

# Mathematics Learning Community Overview

## *Facilitator Materials*

## *Preface*

The Regional Science Resource Center at the University of Massachusetts Medical School (RSRC), in partnership with the Massachusetts Department of Elementary and Secondary Education (ESE), has envisioned and developed the Mathematics Learning Community (MLC) curriculum materials as a means to support job-embedded professional development for teachers of mathematics in grades K – 8. The initial concept development of the MLC was funded by the ESE, along with the Noyce Foundation and the Intel Foundation, as an extension of activities that originated as part of the Massachusetts Partnership Advancing the Learning of Mathematics and Science (PALMS) program, funded by the National Science Foundation.

The materials support productive use of common teacher planning time through establishment and development of a professional learning community with a focus on student learning. Fundamental math concepts addressed in the sessions provide teachers the opportunity to extend mathematical and pedagogical understanding, first through direct engagement with the mathematics, and then through looking at student work and exploring how student understanding develops. The MLC model incorporates a gradual release of responsibility, beginning with modules that are highly structured and scripted to support early stages of facilitation. Later sessions are less prescriptive as the MLC facilitator gains experience with the format of the sessions. The early modules incorporate anonymous – though authentic – student work to promote objectivity in discussions of student understanding. As the sessions progress, teachers look at their own students' work using the descriptive, evidence-based MLC protocol.

Although it would be possible to use the MLC materials with no prior orientation or training, the developers recommend Facilitator Training to ensure a successful, robust implementation. The training addresses practical aspects of MLC implementation as well as promoting a metacognitive perspective that connects MLCs to mathematics curriculum, professional development and initiatives. For more information on MLC Facilitator Training seminars, please contact Sandra Mayrand at 508-856-5097 or [sandra.mayrand@umassmed.edu](mailto:sandra.mayrand@umassmed.edu).

Dona Apple and Wendy Cleaves, the MLC primary authors, wish to thank the mathematics coaches and teachers of Springfield, New Bedford and Worcester who were part of the MLC pilot program. The sharing of their MLC experiences, the thoughtfulness of their mathematics dialog, and their insights and suggestions were invaluable. The authors also thank the many Massachusetts teachers whose classes provided the thought-provoking student work; WestEd as the evaluator for providing crucial formative feedback; Barbara Libby, Life LeGeros, David Parker, and Emily Veader for coordination at the ESE; and Sandra Mayrand, the Director of RSRC, for overall project management.

It is the sincere hope of the MLC development team that these materials will promote student success in mathematics through supporting knowledgeable and reflective teaching and learning.

## *Introduction to the Mathematics Learning Community*

A growing consensus in the research points to the need for coordinated, classroom- and content-focused professional development that links challenging learning standards and curriculum to solve the specific problems hindering the advancement of students of mathematics (Bransford et.al., 2000; Olson, 2002; Smith, 2001; Hill and Cohen, 2000; Whitehurst, 2002; Wiley and Yoon, 1995; Kennedy, 1998; NRC Mathematics Learning Study Committee, 2000; DuFour, 1998). These professional development curriculum materials are intended to help teachers and schools make these critical connections.

These materials provide the structure and content to facilitate school-based discussions about how students understand, express and think about mathematics. Three modules allow teachers to probe student thinking about proportional reasoning, data analysis, and number sense & pre-algebra by learning how to examine student work with a diagnostic eye followed by meaningful collegial conversations. These online materials comprise the fifteen sessions within the number sense & pre-algebra MLC module.

The protocols, student work and guiding questions allow groups of teachers to uncover and explore collaboratively how students think and learn about specific mathematics concepts. With this deeper understanding of how students perceive and apply a concept, teachers are better able to further develop students' understanding in a classroom setting. To clarify goals related to the improvement of instructional practice, the MLC materials are directly linked to the Characteristics of an Effective Standards-Based Mathematics Classroom (a condensed version of which can be found on pages 7 and 8 or download the full version at: [http://www.doe.mass.edu/omste/news07/mathclass\\_char.doc](http://www.doe.mass.edu/omste/news07/mathclass_char.doc)).

It is recommended that the MLC is implemented as a complement to professional development courses that focus on increasing mathematical content knowledge, so that teachers can directly connect subject matter knowledge to the development of mathematical thinking in students. To this end, a version of the MLC that is aligned to the Math Progressions course (formerly known as Intel Math) is available (see Sandra Mayrand's contact information below).

### *MLC Facilitator Training Seminars*

The MLC is meant to serve as a long-term, job-embedded, practical approach to professional growth centered around student thinking and learning. In order to make the MLC as effective as possible, skilled facilitation is required. Math coaches, specialists, and lead teachers often fill MLC facilitator roles. It is strongly recommended that an MLC be co-facilitated for many reasons, including sharing of the workload, continuity of content in case of sickness or emergency, greater depth of knowledge, and the ability to listen more attentively to the voices of the MLC members.

