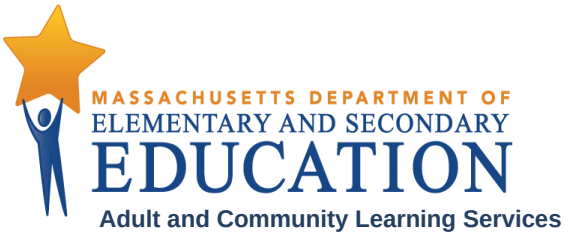

Mathematics Proficiency Guide for Teachers of Adult Basic Education

A Companion to the
Massachusetts Professional Standards
for
Teachers of Adult Basic Education

Revised August 2022

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It was originally developed by the SABES Mathematics and Adult Numeracy Curriculum and Instruction Professional Development Center with Donna Curry, Barbara Fox, Connie Rivera, and Sally Waldron taking on the huge task of figuring out how to make the *MA Professional Standards for Teachers of Adult Basic Education* (MA Department of Elementary and Secondary Education, MA ESE, 2017, revised 2021) come alive in math classes. A very special thank you to Jane Schwerdtfeger for bravely joining our math team to ensure that SABES and ACLS provide a consistent vision for adult education numeracy instruction.

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Thank you to all who helped bring this document to life!

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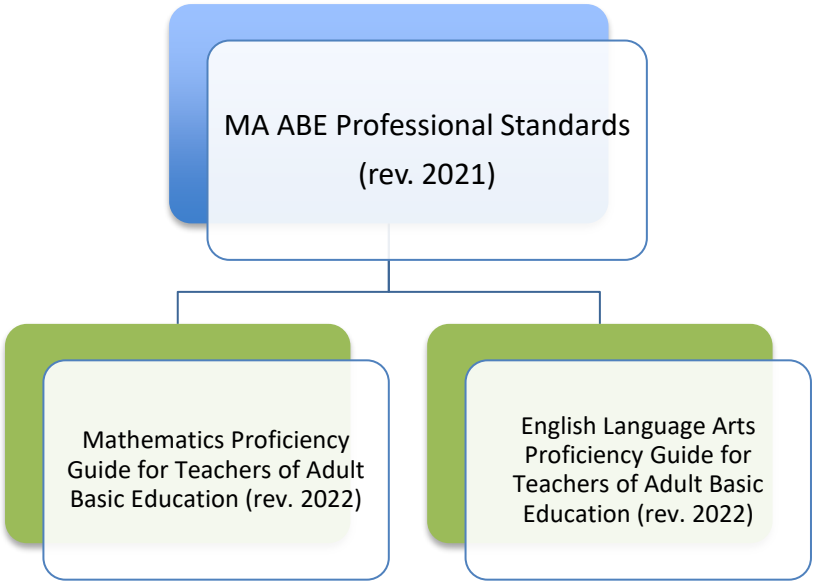
Introduction to the Mathematics Proficiency Guide

Effective teachers and leaders matter. No other program-based factor has as great an influence on student achievement as an effective teacher[1]. Likewise, effective educational leaders foster the conditions that enable powerful teaching and learning to occur. Ensuring that every student is taught by effective teachers and attends an adult education program led by an effective program director is key to preparing all students for success.

This document, the *Mathematics Proficiency Guide for Teachers of Adult Basic Education*, hereafter referred to as the Math Proficiency Guide, is one of several companion pieces to the MA Professional Standards for Teachers of Adult Basic Education (2017, revised 2021), hereafter referred to as the ABE Professional Standards. Two proficiency guides have been created, one for English Language Arts (ELA) and one for mathematics (see Figure 1). This one serves as a resource for adult numeracy teachers and their program directors.

Teachers and directors arrive in adult education programs from a variety of entry points, which ultimately enriches the field and the learning experiences of their students. The Math Proficiency Guide serves to focus this diverse set of educators on a common vision for math by specifying what teachers should know and be able to do related to adult math instruction.

Figure 1. Relationship of the MA ABE Professional Standards and the Proficiency Guides



[1] Hightower, A.M., Delgado, R.C., and Lloyd, S.C. (2011). *Improving student learning by supporting quality teaching*. Editorial Projects in Education, Inc. Bethesda, MD 20814, p. 2; McCaffrey, J. R., Lockwood, D. F., Koretz, D. M., & Hamilton, L. S. (2003). *Evaluating value added models for teacher accountability* [Monograph]. Santa Monica, CA: RAND Corporation; Rowan, B., Correnti, R., & Miller, R. J. (2002). "What large-scale survey research tells us about teacher effects on student achievement: Insights from the Prospects study of elementary schools." *Teachers College Record*, 104, 1525-1567.

