Exploration: A Deeper Dive into the Ratio and Proportions Model Curriculum Unit (grade 6) for Mathematics
Exploration Outline

★ Key Shifts in the 2011 New MA Curriculum Framework for Mathematics

★ Overview of the Model Curriculum Unit (MCU) Project

★ Deep dive into a publicly released *Ratio and Rates* Grade 6 Mathematics Prototype Unit

[http://www.doe.mass.edu/candi/model/sample.html](http://www.doe.mass.edu/candi/model/sample.html)
Overarching Goal

To prepare all students for success after high school
New 2011 MA Framework for Mathematics – Key Shifts
Key Mathematics Shifts in the Standards

- **Focus:** Deeper look into fewer standards
- **Coherence:** Think across grades, and link to major topics
- **Rigor:**
  - Expectation of conceptual understanding, procedural skill and fluency, and application
  - New Standards for Mathematical Practice
- **Clarity**
## Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
Model Curriculum Units (MCU) Project Overview
Model Curriculum Units

Provide districts and teachers with high quality and rigorous units they can choose to teach and/or use to advance their own curriculum development efforts.
Model Curriculum Units

- MCU project is a Race to the Top (RTTT) initiative for ESE, which has previously concentrated on standards and assessments, not curriculum materials
- 100 PK-12 units in ELA/literacy, mathematics, science, and social studies by 2014
- 35 units will be tried out during 2012-2013 in RTTT districts
- Exemplify the shifts in the 2011 Frameworks
- Understanding By Design (UbD) model with lesson plans and print/digital media resources
- Extensive unit review process
- WGBH documenting the process
Why these Math Concepts were Chosen for MCUs

- Identified as critical areas/priority concepts and skills in the Common Core and PARCC
- Focus on a progression over grades (e.g., gr. 3-5 fractions)
- Standards that are challenging for teachers to teach and students to learn, e.g. fractions
MCU Process

COHORT 1 MCU DEVELOPMENT
- MCU development (grade-level content teams)
- Extensive PD (UbD, CEPA)
- Design teams (content & grade level)
- Educators’ selection (RttT districts)

COHORT 2 MCU DEVELOPMENT
- MCU development
- PD (UbD, CEPA)
- Design teams (content & grade level)
- Educators’ selection

Launch
Revisions / Edits
Tryout feedback
Tryouts
Evaluation (Tri-State Rubric)
Ensuring Quality MCUs
Tri-State Quality Review Rubric for Lessons and Units

★ Developed by the Tri-State Collaborative (MA, NY, RI)

★ Used to evaluate all mathematics model units

★ Can be used to:
  ★ create high quality model units
  ★ review existing units for quality
  ★ revise units
## Structure of the Rubric

<table>
<thead>
<tr>
<th>I. Alignment to the Depth of the CCSS</th>
<th>II. Key Shifts in the CCSS</th>
<th>III. Instructional Supports</th>
<th>IV. Assessment</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>The lesson/unit aligns with the letter and spirit of the CCSS:</strong></td>
<td><strong>The lesson/unit reflects evidence of key shifts that are reflected in the CCSS:</strong></td>
<td><strong>The lesson/unit is responsive to varied student learning needs:</strong></td>
<td><strong>The lesson/unit regularly assesses whether students are mastering standards-based content and skills:</strong></td>
</tr>
<tr>
<td>- Targets a set of grade level CCSS mathematics standard(s) to the full depth of the standards for teaching and learning.</td>
<td>- Focus: Lessons and units targeting the major work of the grade provide an especially in-depth treatment, with especially high expectations. Lessons and units targeting supporting clusters have visible connection to the major work of the grade and are sufficiently &quot;brief.&quot; Lessons and units do not hold students responsible for material from later grades.</td>
<td>- Includes clear and sufficient guidance to support teaching and learning of the targeted standards, including, when appropriate, the use of technology and media.</td>
<td>- Is designed to elicit direct, observable evidence of the degree to which a student can independently demonstrate the targeted CCSS.</td>
</tr>
<tr>
<td>- Standards for Mathematical Practice that are central to the lesson are identified, handled in a grade-appropriate way, and well connected to the content being addressed.</td>
<td>- Coherence: The content develops through reasoning about the new concepts on the basis of previous understandings and provides opportunities for students to transfer knowledge and skills within and across domains and learning progressions.</td>
<td>- Uses and encourages precise and accurate mathematical, academic language, terminology, and concrete or abstract representations (e.g., pictures, symbols, expressions, equations, graphics, models) in the discipline.</td>
<td>- Assesses student proficiency using methods that are accessible and unbiased, including the use of grade level language in student prompts.</td>
</tr>
<tr>
<td>- Presents a balance of mathematical procedures and deeper conceptual understanding.</td>
<td>- Rigor: Requires students to engage with and demonstrate challenging mathematics with appropriate balance among the following:</td>
<td>- Engages students in productive struggle through relevant, thought-provoking questions, problems, and tasks that stimulate interest and elicit mathematical thinking.</td>
<td>- Includes aligned rubrics, answer keys, and scoring guidelines that provide sufficient guidance for interpreting student performance.</td>
</tr>
</tbody>
</table>

