# Mathematics Learning Community
## *Number Sense*
## Session 15

**Title:** *Working with Fractions, Decimals & Percents*

**Common Core State Standards Addressed in the LASW Problem:**

**7.EE.3**     Solve multi-step real-world and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

**MA.6.SP.4a**   Read and interpret circle graphs.

**Standards for Mathematical Practice Addressed in the MLC Session:**
**1:** Make sense of problems and persevere in solving them.      **2:** Reason abstractly and quantitatively.
**7:** Look for and make use of structure.

MLC members use quantitative reasoning to mentally solve problems which require them to move in and out of various representations (fraction, decimal and percent).  The LASW Problem requires students to use the same set of skills to solve a problem involving data represented within a circle graph. Students also seek out structure as they take given information presented visually in percents and use it determine unknown quantities.

**Standards-Based Teaching and Learning Characteristics in Mathematics Addressed in the MLC Session:**
* 5.1 Depth of content knowledge is evident throughout the presentation of the lesson.
* 5.3 Students' prior knowledge is incorporated as new mathematical concepts are introduced.
* 5.4 Student misconceptions are anticipated /identified and addressed.
* 5.5 Classroom strategies incorporate multiple forms of representation.

**Session Agenda:**

| Part I: Mathematical Background |
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| Part II: Math Metacognition |
| Part III: Looking at Student Work<br>  • *Fall Festival* Problem (Grade 7) |
| Part IV: Our Learning |
| Part V: Feedback and Wrap-up |

**Materials Needed for this Session:**
✓ Nametags     ✓ Chart paper and markers     ✓ Copies of handouts     ✓ Student Work Samples
✓ Index cards     ✓ Refreshments     ✓ Highlighters

**Possible Ways to Personalize this Session:**
* Student work samples are no longer provided for the LASW problem in this session. Instead, MLC members will provide the samples for the given problem.  See Page 5 in Part III for more details.
* Two Guiding Questions are provided in the Student Work Analysis Grid – the other two questions can be selected by you or by the group.
* If your group needs work with multiplying or dividing decimals, additional practice problems are provided on Page 3 in Part II.

## Part I: Mathematical Background

*Approximate Time*: 20 Minutes
*Grouping*: Whole Group

A. **Welcome** members of your group to the Math Learning Community.  Remind group of **established norms**.

B. **Today's Content**:
   a. The mathematics during this session focuses on converting amongst fractions, decimals, and percents.
   b. What do we need to know in order to be able to represent quantities using these different forms?
   c. Chart ideas to refer to during the Protocol for LASW.
   d. Consider using these prompts to get group members thinking:
      i. What are the differences amongst $\frac{75}{100}$ and $\frac{3}{4}$ and 0.75?   What are the similarities?
      ii. Why do all three numerical representations exist? What are the advantages and disadvantages to each of the three representations (fractions, decimals, and percents) discussed during this session?
      iii. If students understand the number relationships within our base-ten number system as they apply to whole numbers, then they can connect this prior knowledge to decimal numbers.  For example, consider the number 127.127 as represented in the table below.  What do each of the digits represent in this quantity? What number relationships exist?

| Digit | Quantity Expressed as an Operation | Quantity Expressed with Exponents | Place Value |
|---|---|---|---|
| 1 | 1 x 100 | $1 \times 10^2$ | Hundreds <br> 1 _ _ . _ _ _ |
| 2 | 2 x 10 | $2 \times 10^1$ | Tens <br> _ 2 _ . _ _ _ |
| 7 | 7 x 1 | $7 \times 10^0$ | Ones <br> _ _ 7 . _ _ _ |
| 1 | $1 \times \frac{1}{10}$ | $1 \times 10^{-1}$ | Tenths <br> _ _ _ . 1 _ _ |
| 2 | $2 \times \frac{1}{100}$ | $2 \times 10^{-2}$ | Hundredths <br> _ _ _ . _ 2 _ |
| 7 | $7 \times \frac{1}{1000}$ | $7 \times 10^{-3}$ | Thousandths <br> _ _ _ . _ _ 7 |