The document is designed to:

- guide the practice of new, developing, and advanced teachers who teach math to adult learners
- foster communities of practice among teachers and directors across the state by promoting a shared understanding of effective adult numeracy practice, and
- advancing the adult education field in Massachusetts.

Why the Revised Math Proficiency Guide?

The original version of this document was developed in 2017 and updated in 2019. This 2022 version aligns with the revised ABE Professional Standards (2021) that more transparently recognized English learners as a major population served in adult education contexts. Additionally, the global COVID-19 pandemic also propelled the adult education field into online and blended forms of delivery, highlighting the need for both instructors and learners to learn and apply new technologies. As a result of these new priorities and the implementation of the Teaching Skills That Matter (TSTM) framework in MA adult education, concepts related to second language acquisition, culturally responsive teaching, digital literacy, and remote instruction are emphasized in this 2022 version of the document.

The authors drew heavily on the following resources in crafting this document:

- Massachusetts Professional Standards for Teachers of Adult Basic Education and Indicators of Proficiency Rubric (MA DESE, rev. 2021)
- Massachusetts Professional Standards for Teachers of Adult English Speakers of Other Languages (MA DESE, rev. 2021)
- ELA/Literacy professional development materials created by the federal College and Career Readiness Standards-in-Action project (Standards Work, Inc. for US Department of Education, Office for Career, Technical, and Adult Education [OCTAE], 2016–2021)
- Teaching Skills That Matter (TSTM) initiative from the American Institute for Research (AIR) and OCTAE, promoting contextualized approaches to teaching essential skills and knowledge in adult education

Relationship of the Mathematics Proficiency Guide and the Educator Growth and Effectiveness (EGE) Model

This document is best used in conjunction with the Educator Growth and Effectiveness (EGE) Model[2]but is applicable to all teachers of adult education mathematics. The EGE Model is based on the MA ABE Professional Standards and was developed to support teachers and directors in reflecting upon and taking an active role in improving instructional practices. It is grounded in three key questions:

[2] This document is adapted from the K-12 one titled, *Massachusetts System of Educator Effectiveness,* found at http://www.doe.mass.edu/edeval/.

1. *Are students learning?*
2. *What is the teacher doing that contributes to and supports that learning?*
3. *What else might the teacher do to enhance student learning?*

The EGE Model guides teachers through a continuous learning cycle and provides a process that can be adopted or adapted according to local program and staff needs and resources (see Figure 2).

Figure 2. Six Step EGE Continuous Learning Cycle

The EGE Cycle begins with a planning step that invites programs to learn about and tailor the EGE process to function effectively within the context of their own programs (Step 1). When the logistics are arranged and an EGE team leader ensures that supports are in place, the teacher and coach begin working together. With the guidance of an experienced coach, the teacher reviews the MA ABE Professional Standards and the relevant proficiency guide and reflects on their teaching and their professional learning goals (Step 2). Based on their reflection, teachers draft a professional learning plan that will anchor their professional development throughout the EGE Cycle (Step 3). With the support of the coach and EGE team leader, teachers seek out professional learning opportunities, implement new approaches, and reflect on the impact of their changed practice (Step 4). They meet with the coach periodically to assess their progress and receive support and encouragement (Step 5). At the end of the cycle, the teacher and coach review the cumulative evidence of applied learning, assess progress toward the professional learning goals, and determine next steps (Step 6).

Within the EGE Model is a rubric based on the MA ABE Professional Standards that describes effective teaching practices. Because directors and supervisors typically work with teachers across multiple content areas, the *Indicators of Proficiency Rubric* (MA ESE, 2017, revised 2021) is an optional reference tool that targets effective adult education teaching practices at the "macro" level with descriptions of effective teaching practices that apply to mathematics and English Language Arts content areas. The Math Proficiency Guide serves as a bridge from these macro-level descriptions to adult numeracy instruction, providing a clear road map for what these practices look like for math teachers.

Organization of the Math Proficiency Guide

The structure of the Math Proficiency Guide builds on the organization of the MA ABE Professional Standards, making explicit the relationship between the two (see Figure 3).