Facilitator Training Seminars are built on the premise that the training group acts as a model MLC. Through this approach, facilitators can experience first-hand the types of questions that will be raised, solutions that will be shared, student ideas that will be analyzed, and mathematics that will be discussed. Training participants practice their facilitation skills within this safe learning environment. Common facilitation dilemmas are addressed, along with background knowledge of professional learning communities. Video clips of MLCs in action, along with a case study of an MLC member and insights from current MLC Facilitators round out the training. Facilitator Training Seminars can be conducted in-district or at a more cost-effective setting with multiple districts in attendance in Shrewsbury, MA.

For more information on MLC Facilitator Training Seminars, contact Sandra Mayrand at 508-856-5097 or [sandra.mayrand@umassmed.edu](mailto:sandra.mayrand@umassmed.edu).

# Mathematics Learning Community

## MLC

The MLC uses student work as a means to connect professional development to the classroom and to stimulate authentic discussions about how students learn mathematics. The goal of the MLC is to improve students' understanding of mathematics.

### What is the MLC?

The purpose of the MLC is to provide teachers of mathematics with an opportunity to work together with colleagues to relate important content and pedagogy to their own classroom practice. Over the course of the school year, MLC members engage in up to 15 school- or district-based sessions facilitated by a math coach or lead teacher. Each of the MLC sessions consists of a math discussion, mental math tasks, problem solving and the examination of student work. Student work samples examined during MLC sessions represent various grade levels, yet each session's content is always connected to three grade bands: K – 2, 3 – 5, and 6 – 8. Three key mathematical ideas – counting, composition, and context – act as threads tying all content together within the MLC sessions.

### Goals of the MLC

- To form a collaborative learning community that focuses on student learning
- To understand mathematical content in a deeper, more conceptual way
- To closely examine student work and formative assessments in order to determine students' progress and instructional needs
- To reflect on our classroom practice, share strategies, discuss best practices, and expand our professional expertise

### MLC Protocol for Looking at Student Work

- ✓ Read the problem and discuss what it is assessing
- ✓ Solve the problem individually
- ✓ Share your thinking with a partner
- ✓ Discuss the mathematics of the problem as a whole group
- ✓ Look at how students solved the same problem
- ✓ Identify evidence of understanding by using guiding questions
- ✓ Discuss evidence of student understanding as a whole group