A unit or longer lesson should:

- Use varied modes of curriculum embedded assessments that may include pre-, formative, summative and self-assessment measures.
Tri-State Quality Review Rubric

Purposes

★ Provide clear, descriptive criteria for alignment of units to the new Math Curriculum Framework
★ Designed for determining quality of units only
★ Provide meaningful, constructive feedback to developers of units
★ Identify units that can serve as models
★ Guide collegial review and jurying processes
In November 2012

★ Four prototypes were *publicly* released at the Curriculum and Instruction Summit, one of which we will take a deeper dive into today.

★ Over 30 Model Curriculum Units were released to RTTT districts to be tried out this year (2012-13).
Model Curriculum Units
Tryouts in RTTT Districts

- The purpose of the tryouts is to:
  - Collect qualitative feedback to ensure MCUs are high quality
  - Inform final editing and refinement of MCUs prior to publication

- Feedback requested from teachers who try out the units
  - Provide general feedback to us
  - Complete the online survey
<table>
<thead>
<tr>
<th>Grade &amp; Unit Name</th>
<th>Standard(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gr. 1 Power of Ten</td>
<td>1.NBT.2a-c, 1.NBT.3, SMP.2, SMP.7, SMP.8 ELA: SL.1.4, SL.1.5</td>
</tr>
<tr>
<td>Gr. 2 To Compose or Decompose</td>
<td>2.NBT.7, 2.NBT.9, SMP.3, SMP.4, SMP.6, SMP.7, SMP.8 ELA: SL.2.1.a-c, SL.2.3</td>
</tr>
<tr>
<td>Gr. 3 Exploring Fractions</td>
<td>3.NF.1, 3.G.2, 3.NF.2a-b, SMP2, SMP.4, SMP.7</td>
</tr>
<tr>
<td>Gr. 4 Fractions Size Matters</td>
<td>4.NF.1, 4.NF.2, SMP.2, SMP.3, SMP.7 ELA: 4.W.2.D, 4.RI.7</td>
</tr>
<tr>
<td>Gr. 5 Adding Subtracting Fractions</td>
<td>5.NF.1, 5.NF.2, SMP.1, SMP.2, SMP.3, SMP.4, SMP.8</td>
</tr>
<tr>
<td>Gr. 6 Ratios and Rates</td>
<td>6.RP.1, 6.RP.2, 6.RP.3a-c, SMP.1, SMP.2, SMP.3, SMP.4, SMP.7, ELA: RST.4, WHST.2a-e, WHST.4</td>
</tr>
<tr>
<td>Proportions and Proportional Reasoning (Gr. 7)</td>
<td>7.RP.1; 7.RP.2 a, b, c, d; 7. RP.3; SMP.1, SMP.2, SMP.3, SMP.4, CMP.6; 6-8.WHST.1; 6-8.WHST.2.D; 6-8.WHST.2.F</td>
</tr>
<tr>
<td>Connecting Proportions, Lines and Linear Equations (Gr. 8)</td>
<td>8.EE.5, 8.EE.6 , SMP.1. SMP3, SMP4, SMP 6, ELA - SL8.4:</td>
</tr>
<tr>
<td>Reasoning with Equations (HS, Algebra I)</td>
<td>A.REI.1, A.REI.3, A.CED.1, SMP1., SMP.3, SMP.6, SMP.7</td>
</tr>
<tr>
<td>Rational Function Models (HS Algebra II)</td>
<td>F.IF.4, F.IF.9, F.BF.1.b , SMP.1, SMP.3, SMP.4, R2, R7, W1.</td>
</tr>
<tr>
<td>Experimenting with Congruency Transformations in Geometry (HS Geometry)</td>
<td>G. CO.2; G. CO.3;G. CO.4;G. CO.5;G. CO.6; SMP.4; SMP.5; SMP.6;R7. R8. W2</td>
</tr>
</tbody>
</table>
# Math MCUs in Development*