Figure 3. At-A-Glance: ABE Professional Standards

Professional Knowledge Domain (K)	*Instructional Practice Domain (P)*	*Continuous Improvement Domain (C)*
Standard K1. Content, Theory, and Research *Indicators* K1.1 Adult Basic Education (ABE) K1.2 English Language Acquisition* K1.3 Adult Teaching and Learning	**Standard P1. Design and Instruction** *Indicators* P1.1 Standards-based Units P1.2 Well-structured Lessons P1.3 Student Engagement P1.4 Meeting Diverse Needs	**Standard C1. Growth Mindset** *Indicators* C1.1 High Expectations C1.2 Student Ownership C1.3 Lifelong Learning
Standard K2. Standards *Indicators* K2.1 ABE Professional Standards K2.2 College and Career Readiness Standards for Adult Education	**Standard P2. Assessment** *Indicators* P2.1 Assessment Methods P2.2 Modifying Instruction P2.3 Student Progress	**Standard C2. Reflective Practice** *Indicators* C2.1 Self-assessment C2.2 Goal Setting C2.3 Professional Development

****Note:** Indicator K1.2 applies to ABE teachers who have English learners in their ABE classes.*

Just as in the MA ABE Professional Standards, the Math Proficiency Guide also uses these components:

- **Domain**: Domains are the overarching categories of professional practices relevant to effective teaching in adult education contexts. There are three domains for teachers of adult education: *Professional Knowledge, Instructional Practice,* and *Continuous Improvement.*
- **Standard:** Standards are broad statements about the knowledge and behaviors of effective adult numeracy practitioners. Each domain has two standards.
- **Indicator:** Indicators elaborate on specific aspects of a standard and serve as checkpoints to measure progress toward meeting that standard.
    - Below each indicator are examples of what effective math teachers know and what they do. There is also a third column to help teachers *focus* on areas they might want to improve in their practice. This can be useful when setting goals and developing a professional learning plan.

The following sections have been used in the Math Proficiency Guide to broaden understanding of effective math teaching:

- **Supporting Explanation for Each Standard:** A brief narrative introduces each standard, grounding the expectation in research and theory. The explanation provides an overview of why the standard and its indicators are relevant to adult numeracy instruction and points practitioners to areas to explore in more depth.

- **What Effective Math Teachers Know/Do:** These items provide concrete examples of the knowledge and skills needed by math teachers for each indicator. They are not meant to be exhaustive in scope.
- **Vignettes**: These short descriptions bring to life the teaching and learning of math in adult education contexts.
- **Research/Resources for Each Domain:** At the end of the section for each domain is a list of resources that practitioners might find helpful in furthering their own professional development in the specific area of adult numeracy.

About Repeated Ideas

You will note as you progress through the Math Proficiency Guide that some ideas are repeated across various items. For instance, *differentiation* is mentioned in several places. References to *open-ended, authentic problems,* and *formative assessment* are also found multiple times. This redundancy is due to:

1. Repetition of certain concepts was built into the MA ABE Professional Standards, upon which the proficiency guide is based.
2. Key concepts related to adult numeracy instruction warrant repetition and reinforcement, so they become embedded in teaching.

Suggestions for Using This Document

Adult education teachers will find several ways this document can contribute to their teaching. Use the Math Proficiency Guide and its resources to:

- See how the MA ABE Professional Standards apply in the adult numeracy classroom
- Reflect on what effective adult numeracy teachers know and do. Compare these findings to your own practice and skills as a self-assessment. Use what you learn to shape professional development plans and guide collaboration with other practitioners
- Explore targeted topics in more depth
- Improve and accelerate students' outcomes

For directors and evaluators, use the Math Proficiency Guide and its resources to:

- Aid in hiring, supervising, and evaluating staff
- Facilitate collaboration to enhance adult numeracy instruction within programs, targeting professional development activities that teachers might engage in as teams to enrich their experience and to build leadership.

Where to Find Support

Use of the Math Proficiency Guide is intended to be supported by professional development and training. Teachers, directors, supervisors, and evaluators should pursue ongoing professional learning to stay current on new approaches, policies, and materials and to maintain collaborative networks statewide.

The SABES Mathematics and Adult Numeracy Curriculum and Instruction PD Center, hereafter referred to as the SABES Math C&I Center, is the state's go-to provider of high-quality professional development and resources aligned with the *Math Proficiency Guide*. Offerings address standards-focused numeracy instruction, content-specific workshops, and courses designed to help teachers build their own content knowledge as they learn new strategies for teaching more conceptually; a series of workshops on visual tools; and a series on formative assessment strategies. These may be accessed in online, face-to-face, and blended formats.

- For descriptions and currently scheduled offerings, visit the SABES Math C&I PD Center's webpage
- For more information about the SABES Math C&I PD Center, contact adultnumeracy@terc.edu

The SABES Program Support Professional Development Center provides support, guidance, and professional development to program directors and EGE team leaders to assist them in adopting and adapting the EGE model according to specific needs and available resources. Additional support related to the Math Proficiency Guide is available through the Massachusetts Department of Elementary and Secondary Education's Office of Adult and Community Learning Services (ACLS). All resources meet the states' standards for high quality professional development and incorporate current research and evidence-based instruction.