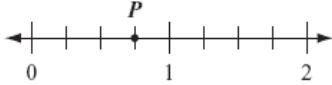
# MLC Number Sense Module

## Overview of Sessions 1 – 10

Session Title	Math Metacognition Problem(s)	LASW Problem(s)
<b>1</b> Getting Started with the MLC	N/A	<b>Grade 6:</b> Lisa has three jobs: walking the family dog, washing the dishes, and vacuuming. She walks the family dog once every 3 days. She washes the dishes once every 4 days. She vacuums once every 6 days. On which day will Lisa again do all three jobs on the same day?
<b>2</b> From Counting and Cardinality to Number in Base Ten	<p>Determine how long it is until your birthday. Then, give that answer to another person and see if s/he can determine when your birthday is.</p> <p>Jenny is enrolling her son at Little Kids Preschool in the fall. The guidelines listed on the school’s website state the following:</p> <ul style="list-style-type: none"> <li>• Little Critters: 2-2.9 year olds, 8:30-12:00</li> <li>• Puddle Jumpers: 2.9-3.5 year olds, 8:30-12:30</li> <li>• Leap Froggers: 3-4 year olds, 8:30-12:30</li> <li>• Pre-Kindergarten: 4-5 year olds, 8:30-12:30</li> </ul> <p>Her son will turn 3 years old on November 28th. Which program can he enter in September? How do you know?</p>	<p><b>Grade 1:</b> Sue and Jack were playing a game with number cards. Each student has four cards and needs to make the greatest two-digit number possible using any two of the four cards. Here are Sue’s four cards: 7      2      4      9</p> <p>Sue said that the number 9 would need to go in the tens place to make the largest number.</p> <p>Is she correct? Can you say why or why not?</p>
<b>3</b> Working with Addition	$78 + 43 = ?$ $192 = 67 + ?$	<b>Grade 2:</b> Write a story problem for this number sentence: $18 + \underline{\quad} = 72$
<b>4</b> The Relationship between Addition and Subtraction	<p><i>Represent each of the following as a number sentence:</i></p> <p>A. Sandy has 43 flowers in her garden. 25 are red, the rest are yellow. How many are yellow?</p> <p>B. I have some crayons. I gave 13 to Beth. Now I have 38. How many crayons did I start with?</p>	<b>Grade 2:</b> Jackie and Sara collected shells at the beach. Jackie collected 48 shells in all. She collected 13 more shells than Sara. How many shells did Sara collect?
<b>5</b> Subtraction Strategies	$52 - 17 = ?$ $603 - 456 = ?$ $1001 - 439 = ?$	<b>Grade 7:</b> -16 is 9 less than a number. Find the number. Show your thinking using a picture or a number line.
<b>6</b> Multiplication Strategies	$29 \times 4 = ?$	<b>Grade 4:</b> Haley swam 22 laps each day for 18 days. Then she swam 25 laps each day for 10 days. What was the total number of laps she swam over the 28 days?
<b>7</b> The Distributive Property	$39 \times 22$ $325 \times 12$	<b>Grade 7:</b> $(20 + 4)(30 + 5)$ Write a word problem that represents this expression. Solve the problem and show your work.
<b>8</b> Dealing with Division	$182 \div 15 =$ Make note of the first step you think about to begin this problem	<p><b>Grade 4:</b> Solve each problem using a method other than the traditional algorithm for division.</p> $56 \div 3$ $70 \div 9$ $128 \div 6$ $482 \div 5$ $972 \div 2$ $371 \div 8$
<b>9</b> Partitive and Quotative Division	$15 \div 5 = 3$ 1. Model the action in the problem above in two different ways. 2. Write a word problem that represents each of the actions modeled.	<p><b>Grade 5:</b> Twenty divided by 5 is 4.</p> <ol style="list-style-type: none"> <li>1. Represent this statement as a division problem.</li> <li>2. Write a word problem that models this division problem. Be sure to include labels for each of the three numbers.</li> <li>3. Draw a picture that models your word problem. Be sure to include labels in your picture.</li> </ol>
<b>10</b> Interpreting Remainders in Division Contexts	Write a division problem (two-digit by a one-digit number) where you have a remainder. Write a word problem that models this division problem. Write a brief explanation interpreting your remainder.	<p><b>Grade 7:</b> Write a word problem for <math>44 \div 6</math> where the answer to the problem is <math>7\frac{1}{3}</math>.</p> <p>Write a word problem for <math>44 \div 6</math> where the answer to the problem is 7 or 8.</p>

## MLC Number Sense Module

### Overview of Sessions 11 – 15

Session Title	Math Metacognition Problem(s)	LASW Problem(s)
<b>11</b> <b>Representing and Interpreting Fractions</b>	Point $P$ is located on the number line shown below.    Which of the following fractions best represents the location of point $P$ ?  A) $\frac{1}{4}$ B) $\frac{3}{8}$ C) $\frac{3}{4}$ D) $\frac{4}{5}$	<b>Grade 5:</b> The table below shows the amount of time each of four students spent on a mathematics test yesterday.  Time Spent on Mathematics Test (Time in hours):  Joe: $\frac{1}{2}$ , Keith: $\frac{2}{3}$ , Lena: $\frac{1}{4}$ , Mia: $\frac{2}{5}$  Which student spent the <b>greatest</b> amount of time on the test? Show how you figured out your answer.
<b>12</b> <b>Adding and Subtracting Fractions</b>	Examine the following pairs of problems. Keep track of your thinking as you determine which sum or difference is greater.  $\frac{8}{7} - \frac{3}{4}$ or $\frac{4}{5} - \frac{6}{11}$  $\frac{1}{15} + \frac{1}{11}$ or $\frac{4}{3} - \frac{3}{4}$  $\frac{3}{4} + \frac{6}{7}$ or $\frac{4}{5} + \frac{5}{6}$	<b>Grades 6 – 8:</b> A new fitness center is going to be built soon in your city. <ol style="list-style-type: none"> <li>Label each section with its fraction of the total area of the fitness center.</li> <li>The owners of the fitness center have now decided to use <math>\frac{1}{8}</math> of the Pool's area to install showers. What fraction of the total area of the fitness center will the Pool now cover? Explain your thinking.</li> <li>The Weights, Aerobics, and Treadmills sections will be renamed as one section called the Workout Zone. What fraction of the total area of the fitness center will the Workout Zone now cover?</li> </ol>
<b>13</b> <b>Multiplying Fractions</b>	Estimate the following products.  $2\frac{1}{3} * \frac{5}{8}$  $4\frac{1}{2} * 3\frac{12}{13}$	<b>Grade 6 – 8:</b> I baked a rectangular sheet cake for a party. $\frac{3}{8}$ of the cake was left after the party. Draw and label how much of the cake was left. My son came to visit and ate $\frac{1}{3}$ of the remaining cake.  Draw and label how much of the remaining cake he ate. What portion of the whole cake did he eat?
<b>14</b> <b>Dividing Fractions</b>	Consider the following problem:  How many $\frac{2}{3}$ 's are in 3?  Be prepared to explain how you solved the problem.	<b>Grades 6 – 8:</b> Solve each of the following problems. Draw a model to show your thinking. <ol style="list-style-type: none"> <li>Juan has 3 gallons of yellow paint that needs to be put into new containers for an art project. Each container holds <math>\frac{2}{7}</math> of a gallon of paint. How many containers can Juan fill if he uses up all of the paint?</li> <li>The art class has <math>\frac{5}{6}</math> of a gallon of paint that needs to be shared equally among 3 students. How much paint will each student get?</li> <li>The art class wants to make banners out of construction paper. Each banner takes <math>\frac{3}{5}</math> yard of paper. How many banners can they make if they have <math>2\frac{1}{2}</math> yards of paper to use?</li> </ol>
<b>15</b> <b>Working with Fractions, Decimals &amp; Percents</b>	Mentally determine each of the following:  $12\frac{1}{2}\% + 0.50 + \frac{5}{8} = ?$  $0.15 + \frac{4}{9} + 67\% = ?$  $\frac{2}{8} + 0.435 - 25\% = ?$	<b>Grade 7:</b> The circle graph below shows the student attendance at the Central Middle School Fall Festival. <ol style="list-style-type: none"> <li>What percent of the students who attended the Fall Festival were grade 7 girls? Show or explain how you got your answer.</li> <li>What part of the students attending the Fall Festival were girls? Write your answer as a fraction. Show or explain how you got your answer.</li> <li>There were 32 grade 7 girls who attended the Fall Festival. What was the total number of students who attended the Fall Festival? Show or explain how you got your answer.</li> </ol>