*as of 2/1/2013*

<table>
<thead>
<tr>
<th>Unit Name &amp; Grade</th>
<th>Standard(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>PreK</strong> Putting Together and Taking Away Through Five</td>
<td>PK.OA.MA.1, PK.CC.MA.5, SMP.1, SMP.7, SL.4, SL.5</td>
</tr>
<tr>
<td><strong>Gr. K</strong> Composing and Decomposing the Tweens and Teens</td>
<td>K.NBT.1, SMP.2, SMP.3, SMP.7, SMP.8, SLK.5, SLK.6.</td>
</tr>
<tr>
<td><strong>Gr. 1 Math Detectives: Using Properties of Operations to Add &amp; Subtract</strong></td>
<td>1.OA.1, 1.OA.3, 1.OA.4, 1.OA.7, 1.OA.MA.9, SMP.1, SMP.2, SMP.3, SMP.4</td>
</tr>
<tr>
<td><strong>Gr. 2 Problem Solving Using Addition &amp; Subtraction Situations</strong></td>
<td>2.OA.1, 2.MD.5, 2.MD.6, SMP.1, SMP.2, SMP.4, SMP.5, SMP.7</td>
</tr>
<tr>
<td><strong>Gr. 3 Using Properties of Operations to Multiply and Divide</strong></td>
<td>3.OA.5, 3.OA.6</td>
</tr>
<tr>
<td><strong>Gr. 4 Building Fractions from Unit Fractions</strong></td>
<td>4.NF.3a-d, 4.NF.4a-c, SMP.2, SMP.4, 4.RI.7</td>
</tr>
<tr>
<td><strong>Gr. 5 Multiplication and Division of Fractions</strong></td>
<td>5.NF.4a-b, 5.NF.7a-c, SMP.2, SMP.4, SMP.7, 5.RI.4</td>
</tr>
<tr>
<td><strong>Solving One Variable Equation (Gr. 6)</strong></td>
<td>6.EE.5, 6.EE.6, 6.EE.7, 6.EE.8, 6.EE.9, SMP.1, SMP.2, SMP.4, SMP.5, SMP.6, SMP.7, SMP.8, ELA: RST.4, RST.7, WHST.2D</td>
</tr>
<tr>
<td><strong>Equivalent Expressions (Gr. 7)</strong></td>
<td>7.EE.1, 7.EE.2, 7.EE.3, 7.EE.4a-c, 7.EE.MA.4c, SMP.1, SMP.2, SMP.3, SMP.4, SMP.5, SMP.6, SMP.7, SMP.8 ELA: WHST.1, WHST.2d, WHST.2f</td>
</tr>
<tr>
<td><strong>Solving Simultaneous Equations (Gr. 8)</strong></td>
<td>8.EE.7, 8.EE.8</td>
</tr>
<tr>
<td><strong>Interpret and Create Different Models of Functions (HS Algebra I)</strong></td>
<td>F.IF.1-3, F.LE.1-3</td>
</tr>
<tr>
<td><strong>Analyzing Functions Using Different Representations - HS Algebra II</strong></td>
<td>F.IF.7e, F.IF.9, F.LE.4, SMP.3, SMP.4, SMP.7</td>
</tr>
<tr>
<td><strong>Similarity and Transformations in Geometry - HS</strong></td>
<td>SRT.1, SRT.2, SRT.3, SRT.5, SMP.2, SMP.3, SMP.7</td>
</tr>
<tr>
<td><strong>Reality Check - Financial Literacy - HS</strong></td>
<td>FRDM.2, FRDM.4, RST.10, +F-BF.1c, +F-BF.5, +S-MD.5, *A-SSE.4, RST.CS.4, SMP.3, SMP.4, SMP.5, PMM.1, PMM.4, PMM.6, CD.1, CD.3, E.1.10, E.6.5, SI.1, SI.3</td>
</tr>
</tbody>
</table>
A Deep Dive into a Mathematics MCU: Grade 6: *Ratio and Rate*
Why Ratio and Rates?