PROFESSIONAL KNOWLEDGE DOMAIN (K)

Effective teachers draw on a body of professional knowledge, research, and standards to respond to the needs of their students within their educational contexts. Teachers have considerable knowledge of what they teach (i.e., mathematics, ELA, ESOL). They know and understand the content and underlying concepts relevant to what they teach. They understand what constitutes effective, developmentally appropriate teaching strategies and use this knowledge to make the content meaningful to students.

Effective teachers know their students well, including their diverse linguistic, cultural, and educational backgrounds. They know how the experiences that adult learners bring to the classroom affect their continued learning. They know how to structure their lessons to meet the social and intellectual development and characteristics of adult learners so they can succeed academically.

STANDARD K1: Content, Theory and Research
Knows the subject matter well, understands how adults learn, and draws upon relevant theories and research in adult education. Applies this knowledge to the design of rigorous learning experiences that enable students to acquire increasingly complex knowledge and skills.

Supporting Explanation for Standard K1

Adult Basic Education

Unquestionably, content knowledge of mathematics as a subject area is crucial to curriculum and planning. This is true for ABE teachers as well as ESOL teachers who can also incorporate high quality math instruction when provided with appropriate professional development, math instructional materials, and support. However, math content knowledge is only one component of an effective teacher's proficiencies.

Mathematical knowledge, "the mathematical knowledge that teachers need to carry out their work as teachers of mathematics," (Ball, p. 395) is more than just knowledge of the content. Clearly, understanding of mathematics content is a cornerstone for teaching mathematics. But *merely knowing the material* is not sufficient for *teaching mathematics.* Effective teaching requires additionally that teachers recognize not only how to do mathematics themselves, but also know the content well enough to improve student learning.

They need to know how to interpret student errors, discern student understanding, ask effective questions, help students make connections to prior knowledge, bring meaning to algorithms, modify the difficulty of a math task, and provide appropriate scaffolding of math, particularly for English learners (ELs) who may need greater support with vocabulary, reading comprehension, and understanding of mathematical symbols that could differ from those used in their countries.

They need to recognize the mathematics understanding in student work and explanations and know how to move students towards deeper knowledge. Acquiring this knowledge comes from studying the mathematics to be taught, anticipating student misunderstandings, and identifying prior knowledge embedded in the tasks. It comes from listening to how students understand mathematics, learning to ask probing questions, and being flexible in responding.

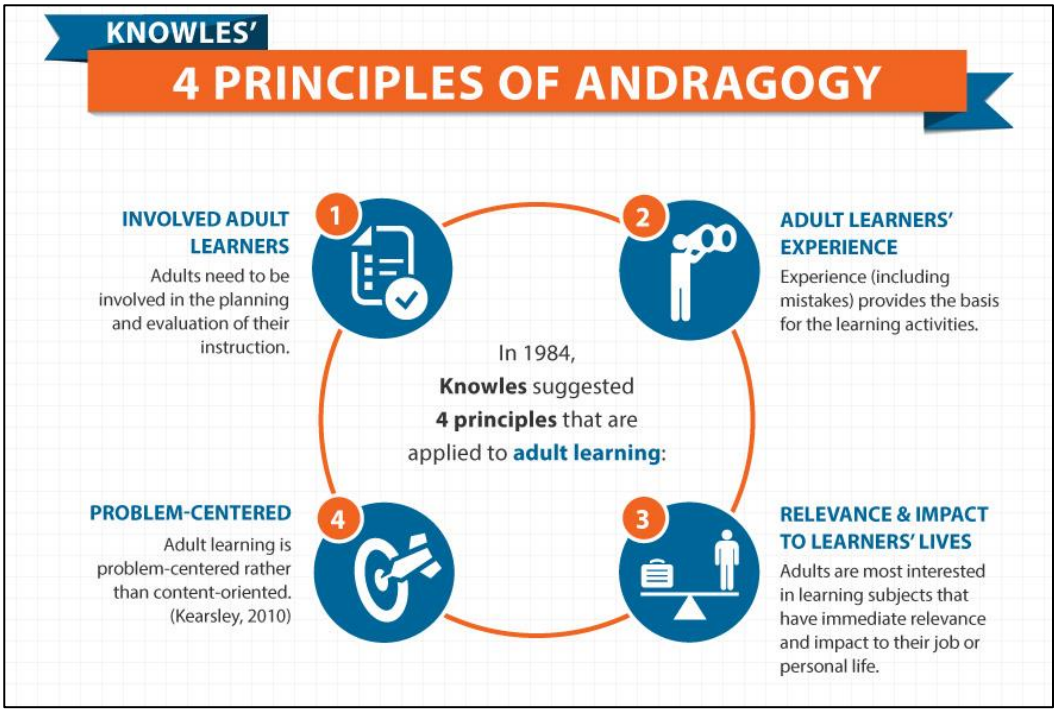
Understanding the concepts behind the math we teach is crucial. The skills we need for teaching math are different than knowing procedures or being able to solve a problem for ourselves. We must know the math well enough to be flexible. This knowledge does not necessarily come from college courses or a subject-area teaching certificate, but it does come from spending time studying the subject and developing the complex skills it takes for successful instruction.