# Standards-Based Teaching and Learning Characteristics in Mathematics

To support efforts to improve the teaching and learning of mathematics across the state, the summer of 2005 the Massachusetts Department of Education (MADOE) launched the Comprehensive School Reform Math Initiative. Each district participating in this initiative receives grant funding to support a fulltime staff position dedicated to leadership of the district's math initiative. These leaders, in cooperation with MADOE, come together regularly as the Math Support Specialist (MSS) Network to share ideas, resources, and strategies related to K-8 mathematics education reform.

MADOE's Mathematics Targeted Assistance team and the MSS Network participants have developed a shared vision of standards-based mathematics teaching and learning to guide this new collaboration. Based on this vision, we have articulated the characteristics of an effective standards-based mathematics classroom and their corresponding indicators to serve as a reference for instructional planning and observation. This document represents the present state of this work. It is intended to support activities that advance standards-based educational practice, including formal study, dialogue and discussion, classroom observations, and other professional development activities.

## **A Shared Vision of Standards-Based Mathematics Teaching and Learning**

***Standards-based mathematics teaching and learning is a cooperative effort by teachers and students to actively engage in purposeful learning experiences that stimulate curiosity, enjoyment, and deep understanding of the mathematical concepts outlined in the Massachusetts Mathematics Curriculum Framework. Teachers and students are knowledgeable about learning objectives, and have ownership of and are accountable for learning outcomes.***

# Standards-Based Teaching and Learning Characteristics in Mathematics

## 1. STUDENT LEARNING STANDARDS

- 1.1. The mathematics standards being addressed in the lesson are evident and clear to the students
- 1.2. Exemplars demonstrate expectations of student achievement.

## 2. ORGANIZATION OF THE LESSON

- 2.1. The lesson is well planned and organized. The objectives of the lesson are clearly stated and connected with the learning standards of the larger unit of which it is a part. The lesson develops in a clear, logical manner.
- 2.2. Time is used effectively and purposefully.
- 2.3. Multiple grouping strategies are used to achieve the learning that is the object of the lesson (e.g., individual, small groups, whole class, teacher-student).

## 3. CLASSROOM ENVIRONMENT

- 3.1. It is clear that the students appear to feel safe and are willing to take risks.
- 3.2. The appearance and physical organization of the classroom contribute to a positive learning environment.

## 4. STUDENT ENGAGEMENT

- 4.1. Students are actively engaged in all aspects of the lesson. Behavior is appropriate for the lesson/activities.
- 4.2. Students are engaged in understanding and learning mathematics in various ways that include skill building, conceptual understanding, applying multiple problem-solving strategies, and real-world applications.
- 4.3. Students consciously examine their thinking by questioning their understanding of the mathematics presented. Students support and defend their reasoning with data while using appropriate mathematical language.

