D. Weaver, T. Dick, and N. Rigelman, Oregon Mathematics Leadership Institute Partnership, 2006.
  - Research report designed to address the following question:
    - Can student achievement in mathematics be significantly improved by increasing the quantity and quality of meaningful mathematical discourse in mathematics classrooms?
  - Provides a classroom observation protocol, along with definitions of terms around discourse (i.e., discourse modes – T (student addresses teacher), S (student addresses student), G (student addresses group or whole class), and I (individual – student reflects in writing)
  - Describes what is and is not student mathematical discourse in table form
  - Provides codes along with a key for note-taking in the classroom
  - Examples of evidence note-taking are given
  - Guidelines for observing lessons along with forms, surveys, and other tools to summarize classroom observations
  
- **“Beyond Exhortations Not to Tell: The Teacher’s Role in Discussion-Intensive Mathematics Classes,”** by D. Chazan and D. Ball, National Center for Research on Teacher Learning, Michigan State University, 1995.
  - Discusses the ways in which teachers need to redesign their role to shift more authority and autonomy to students
  - Provides two different detailed classroom discussions, one of which is a Grade 3 discussion on locating fractions on a number line.
  - Also discusses how to manage classroom disagreements as a resource for student learning
  
- ***Classroom Discourse: The Language of Teaching and Learning*** by C. Cazden, Henieman, 2001

- *Classroom Discussions: Using Math Talk to Help Students Learn, Grades 1 – 6* by S. Chapin, C. O'Connor, and N. Anderson, Math Solutions Publications, 2003
  
- *Literacy Strategies for Improving Mathematics Instruction* by J. Kenney et. al., ASCD, 2005
  - Chapter 5: Discourse in the Mathematics Classroom
    - What Discourse Looks Like
    - Creating Discourse-Friendly Classrooms
    - Discourse and Computation
    - Discourse and Problem Solving
    - Discourse and Vocabulary
    - Using Concept Maps to Foster Discourse
  
- “Student Interaction in the Math Classroom: Stealing Ideas or Building Understanding,” by Dr. C. Bruce, The Literacy and Numeracy Secretariat, (Ontario), 2007
  - <http://www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/Bruce.pdf>
  - Provides some basic general information on promoting student discourse, with specific strategies for ways to get students “talking math.”

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