Another essential element of curriculum planning is an understanding of the adult learner. The practice of andragogy, the art and science of helping adults learn, lead us to design a climate of respect in which adult students participate by:

- Determining their learning needs
- Planning their learning experiences
- Evaluating their learning.

According to Malcolm Knowles (1984), andragogy differs from pedagogy. Adults have workplace and life experience to draw on and expect to be applying their knowledge to a problem rather than learning a subject for future application. Group work and making connections across relevant topics is necessary with adult students. Adult learners need to play an active role in guiding their learning, from diagnosing needs, to planning their learning experiences, to evaluating those experiences.

Figure 4. Knowles' Principles of Andragogy



http://elearninginfographics.com/adult-learning-theory-andragogy-infographic

Another key to facilitating adult learning includes being mindful of adults' desire to know "why" and a desire for immediate application of what they are learning. Adults have goals, from passing a high school equivalency test, to getting into college, to training and successfully completing a course of study to obtain meaningful and family-sustaining employment.

Support for English Learners

The adult education system in MA has both ABE/ASE classes and ESOL (English for Speakers of Other Languages) classes. While ESOL classes focus exclusively on teaching those who need to learn the English language, many ABE classes serve both students who speak English as a first language and students whose English is their second or third language. The latter are students who either graduated from ESOL classes and are moving into ABE content classes or students whose level of English is too high for ESOL classes, but who still need support in building knowledge and skills especially in the academic content areas.

Because this population of English Learners (ELs) comprise such a significant portion of ABE classes, math teachers need to provide additional instructional scaffolds to support the math knowledge and skills of these students.

Considerable overlap exists between good teaching as it applies to speakers of English as a first language and good teaching of ELs. However, effective math teachers think specifically about why certain approaches may or may not be useful for ELs, making instructional moves as needed to address their unique challenges.

Some examples of instructional moves include[3]:

1. Scaffolding tasks and language so that students can make their own meaning
2. Providing opportunities and supports for students to clearly describe their mathematical thinking to others, orally, visually, and in writing
3. Providing opportunities and supports for constructive mathematical conversations
4. Strengthening the" meta-" connections and distinctions between mathematical ideas, reasoning, and language.

Standard K1.2 chart following this section provides detailed sample math applications to illustrate these four instructional moves.

[3] Principles for the Design of Mathematics Curricula: Promoting Language and Content Development, Understanding Language, Stanford Center for Assessment, Learning and Equity, February 28, 2017

INDICATOR	**K1.1. Adult Basic Education** Demonstrates knowledge of current research and a comprehensive understanding of the underling concepts, procedural knowledge, and contextualized application of the subject matter by engaging students in evidence-based instruction that enables them to acquire increasingly complex knowledge and skills.

Sample Math Applications

What Effective Math Teachers KNOW	What Effective Math Teachers DO	✓ Focus
a. Knowing how to do math is not the same as knowing how to teach math.	• Keep abreast of the latest research on how students learn math • Learn new strategies for teaching so all students have a chance to learn deep math content	
b. Students should not be asked to "suspend reality" and accept unrealistic contexts.	• Use context when it is realistic and powerful • Modify word problems considering why someone would want to ask this information • Ask open-ended questions with multiple correct answers • Ask questions that involve decision-making	
c. Going over a topic does not mean that students have learned it.	• Replace 'showing students how math works' with 'students doing the math to learn how math works' • Strive for deep understanding in a variety of contexts rather than shallow familiarity with many different concepts	
d. Overarching problem-solving skills, not discrete procedures and specific 'tricks,' are what must be taught.	• Explore math concepts, allowing students to make the discoveries • Make connections using contextualized problems and by connecting an understanding of the concepts to procedures	

Vignette:

Aisha was beginning a geometry unit. She assessed student knowledge with a brief diagnostic quiz. She noted that, though students knew formulas for area and perimeter and could solve simple computations (i.e., Find the perimeter of a rectangle with a width of 4m and a length of 8m), they were unable to make sense of and apply this information to solve more complex problems such as:

> *Rashid wants to enclose a rectangular section of his backyard with a fence to protect a vegetable garden. He has 100 meters of fencing. What are the dimensions of the enclosure that will give him the largest growing area for vegetables?*

The problem requires a clear understanding of perimeter (outside dimensions) and area (surface covered) as well as an understanding of how the two relate to each other. What happens to the area when you change the outside dimensions?