## 5. TEACHING

- 5.1. Depth of content knowledge is evident throughout the presentation of the lesson. Mathematical concepts are presented accurately.
- 5.2. Through the use of probing questions and student responses, decisions are made about what direction to take, what to emphasize, and what to extend in order to build students' mathematical understanding.
- 5.3. Students' prior knowledge is incorporated as new mathematical concepts are introduced. When students raise comments, questions, and/or concerns, their perspectives are acknowledged and either redirected or affirmed, linking existing knowledge to new knowledge gained within the lesson.
- 5.4. Student misconceptions are anticipated/identified and addressed.
- 5.5. Classroom strategies incorporate multiple forms of representation (e.g., pictures, words, symbols, diagrams, tables, graphs).

## 6. INSTRUCTIONAL TOOLS

- 6.1. Appropriate tools for learning are provided (e.g., measuring instruments, manipulatives, calculators, computers). All necessary resources for the lesson are easily accessible. Instruction and support are provided for use of tools.

## 7. EQUITY

- 7.1. There are high learning expectations for all students. All students participate, and their ideas are valued. The belief is evident among all in the classroom that effort, not innate ability, is the key to significant mathematical learning.
- 7.2. Various learning experiences are provided that are appropriate for the range of learners in the classroom (i.e., differentiation by content, process, and/or product).

## 8. ASSESSMENT

- 8.1. There is evidence of multiple types (e.g., group/individual presentations, written reflections, tests) of diagnostic and ongoing formative assessment.
- 8.2. Students are engaged in and responsible for their own learning, examining their results with directive feedback that enables revision and improvement.



## MLC Number Sense Module

### Crosswalk of Sessions 1 – 4

Session Title & LASW Problem	CCSS Addressed in the LASW Problem	Standards for Mathematical Practice Addressed in the Session	CCSS: Vertical Connections
<p><b><i>1: Getting Started with the MLC</i></b></p> <p>Lisa has three jobs: walking the family dog, washing the dishes, and vacuuming.</p> <ul style="list-style-type: none"> <li>▪ She walks the family dog once every 3 days.</li> <li>▪ She washes the dishes once every 4 days.</li> <li>▪ She vacuums once every 6 days.</li> </ul> <p>The calendar below shows that Lisa did all three jobs on Monday the 2<sup>nd</sup>. On which day will Lisa again do all three jobs on the same day?</p>	<p><b>6.NS.4</b> Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.</p>	<p><b>5:</b> Use appropriate tools strategically.</p> <p><b>7:</b> Look for and make use of structure.</p>	<p>K.CC.3 2.NBT.2 2.OA.3 MA.2.MD.7a 4.OA.4 6.NS.4</p>
<p><b><i>2: From Counting and Cardinality to Number in Base Ten</i></b></p> <p>Sue and Jack were playing a game with number cards. Each student has four cards and needs to make the greatest two-digit number possible using any two of the four cards. Here are Sue's four cards:</p> <p style="text-align: center;">7    2    4    9</p> <p>Sue said that the number 9 would need to go in the tens place to make the largest number. Is she correct? Can you say why or why not?</p>	<p><b>1.NBT.2</b> Understand that the two digits of a two-digit number represent amounts of tens and ones.</p>	<p><b>2:</b> Reason abstractly and quantitatively</p> <p><b>3:</b> Construct viable arguments and critique the reasoning of others.</p> <p><b>6:</b> Attend to precision</p>	<p>K.CC.1 – 7 K.NBT.1 K.MD.1 1.NBT.1 – 3 2.NBT.1 2.NBT.3 – 7 2.NBT.9 MA.2.MD.7a 3.NBT.1 – 3 3.MD.1 4.NBT.1 – 6 4.MD.1 – 2 5.NBT.1 – 7 5.MD.1 6.NS.2 – 3</p>
<p><b><i>3: Working with Addition</i></b></p> <p>Write a story problem for this number sentence: <math>18 + \underline{\quad} = 72</math></p>	<p><b>2.OA.1</b> Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.</p> <p><b>2.NBT.5</b> Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.</p>	<p><b>2:</b> Reason abstractly and quantitatively</p> <p><b>4:</b> Model with Mathematics</p>	<p>K.CC.4 – 5    7.EE.4a K.OA.1 – 2    8.EE.7a – b 1.OA.1        8.EE.8a – b 1.OA.5 – 6 2.OA.1 2.NBT.5 3.NBT.2 3.OA.8 4.OA.3 4.NBT.4 6.EE.1 6.EE.2a – c 6.EE.3 – 6 7.EE.1 – 3</p>
<p><b><i>4: The Relationship Between Addition and Subtraction</i></b></p> <p>Jackie and Sara collected shells at the beach. Jackie collected 48 shells in all. She collected 13 more shells than Sara. How many shells did Sara collect?</p>	<p><b>2.OA.1</b> Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.</p> <p><b>2.NBT.7</b> Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.</p>	<p><b>1:</b> Make sense of problems and persevere in solving them.</p> <p><b>3:</b> Construct viable arguments and critique the reasoning of others.</p>	<p>K.OA.1 – 2 K.OA.5 1.OA.1 – 6 1.OA.8 MA.1.OA.9 2.OA.1 – 2 2.OA.4 2.NBT.6 – 9 3.OA.7 3.NBT.2 4.NBT.6 5.NBT.6 – 7 7.NS.1a – d 7.NS.2a – d</p>