C. **Relating Content to the Three C's Theme**:
   a. How do the ways in which students learn to express quantities as fractions relate to the ways in which they express quantities as decimals? As percents?
   b.  Which attributes or properties of whole numbers hold true for decimals? Which do not?
   c. Which attributes or properties of whole numbers hold true for percents? Which do not?
   d. Which representation should come first in instruction – fractions, decimals or percents?  Why?
   e. How does context play a role in the development of decimal sense? Percent sense?
   f. How is counting used in problem solving with percents?

**Part II: Math Metacognition**
*Approximate Time*: 25 minutes
*Grouping*: Whole Group

A. **Problem:** The problem used for this session is appropriate for Grades 6 - 8.

> Mentally determine each of the following:
>
> $$12 \tfrac{1}{2}\% + 0.50 + \frac{5}{8} = ?$$
>
> $$0.15 + \frac{4}{9} + 67\% = ?$$
>
> $$\frac{2}{8} + 0.435 - 25\% = ?$$

B. **Solution to Problem:** Problem 1: 1.25; Problem 2: approx. 1.2644 or $1\frac{119}{450}$;

Problem 3: 0.435

C. **Problem Intent:** (*Note: The problem intent for all Math Metacognition problems is the same*). See Session 2 for more information. In addition, this problem serves as a good start to discussing methods used to convert from one numerical representation to another.

D. **Discuss** how each group member solved the problem. Ask group members:
   a. How did you decide what form or representation to use to express your solution? Did the problem format impact this decision at all? (i.e., the first quantity is a percent, so problem #1's solution is a percent)
   b. Within which representation is it most challenging to work? Is this always the case?
   c. How does estimation play a role in a problem involving multiple numerical representations?

E. **Many shortcuts** exist in working with multiplication and division of decimals that often inhibit students' development of decimal sense. In particular, methods in which the decimal point is moved with little or no regard to the quantities given in the problem. Instead, if we try to connect what students know and understand about whole number operations and relationships, they may have better success working with and understanding decimals.
   a. For example, if 10 x 10 = 100 [1 ten x 1 ten = 1 hundred], then .10 x .10 = .0100 [1 tenth x 1 tenth = 1 hundredth].
   b. Another example: 23.7 ÷ .2. Students are often taught to convert this problem to 237 ÷ 2 by "moving the decimal point in order to make the divisor a whole number" when in fact what students are actually doing mathematically is hidden to them. The fact that they are maintaining the same relationship between the two quantities by multiplying both quantities by the same value (some multiple of 10) is masked.
   c. These ideas, and others like it, are more important to teach than rules about the movement of the decimal point and they should be brought out into the open during classroom discussion.

F. If your group is interested in **practicing these ideas**, consider presenting the following problems to try:
   a. 24.6 ÷ .3 = ?
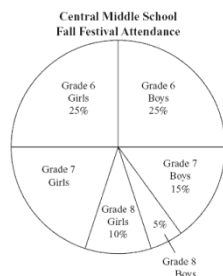   b. Estimate the following: 0.25 x .8.

## Part III: Looking at Student Work (LASW)

*Approximate Time*: 50 minutes
*Grouping*: Refer to protocol

A.  Complete the **MLC protocol** with the group.

B.  **Problem**: The source of the problem used for this session is an open-response MA DESE Released Test Item from the 2006 Grade 7 MCAS.

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**The circle graph below shows the student attendance at the Central Middle School Fall Festival.**

Central Middle School
Fall Festival Attendance

Grade 6 Girls 25% | Grade 6 Boys 25% | Grade 7 Girls | Grade 7 Boys 15% | Grade 8 Girls 10% | 5% | Grade 8 Boys

a.  What percent of the students who attended the Fall Festival were grade 7 girls? Show or explain how you got your answer.
b.  What part of the students attending the Fall Festival were girls? Write your answer as a **fraction**. Show or explain how you got your answer.
c.  There were 32 grade 7 girls who attended the Fall Festival. What was the total number of students who attended the Fall Festival? Show or explain how you got your answer.