Aisha realized that the students had little understanding of the relationship of perimeter and area and formulated a lesson to address this issue. She wanted students to explore the ideas and draw conclusions about what they noticed.

Aisha began the lesson with a short introductory activity to activate the prior knowledge students had demonstrated regarding perimeter and area. This was designed to build confidence since she knew they had recalled formulas needed and were able to solve simple problems. Using the above problem as the context, she asked students to investigate what happens to the area when you use the 100 meters of fencing in different rectangular configurations (i.e., A rectangle 10 x 40 has an area of 400 square meters while one measuring 48 x 2 has only 96 square meters.). She shared centimeter paper with students so they could picture their work. She asked them to record the dimensions of the various rectangles and the area of each "garden."

As Aisha looked at students' work, she noticed some students' organization and choice of numbers showed they recognized that changing the dimensions changed the area and that it happened in a predictable way.

Class discussion showed a growing understanding of what perimeter and area were, how they related to each other, and how real problems could be solved with this understanding (e.g., Rashid decided on a garden enclosure of 25m x 25m, giving him a total area of 625 square meters.).

During additional discussion, Aisha could see that students recognized that keeping the perimeter the same (100 meters) but changing the side dimensions resulted in different areas. They generalized that the square was the greatest area. (Aisha noted that she needed to repeat this problem with numbers that did not yield squares but showed the same idea: the closer the dimensions are to each other, the larger the area will be.) Aisha was pleased with the lesson since, though some students struggled with generalizing, they were grappling with complex ideas and increasing their conceptual understanding of perimeter and area measurements.

INDICATOR	**K1.2 English Language Acquisition** (Note: This indicator applies to ABE teachers who have English learners in their ABE class.) Demonstrates knowledge of current research and a comprehensive understanding of language acquisition, communicative competence, and the structure and conventions of English by designing engaging learning experiences that advance English learners' linguistic and academic skills.

Sample Math Applications

What Effective Math Teachers KNOW	What Effective Math Teachers DO	✓ Focus
a. English language learners at any level can engage with math content when provided with temporary supports and scaffolds.	• Scaffold language and content to support English learners (e.g., teacher modeling, students making charts with information from a word problem, students using manipulatives or graphic organizers) • Encourage and provide opportunities for ELs to produce language in response to math tasks, including with sentence frames, writing activities to explain reasoning and/or math strategies used, and digital tools to express math understanding • Ensure that conceptual understanding is the focus, not decontextualized computation • Provide ample opportunities to apply math in real-life situations • Encourage students to explain their reasoning, giving them further opportunity to use language and demonstrate understanding • Use challenging problems that require students to talk to one another	
b. Many countries have different procedures for doing mathematical computations, and most countries use the metric system.	• Encourage students to use strategies, procedures, and vocabulary they have previously learned • Use students' different strategies and vocabulary as opportunities to connect the reasoning behind each strategy • View students' different procedures, strategies, and vocabulary as resources for connecting concepts to procedures • Use visual models, real-world connections, and cultural differences in math notation to encourage English learners to draw on existing understanding and to establish correlations between new procedures, strategies, and vocabulary	

c. Modeling math concepts with manipulatives helps ELs reason about the concepts using their own words.	• Model operations with visuals, such as base ten blocks and American currency, to build understanding of place value and operations and make sense of memorized procedures and vocabulary like "carrying/borrowing" commonly found in math materials • Give time for students to explore, create, and problem-solve with visual tools, such as fraction strips, to connect math vocabulary, notation, and meaning. For example, the process of creating fraction strips and connecting benchmark fractions to percents helps students connect notation to meaning (e.g., the denominator tells students how many pieces are in one whole)	
d. Pronunciation of numbers can be challenging and is critical to ELs' comprehension and ability to communicate clearly with others.	• Offer strategies to help students distinguish between stressed and unstressed syllables during listening and pronunciation of numbers. Examples include: o Use objects (like blocks of two different sizes) to show where the stress is during pronunciation of numbers. The taller block can represent the stressed syllable o One student says numbers and knocks on a table while doing so, knocking softly when saying unstressed syllables and knocking hard for stressed syllables	