## MLC Number Sense Module

### Crosswalk of Sessions 5 – 8

Session Title & Student Work Problem	CCSS Addressed in the LASW Problem	SMP Addressed in the Session	CCSS: Vertical Connections
<p><b>5: Subtraction Strategies</b></p> <p>16 is 9 less than a number. Find the number. Show your thinking using a picture or a number line.</p>	<p><b>7.NS.1</b> Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.</p> <p>c. Understand subtraction of rational numbers as adding the additive inverse, <math>p - q = p + (-q)</math>. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.</p> <p>d. Apply properties of operations as strategies to add and subtract rational numbers.</p>	<p><b>2:</b> Reason abstractly and quantitatively.</p> <p><b>5:</b> Use appropriate tools strategically.</p>	<p>K.OA.1 – 2    7.NS.2a – d 1.OA.1        7.NS.3 1.OA.3 – 4    7.EE.3 MA.1.OA.9 2.OA.1 3.NF.2a – b MA.5.NS.1 5.OA.1 6.NS.5 6.NS.6a – c 6.NS.7a – d 6.EE.2a 7.NS.1a – d</p>
<p><b>6: Multiplication Strategies</b></p> <p>Haley swam 22 laps each day for 18 days. Then she swam 25 laps each day for 10 days. What was the total number of laps she swam over the 28 days?</p>	<p><b>4.OA.3</b> Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.</p>	<p><b>1:</b> Make sense of problems and persevere in solving them.</p> <p><b>6:</b> Attend to precision.</p>	<p>K.CC.1        4.NF.4a – c 1.OA.5        5.NBT.5 1.NBT.4 – 6    5.NBT.7 2.OA.3 – 4    5.NF.4a – b 2.NBT.2       5.NF.5 – 6 3.OA.1        6.NS.3 3.OA.3 – 5    7.NS.2 a, c 3.OA.7        7.NS.3 3.OA.8        8.EE.1 3.NBT.3       8.EE.4 3.MD.7a – d 4.OA.1 – 4 4.NBT.5 MA.4.NBT.5a</p>
<p><b>7: The Distributive Property</b></p> <p><math>(20 + 4)(30 + 5)</math> Write a word problem that represents this expression. Solve the problem and show your work.</p>	<p><b>7.EE.3</b> Solve multi-step real-world and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.</p>	<p><b>4:</b> Model with Mathematics.</p> <p><b>7:</b> Look for and make use of structure.</p>	<p>K.CC.1        6.NS.3 – 4 1.OA.5        6.EE.1 1.NBT.4 – 6    6.EE.3 – 4 2.OA.3 – 4    7.NS.2a, c 2.NBT.2       7.NS.3 3.OA.1        7.EE.1 – 3 3.OA.3 – 5    8.EE.1 3.OA.7 – 8 3.NBT.3 4.OA.1 – 4 4.NBT.5 MA.4.NBT.5a 5.OA.1 5.NBT.5</p>
<p><b>8: Dealing with Division</b></p> <ul style="list-style-type: none"> <li>▪ <math>56 \div 3</math></li> <li>▪ <math>70 \div 9</math></li> <li>▪ <math>128 \div 6</math></li> <li>▪ <math>482 \div 5</math></li> <li>▪ <math>972 \div 2</math></li> <li>▪ <math>371 \div 8</math></li> </ul>	<p><b>4.NBT.6</b> Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</p>	<p><b>7:</b> Look for and make use of structure.</p> <p><b>8:</b> Look for and express regularity in repeated reasoning.</p>	<p>K.CC.6        7.NS.2b,c K.OA.1        7.NS.3 1.OA.3        7.EE.3 1.NBT.6       8.EE.1 2.OA.2        8.EE.3 2.NBT.5 2.NBT.8 – 9 3.OA.3 – 8 4.OA.2 – 3 4.NBT.6 5.NBT.6 – 7 5.NF.3 5.NF.7 a – c 6.NS.1 – 3</p>