★ First critical area of focus for Grade 6

(1) Connecting ratio and rate to whole number multiplication and division, and using concepts of ratio and rate to solve problems

★ First of the three model units in the learning progression from

★ Grade 6: Ratio and Rates
★ Grade 7: Proportions & Proportional Reasoning
★ Grade 8: Connecting Proportions, Lines, & Linear Equations
Why Ratio and Rates?

★ Standards Addressed in this unit* (Established Goals):

Content
• 6RP1: Understand the concept of ratio and use ratio language...
• 6RP2: Understand the concept of a unit rate...and use rate language
• 6RP3: Use ratio and rate reasoning to solve real world problems...

Practice
• SMP1: Make sense of problems...
• SMP2: Reason abstractly and quantitatively
• SMP3: Construct viable arguments...
• SMP4: Model with Mathematics
• SMP7: Look for and Make use of Structure

*MA Mathematics Framework page 55, Ratio and Rates Unit pages 3-5

Supporting ELA Standards
RST4 Determine the meaning ... in a technical context..., WHST2 Write informative/explanatory texts..., WHST4 Produce clear, coherent writing...
Essential Questions from this Unit, Discuss!

- How are ratios and rates used in everyday life?

- How would life be different without ratios and rates?
Take a Look Inside MCU UbD Format

- Stage 1- Desired Results
- Stage 2- Evidence
- Stage 3- Learning Plan
- Followed by detailed lesson plans
### UbD Backward Design

<table>
<thead>
<tr>
<th>Stage 1 Desired Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identify desired outcomes and results</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Stage 2 - Evidence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Determine what constitutes acceptable evidence of the targeted standards (assessment).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Stage 3 – Learning Plan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plan learning experiences and instructional strategies that prepare students to achieve success on the CEPA and meet the high level of the targeted standards.</td>
</tr>
</tbody>
</table>
Stage 1 Desired Results – what do you see?

### ESTABLISHED GOALS

**6.RP.1**
Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”

**6.RP.2**
Understand the concept of a unit rate \(\frac{a}{b}\) associated with a ratio \(a:b\) with \(b\neq 0\), and use ratio language in the context of a ratio relationship. For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar; so there is \(\frac{3}{4}\) cup of flour for each

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### Stage 1 Desired Results

**Transfer**

Students will be able to independently use their learning to...

Apply mathematical knowledge to analyze and model mathematical relationships in the context of a situation in order to make decisions, draw conclusions, and solve problems.

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### Meaning

**UNDERSTANDINGS**

Students will understand that...

**U1**
A ratio or a rate expresses the relationship between two quantities. Ratio and rate language is used to describe a relationship between two quantities (including unit rates).

**U2**
A rate is a type of ratio that represents a measure, quantity, or frequency, typically one measured against a different type of measure.