e. Some ELs need further instruction and scaffolding of math vocabulary.	• Pre-teach math words when necessary. For example, when students need the words to construct meaning before starting a task, when there is a cognate in their native language and students are familiar with content, or when students know content but not the words to describe it • Teach unfamiliar words while students are doing math (i.e., "just in time" learning, which allows students to connect meaning with words). For example, when students are engaged in math and the teacher sees the opportunity to introduce a relevant math word or when students need words to write about the math they are doing • Teach words after students have explored the math. For example, when students have learned new concepts and are ready to label these concepts with formal terms (e.g., students exploring perimeter by creating a "fence" made of string may use words like *around, all sides, distance around* to describe what is happening with perimeter before they learn the formal term) • Teach words after students have explored the math • Create a (digital) word wall to serve as a visual reference for all students and to share and consolidate vocabulary	
f. Well-timed language placement will help ELs use mathematical language precisely to read, discuss, and reason about math.	• Connect language to math concepts • Allow students to construct meaning • Allow more time for students to be engaged in math • Increase the cognitive demand by allowing students to explore concepts concretely	

Vignette A:

Chitra teaches math in a program that serves ABE students, many of whom are ELs. She has participated in PD on *How Do Adults Learn English? An Exploration of Second Language Acquisition Principles* from the SABES ESOL Curriculum and Instruction PD Center, so she understands the need to be sensitive to students' language development at the same time they are learning math concepts. She followed that PD up with the asynchronous *English Language Learners in the ABE Math Classroom* so she could learn some specific strategies to use with her students.

When Chitra decided to teach a unit on data, she knew she needed to provide multiple opportunities for students to share their understanding, compare their reasoning and strategies with peers, and problem-solve with others with targeted guidance (not answers) from her. This scaffolded approach supports sense-making for everyone, but it is especially helpful for English learners who benefit from having more than one way to access math concepts and terms.

With these supports in mind, Chitra presented a scenario where students had to look at the different performance ratings of three waiters. Students, in groups of three, discussed which waiter they felt was the best. Each had to explain her reasoning to the rest of the group, using visuals or manipulatives, illustrations, or other modes of presentation to aid the understanding of the group as needed. While there was clearly no one right answer, the purpose of the activity was to get students to justify their reasoning and to encourage students to critique others' explanations.

Once all groups had a chance to discuss their reasoning, Chitra used the annotation feature of Zoom to share that many of them had been talking around the idea of mean for a data set although no one used the term. She then introduced the term; students shared what they thought the word meant in their native language just to be sure that they understood the meaning.

Once Chitra was sure that everyone understood the term, she followed up with another activity where students had a chance to further practice finding the mean and using the more formal math language they experienced during the activity.

INDICATOR	**K1.3 Adult Teaching and Learning** Demonstrates knowledge of current research and a comprehensive understanding of andragogy and the learning processes of adults. Designs engaging learning experiences that honor the life experiences of adult learners.

Sample Math Applications		
What Effective Math Teachers KNOW	**What Effective Math Teachers DO**	✓ Focus
a. Adult learners most likely have not had math instruction designed to develop mathematical understanding.	• Focus on mental math strategies for calculations such as counting up to count back change or breaking down a number by place value • Provide opportunities for students to solve problems using two different methods • Begin with concrete experiences before turning to abstract procedures • Connect conceptual learning to algorithms to explain why they work	
b. Many adult learners have experienced failure in mathematics resulting in negative attitudes and anxiety about mathematics learning.	• Create safe learning environments where students can gain confidence in their ability to learn mathematics • Develop learning environments which value mistakes and misunderstandings as ways to deeper learning	
c. Adult learners need to see the usefulness of mathematics in their daily personal and professional lives.	• Present mathematical tasks and challenges in a meaningful context, such as buying a car, choosing the most affordable daycare/childcare option, deciding between two job offers, comparison shopping, etc. • Present mathematical tasks and challenges that can be applied to daily life, such as estimating travel time to/from work/school/daycare, deciding to pay daily for transportation or to buy a weekly or monthly pass, choosing the best times of day and which days to be on-duty as a rideshare driver	
d. Learners must have a concrete experience before moving to abstract concepts.	• Begin new topics with an activity that requires students to show or explain their understanding with concrete materials, pictures, or other representations (e.g., tables, diagrams, graphs) before moving to a symbolic representation • Use and expect a variety of representations of concepts so that all learners have a way to access and express the concept (e.g., visual, manipulatives, verbal explanation, real-world example)	

e. Adult learners have life experiences that have involved math content.	• Value the experiences that students bring to the classroom, even if those experiences involve strategies other than "school math" • Seek input from students about how they use math at home or on the job and build this information into the lessons	

Vignette:

Kendra is planning for a new unit. She knows she needs to include students in the planning process to make sure that their voices are heard. She initiates a discussion about where they use math in their daily lives. She captures their ideas where everyone can see them, and students review the list together.