## MLC Number Sense Module

### Crosswalk of Sessions 9 – 12

Session Title & Student Work Problem	CCSS Addressed in the LASW Problem	SMP Addressed in the Session	CCSS: Vertical Connections
<p><b>9: Partitive and Quotative Division</b></p> <p>Twenty divided by 5 is 4.</p> <ul style="list-style-type: none"> <li>▪ Represent this statement as a division problem.</li> <li>▪ Write a word problem that models this division problem. Be sure to include labels for each of the three numbers.</li> <li>▪ Draw a picture that models your word problem. Be sure to include labels in your picture.</li> </ul>	<p><b>5.NBT.6</b> Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</p>	<p><b>2:</b> Reason abstractly and quantitatively.</p> <p><b>4:</b> Model with Mathematics.</p>	<p>K.CC.6      6.EE.2a – b  K.OA.1 – 2      7.NS.2b, c  1.OA.3      7.NS.3  1.NBT.6      7.EE.2  2.OA.1 – 2      7.EE.3  2.NBT.5  2.NBT.7  3.OA.2 – 8  4.OA.2 – 3  4.NBT.6  5.OA.2  5.NBT.6 – 7  5.NF.7a – c  6.NS.1 – 3</p>
<p><b>10: Interpreting Remainders in Division Contexts</b></p> <p>Write a word problem for <math>44 \div 6</math> where the answer to the problem is <math>7\frac{1}{3}</math>.</p> <p>Write a word problem for <math>44 \div 6</math> where the answer to the problem is 7 or 8.</p>	<p><b>4.OA.3</b> Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.</p> <p><b>5.NF.3</b> Interpret a fraction as division of the numerator by the denominator (<math>\frac{a}{b} = a \div b</math>). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.</p>	<p><b>1:</b> Make sense of problems and persevere in solving them.</p> <p><b>3:</b> Construct viable arguments and critique the reasoning of others.</p> <p><b>4:</b> Model with Mathematics.</p>	<p>K.CC.4 – 5      7.NS.2a – d  1.OA.1      7.NS.3  1.G.3      7.EE.1  2.OA.1      7.EE.3  2.G.3      8.EE.3  4.OA.3  5.NF.3  3.OA.2 – 4  3.NF.1  3.G.2  4.NBT.6  4.NF.1  5.NBT.6 – 7  5.NF.7a – c  6.NS.1</p>
<p><b>11: Representing and Interpreting Fractions</b></p> <p>The table below shows the amount of time each of four students spent on a mathematics test yesterday. Which student spent the <b>greatest</b> amount of time on the test? Show how you figured out your answer.</p>	<p><b>4.NF.2</b> Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as <math>\frac{1}{2}</math>. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols <math>&gt;</math>, <math>=</math>, or <math>&lt;</math>, and justify the conclusions, e.g., by using a visual fraction model.</p>	<p><b>2:</b> Reason abstractly and quantitatively</p> <p><b>4:</b> Model with Mathematics.</p>	<p>1.G.3      7.EE.3  2.G.3  3.NF.1  3.NF.2a – b  3.NF.3a – d  3.G.2  4.NF.1 – 2  5.NF.2 – 3  6.RP.3  6.NS.6a – c  6.NS.7  7.NS.1a – d</p>
<p><b>12: Adding and Subtracting Fractions</b></p> <p>A new fitness center is going to be built soon in your city.</p> <ol style="list-style-type: none"> <li>1. Label each section with its fraction of the total area of the fitness center.</li> <li>2. The owners of the fitness center have now decided to use <math>\frac{1}{8}</math> of the Pool's area to install showers. What fraction of the total area of the fitness center will the Pool now cover? Explain your thinking.</li> <li>3. The Weights, Aerobics, and Treadmills sections will be renamed as one section called the Workout Zone. What fraction of the total area of the fitness center will the Workout Zone now cover?</li> </ol>	<p><b>5.NF.2</b> Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.</p>	<p><b>1:</b> Make sense of problems and persevere in solving them.</p> <p><b>2:</b> Reason abstractly and quantitatively.</p> <p><b>4:</b> Model with mathematics.</p>	<p>K.CC.6      7.NS.1a – d  K.OA.2      7.NS.3  1.OA.1      7.EE.3  1.G.2 – 3  2.OA.1  2.G.3  3.NF.1  3.NF.3a – d  3.MD.5a – b  3.MD.6  3.MD.7a – d  3.G.2  4.NF.1 – 2  4.NF.3a – d  5.NF.1 – 3  6.RP.3a – d  6.NS.7  7.RP.2a – d</p>