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C.  **Solution:** a) 20% of students were grade 7 girls; b) $\frac{55}{100}$ were girls; c) 160 students attended the festival.

D.  **Problem Intent**: This problem provides an opportunity to consider how students and adults alike move in and out of numerical representations, namely fractions and percents in this particular problem. In addition, this problem layers on the fact that the numerical data is presented in a circle graph.

E.  **Discuss** the following:
   a.  What strategies were used to solve this problem?
   b.  How much experience have group members had with circle graphs?
   c.  What are common strategies or algorithms taught to students to determine quantities expressed as percents? Do these methods help or hinder the development of percent sense in students?

**Part III: Looking at Student Work (LASW), continued**

F.  Misconceptions/Questions that May Arise:
    a.  M: Refer to Part I for information on common misconceptions when working with numerical representations.
    b.  Q: How many group members simplified the fraction in part b? Is this a necessary step? Is the fraction now out of context if it is simplified? In what other situations do students apply prior knowledge in a way that does not always translate?

Options for Customization

G.  Guiding Questions:  Two Guiding Questions have been provided in the Student Work Analysis Grid for this problem. As a way to customize the LASW process, you (or your group) will need to decide on the remaining two questions. You can use the two questions listed below that are specific to this problem or refer to the list of generic questions found on Page 5 in Session 7.
    a.  What method(s) are used to translate from one representation to another?
    b.  What important mathematical ideas does the student make use of in his/her solution?

H.  Using A Group Member's Student Work:
    a.  *Prior to offering this session to your MLC, you need to collect student work samples for the task:* **Fall Festival.**
    b.  See Page 5 in Session 7 for more details on collecting student work samples. For this task, select 4 samples to discuss during the MLC session. Prior to photocopying samples, mark them as **A, B, C, and D**.

## Part IV: Our Learning

*Approximate Time*: 20 Minutes
*Grouping*: Whole Group

A. **Discussion:** After evidence of student understanding has been discussed as a whole group, you want to facilitate discussion around how the LASW process will impact what teachers do within their classrooms. Some questions to help guide discussion include:
   a. What do we take away after LASW?
   b. What did we learn? About student thinking? About our own knowledge?
      1. Refer back to chart made at the beginning of the session
   c. How does today's session relate to important mathematical content and pedagogy?
   d. How does it impact **your** practice at **your** grade level? *(Note: In order to help teachers connect this session to the mathematics within their own grade level, refer to the information below).*

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**Making Connections** Across the Grade Levels

**K – 2**: Common fractions and money are both are introduced in this grade band, which will then lead to work with percents and decimals in future grades. Understanding of place value to the left of the decimal point (i.e., whole numbers) develops here and will pave the way for work to the right of the decimal point (i.e., decimal numbers). In addition, drawing conclusions about data using graphs is a skill that begins as early as Grade 1. (K.NBT.1, K.MD.3, 1.NBT.2 a – c, 1.MD.4, MA.1.MD.5, 1.G.3, 2.NBT.1 a – b, 2.MD.8, 2.MD.10, 2.G.3)

**3 – 5**: In this grade band, students are formally introduced to decimals, including contexts involving money. Percents are also introduced, and students are expected to identify [read, name, and write], determine, and compare common fractions, decimals and percents. (3.NF.1, 3.NF.3 a – d, 3.MD.3, 3.G.2, 4.NF.1, 4.NF.2, 4.NF.3 a – d, 4.NF.5, 4.NF.6, 4.NF.7, 4.MD.4, 5.NBT.3 a – b, 5.NBT.4, 5.NF.2, 5.MD.2)