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### ESSENTIAL QUESTIONS

**Q1**
When is it useful to be able to relate one quantity to another?

**Q2**
How are ratios and rates used in everyday life? How would life be different without ratios and rates?

*Note:*
# Stage 2 Evidence - What do you see?

<table>
<thead>
<tr>
<th>Evaluative Criteria</th>
<th>Assessment Evidence</th>
</tr>
</thead>
<tbody>
<tr>
<td>See CEPA rubric.</td>
<td></td>
</tr>
</tbody>
</table>

**Curriculum Embedded Performance Assessment**

**Title:** Pizza Champions  
**Overview**

For this assessment, students assume the role of school cafeteria chef to create a pizza recipe and plan ingredients to make pizza for 12, 60, and 240 students. Students compute unit prices for the ingredients, prepare a budget, and calculate the cost to feed varying numbers of people. Using ratio/rate language, each student writes a proposal to persuade the cafeteria manager to use the recipe.

**List of Large-Scale Tasks**

- CEPA Task I: Cost to Make One Pizza  
- CEPA Task II: Cost to Feed 12, 60, and 240 Students  
- CEPA Task III: Proposal to the Cafeteria Manager  

All materials for this CEPA can be found at the end of the unit.

*Rubric is on pg. 130 in the unit*  
*Full CEPA pg. 121 in the unit*
Stage 2 - Evidence

What do you see?

- Curriculum Embedded Performance Assessment (CEPA)
  - Designed with the end in mind
  - Requires students to *independently* apply and demonstrate their understanding through complex performance
  - Goal is for students to independently complete the CEPA(s)
- Other assessments, evaluative criteria
## Stage 3 – Learning Plan

**What do you see?**

### Summary of Key Learning Events and Instruction

Note: In Grade 6, the focus is on ratios, equivalent ratios, and rates. In Grade 7, students will study proportions and proportional reasoning. All sessions are 50 minutes each.

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Topic</th>
<th>Duration</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lesson 1</td>
<td>Introduction to Ratios</td>
<td>1 session</td>
<td>P. 8</td>
</tr>
<tr>
<td>Lesson 2</td>
<td>Writing Ratios</td>
<td>1 session</td>
<td>P. 13</td>
</tr>
<tr>
<td>Lesson 3</td>
<td>Equivalent Ratios Part 1</td>
<td>1 session</td>
<td>P. 21</td>
</tr>
<tr>
<td>Lesson 4</td>
<td>Equivalent Ratios Part 2</td>
<td>1 session</td>
<td>P. 28</td>
</tr>
<tr>
<td>Lesson 5</td>
<td>Solving Mathematical and Real-Life Problems with Ratios</td>
<td>2 sessions</td>
<td>P. 36</td>
</tr>
<tr>
<td>Lesson 6</td>
<td>Ratios: Review and Assessment</td>
<td>2 sessions</td>
<td>P. 45</td>
</tr>
<tr>
<td>Lesson 7</td>
<td>Understanding Rates and Unit Rates</td>
<td>1 session</td>
<td>P. 53</td>
</tr>
<tr>
<td>Lesson 8</td>
<td>Solving Problems with Unit Rates</td>
<td>1 session</td>
<td>P. 58</td>
</tr>
<tr>
<td>Lesson 9</td>
<td>Using Rates / Unit Rates to Make Good Consumer Decisions</td>
<td>1 session</td>
<td>P. 62</td>
</tr>
<tr>
<td>Lesson 10</td>
<td>Rates and Unit Rates Assessment</td>
<td>1 session</td>
<td>P. 66</td>
</tr>
<tr>
<td>Lesson 11</td>
<td>What is a Percent?</td>
<td>1 session</td>
<td>P. 70</td>
</tr>
<tr>
<td>Lesson 12</td>
<td>Percents and Tape Diagrams</td>
<td>1 session</td>
<td>P. 86</td>
</tr>
<tr>
<td>Lesson 13</td>
<td>Solving Percent Problems: Missing Part, Missing Percent</td>
<td>1 session</td>
<td>P. 91</td>
</tr>
<tr>
<td>Lesson 14</td>
<td>Solving Percent Problems: Missing Whole</td>
<td>2 sessions</td>
<td>P. 95</td>
</tr>
<tr>
<td></td>
<td>Curriculum-Embedded Performance Task</td>
<td>3-5 sessions</td>
<td>P. 106</td>
</tr>
</tbody>
</table>
Stage 3 - Learning Plan