## MLC Number Sense Module

### Crosswalk of Sessions 13 – 15

Session Title & Student Work Problem	CCSS Addressed in the LASW Problem	SMP Addressed in the Session	CCSS: Vertical Connections
<p><b>13: Multiplying Fractions</b></p> <p>I baked a rectangular sheet cake for a party. <math>\frac{3}{8}</math> of the cake was left after the party. Draw and label how much of the cake was left. My son came to visit and ate <math>\frac{1}{3}</math> of the remaining cake. Draw and label how much of the remaining cake he ate. What portion of the whole cake did he eat?</p>	<p><b>5.NF.4</b> Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.</p>	<p><b>2:</b> Reason abstractly and quantitatively.</p> <p><b>4:</b> Model with mathematics.</p>	<p>K.CC.5 – 6 1.OA.1 1.G.3 2.OA.3 – 4 2.G.2 – 3 5.NF.4 a – b 3.OA.1 3.OA.3 3.OA.5 3.OA.7 – 8 3.NBT.1 3.NF.1 3.NF.3 a – d 3.MD.5a – b 3.MD.6 3.MD.7a – d 3.G.2 4.OA.3 4.NBT.5 4.NF.2</p> <p>4.NF.4 a – c 5.NBT.7 5.NF.2 – 3 5.NF.5 a – b 5.NF.6 6.RP.3 a – d 6.NS.1 6.NS.6 a – c 6.NS.7 6.EE.2 a – c 6.EE.3 6.G.1 7.RP.2 a – d 7.NS.2 a – d 7.NS.3 7.EE.3 7.G.6</p>
<p><b>14: Dividing Fractions</b></p> <p><i>Solve each of the following problems. Draw a model to show your thinking.</i></p> <p>A. Juan has 3 gallons of yellow paint that needs to be put into new containers for an art project. Each container holds <math>\frac{2}{7}</math> of a gallon of paint. How many containers can Juan fill if he uses up all of the paint?</p> <p>B. The art class has <math>\frac{5}{6}</math> of a gallon of paint that needs to be shared equally among 3 students. How much paint will each student get?</p> <p>C. The art class wants to make banners out of construction paper. Each banner takes <math>\frac{3}{5}</math> yard of paper. How many banners can they make if they have <math>2\frac{1}{2}</math> yards of paper to use?</p>	<p><b>6.NS.1</b> Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.</p>	<p><b>1:</b> Make sense of problems and persevere in solving them.</p> <p><b>8:</b> Look for and express regularity in repeated reasoning.</p>	<p>K.CC.5 – 6 K.OA.2 1.OA.1 1.G.3 2.OA.1 2.OA.3 – 4 2.G.2 – 3 3.OA.2 – 3 3.OA.6 – 8 3.NF.1 3.NF.3a – d 3.MD.5a – b 3.MD.6 3.MD.7a – d 3.G.2 4.OA.2 – 3 4.NBT.6 4.NF.1 4.NF.3a – d 5.NBT.6</p> <p>5.NF.3 5.NF.7a – c 6.RP.3a – d 6.NS.1 – 2 6.NS.7 7.RP.2 a – d 7.NS.2 a – d 7.NS.3 7.EE.3 7.G.6 8.NS.1</p>
<p><b>15: Working with Fractions, Decimals &amp; Percents</b></p> <p>The circle graph below shows the student attendance at the Central Middle School Fall Festival.</p> <p>a) What percent of the students who attended the Fall Festival were grade 7 girls? Show or explain how you got your answer.</p> <p>b) What part of the students attending the Fall Festival were girls? Write your answer as a fraction. Show or explain how you got your answer.</p> <p>c) There were 32 grade 7 girls who attended the Fall Festival. What was the total number of students who attended the Fall Festival? Show or explain how you got your answer.</p>	<p><b>7.EE.3</b> Solve multi-step real-world and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.</p> <p><b>MA.6.SP.4a</b> Read and interpret circle graphs.</p>	<p><b>1:</b> Make sense of problems and persevere in solving them.</p> <p><b>2:</b> Reason abstractly and quantitatively.</p> <p><b>7:</b> Look for and make use of structure.</p>	<p>K.NBT.1 K.MD.3 1.NBT.2a – c 1.MD.4 MA.1.MD.5 1.G.3 2.NBT.1a – b 2.MD.8 2.MD.10 2.G.3 3.NF.1 3.NF.3a – d 3.MD.3 3.G.2 4.NF.1 – 2 4.NF.3a – d 4.NF.5 – 7 4.MD.4 5.NBT.3a – b 5.NBT.4 5.NF.2 5.MD.2</p> <p>6.RP.1 6.RP.3a – d 6.SP.4 MA.6.SP.4a 7.RP.2a – d 7.RP.3 7.NS.3 7.EE.3 8.NS.1 8.NS.2 8.EE.2 8.EE.5</p>