**6 – 8**: The LASW problem is appropriate for Grades 6 to 8 to explore and specifically addresses learning standard 7.EE.3. In addition, the circle graph as a form of data representation specifically addresses learning standard MA.6.SP.4a. This session's mathematics highlights the emphasis placed in middle school on the development of fraction-decimal-percent "sense." This should include the ability to translate from one representation to another, but it should be coupled with an understanding of how the three forms relate and why all three ways to represent quantities exist. In addition, the LASW problem can be solved using proportional reasoning, which is often considered the cornerstone of algebraic thinking. Decimal and percent computation are also formalized in this grade band. (6.RP.1, 6.RP.3 a – d, 6.SP.4, 7.RP.2 a – d, 7.RP.3, 7.NS.3, 8.NS.1, 8.NS.2, 8.EE.2, 8.EE.5)

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B. **Writing a Problem or Task:** As a way to synthesize learning from today's session, ask MLC members to come up with a math problem or task that would embody the ideas discussed today. The problem should be appropriate to use at their grade level. Writing these problems will help both you as the facilitator and the other group members to develop a stronger sense of how these mathematical ideas show up in classrooms from grades K – 8. *(Note: See Part IV in Session 1 for more details).*

## Part V: Feedback & Wrap-up
*Approximate Time*: 5 Minutes
*Grouping*: Individual

A. **Closing:** Close the session with a message such as: "Hope you leave here with more questions – about student thinking, about your teaching, and ways that we as a group can help support one another." Have group members keep in mind the following: Dialogue, Reflection, and Inquiry are the keys to successful learning.

B. **Exit Cards:** Pass out exit cards for group members and ask them to provide some feedback to you as the facilitator. Select one or two questions from the list below to help them summarize their thinking about the mathematics from today's session. Collect exit cards so that a summary can be shared during the next session.

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### Feedback / Exit Card Questions

- How does the mathematics that we explored connect to your own teaching?

- How do I see what we've done today relate to key mathematical ideas or pedagogical content knowledge?

- What idea or discussion topic did you find most interesting from today's session. Why?

- How was this session for you as a learner?

- What ideas were highlighted for you in today's session that you had not previously considered?

- What are you taking away from today's session?

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# Mentally determine each of the following:
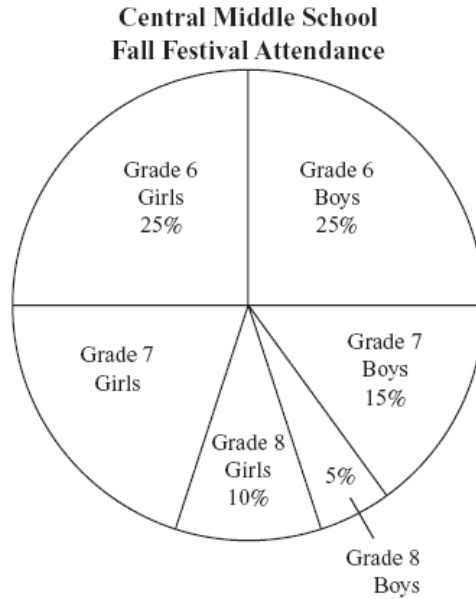
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$$0.15 + \frac{4}{9} + 67\% = ?$$

$$\frac{2}{8} + 0.435 - 25\% = ?$$

The circle graph below shows the student attendance at the Central Middle School Fall Festival.

**Central Middle School Fall Festival Attendance**

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Grade 7 Girls

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Grade 8 Girls 10%

5%

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a. What percent of the students who attended the Fall Festival were grade 7 girls? Show or explain how you got your answer.

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c. There were 32 grade 7 girls who attended the Fall Festival. What was the total number of students who attended the Fall Festival? Show or explain how you got your answer.

# Student Work Analysis for: Fall Festival

| Student | How do you see the student making connections between fractions and percents? | What evidence is there of proportional reasoning? | | |
|---------|---|---|---|---|
| A | | | | |
| B | | | | |
| C | | | | |
| D | | | | |