*What do you see?*

- A “roadmap” of the learning experiences throughout the unit as defined in the targeted standards
- Followed by detailed lesson plans
Ratio and Rates Unit
Lesson Components

Turn to Lesson 1, page 10.

- Essential Questions and standards
- Assumptions of what students need to know coming into the unit
- Outcomes
- Instructional resources/tools
- Anticipated student Preconceptions/Misconceptions
- Assessments
- Lesson sequence and description with Teacher Notes and Technology resources
- Closure
- Teacher reflections
2. Here is a grid with different kinds of boxes.

A. Using the colon form (:) write the ratio of 🌟 to ↑ ____________
B. Using the fraction form, write the ratio of 🙂 to the total number of boxes in the grid. _____

C. Write the ratio of ↑ to the total number of boxes in the grid. ______
D. Write the ratio of empty boxes to the total number of boxes in the grid. _____
E. Write the ratio of 🌟 to 🙂 ______
F. Which two kinds of boxes have a ratio of 2:3? Justify your answer.
Ratio and Rates

★ Standards Addressed in this unit* (Established Goals):

Content
• 6RP1: Understand the concept of ratio and use ratio language...
• 6RP2: Understand the concept of a unit rate...and use rate language
• 6RP3: Use ratio and rate reasoning to solve real world problems...

Practice
• SMP1: Make sense of problems...
• SMP2: Reason abstractly and quantitatively
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• SMP4: Model with Mathematics
• SMP7: Look for and Make use of Structure

Supporting ELA Standards
RST4 Determine the meaning ... in a technical context..., WHST2 Write informative/explanatory texts..., WHST4 Produce clear, coherent writing...

*MA Mathematics Framework page 55, Ratio and Rates Unit pages 3-5
As you work on the problem, consider...

⭐ What standards, goals or math concepts are students addressing in the task? (refer to next slide)

⭐ Why is this task a good task for students?
Exploring the MCU SCAPE!
Exploring the *MCUScape* (see handout)

Use this guide to identify SPECIFIC pages and references in the Grade 6 Ratio and Rates unit where can you find:

<table>
<thead>
<tr>
<th>Example of rigor</th>
<th>Task that addresses student discourse.SMP3*</th>
<th>The CEPA rubric</th>
<th>Two examples of visual models</th>
<th>An example of real-world ratio (application)</th>
</tr>
</thead>
<tbody>
<tr>
<td>An example of suggested extended learning activity</td>
<td>Teacher background content/notes with definitions for ratio and unit rate</td>
<td>Reference to the use of the double number line</td>
<td>An example of possible formative assessment</td>
<td>A task in a lesson that addresses SMP4*</td>
</tr>
<tr>
<td>Reference to the use of a tape diagram</td>
<td>An example of assumed prerequisite student knowledge needed for a lesson</td>
<td>Guidance for possible student misconception</td>
<td>An example of possible formative assessment</td>
<td>An example of suggestions for differentiation</td>
</tr>
</tbody>
</table>
Reflection

Think - Pair - Share Activity

★ What is new about how the mathematics is taught in this unit?

★ Do you think this unit will help students reach the intended goals for the unit? Why or why not?

★ What is new about the 6th grade math content taught in this unit?

★ What supports might be needed to teach this unit?
Next Steps
Implementation and/or Unit Development
Implementation

★ If presented with this unit, could the teachers in your district/school pick it up and teach it? Justify your response.

★ What professional development would they need to teach one of these model units?
District Unit Development

★ What support would your teachers need to develop such units?
★ How would they ensure quality units are developed?
Questions?