Since many of the students mention their difficulty interpreting the graphs they see at work and in the media, Kendra focuses the next unit on graphic literacy. She involves students from the beginning by asking them to notice a data representation that is meaningful in their life and capture it in order share it with the class.

Kendra makes sure all the graphs are collected so she can share them with the students at the next class. As the students review the graphs and tell "stories", Kendra notes that many students are particularly interested in a graph from the community about the arsenic levels in the soil at a local playground where some of the students' children regularly play. There is an upcoming community meeting to discuss this issue. Kendra decides to use this scenario to have students learn more about why so much hazardous waste has been allowed to be dumped in their communities. She then has students create meaningful data representations. She asks them to work in groups to prepare a presentation for this meeting, which includes one or more graphic representations that will make an impact on the decision makers at the meeting.

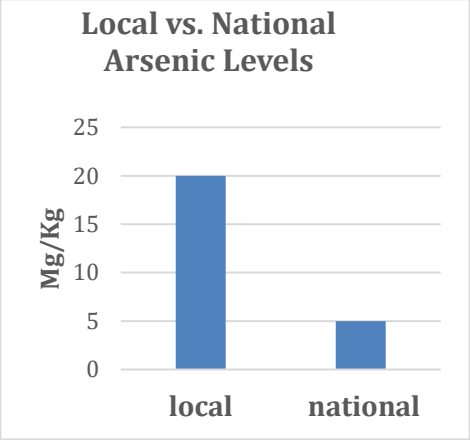
She tells students that the project will end with a simulation of the meeting where each group will present and get feedback on the effectiveness of the graphic representation.

What story does it tell?

Is all necessary information included?

Is the format appealing? Clear?

Does the graph focus on one or more main points of the presentation?



For those students who want to create a professional-looking presentation, she provides time for them to learn how to use Excel or Google Sheets where students can input their data and then choose an appropriate graph design.

Though this will primarily be a class assignment, she suggests that some might actually attend the community meeting and make their points, especially given some of the data as shown in one student example.

STANDARD K2: Standards
Draws upon a comprehensive knowledge of adult education state standards for teaching and learning. Applies this knowledge to the design and implementation of rigorous units with lessons and learning experiences that enable students to acquire the knowledge and skills needed for postsecondary education, training, and careers.

Supporting Explanation for Standard K2

The MA ABE Professional Standards outline what practitioners of any discipline need to know and be able to do to provide high quality instruction to adult learners. These standards are based on the MA EGE Model for grades K-12 teachers and have been piloted in a range of adult education programs. The Mathematics Proficiency Guide (this document) was developed as a resource to help practitioners apply the professional standards in their own teaching environment.

Another set of standards that are critical for the adult educator to know and understand is the College and Career Readiness Standards for Adult Education (CCRSAE), adopted by DESE/ACLS in 2013. Based on the 2010 *Common Core State Standards* (*CCSS*) developed for K-12, the *CCRSAE* provide the framework for curriculum development, instructional design, and lesson planning, used alongside each program's scope and sequence.

The CCRSAE for Mathematics emphasize three key shifts in mathematics instruction:

- **Focus** (focusing strongly where the standards focus)
- **Coherence** (designing learning around coherent progression from level to level)
- **Rigor** (pursuing conceptual understanding, procedural skill and fluency, and application – all with equal intensity).

As the document states, "At the heart of these shifts is a focus in mathematics instruction on delving deeply into the key processes and ideas upon which mathematical thinking relies. The shifts … therefore center on the knowledge and skills students must master to be adept at understanding and applying mathematical ideas." (CCRSAE, p. 44-45)

The CCRSAE for Mathematics also include **eight overarching standards for mathematical practices** (CCRSAE, p. 48-50):

MP.1 Make sense of problems and persevere in solving them
MP.2 Reason abstractly and quantitatively
MP.3 Construct viable arguments and critique the reasoning of others
MP.4 Model with mathematics
MP.5 Use appropriate tools strategically
MP.6 Attend to precision
MP.7 Look for and make use of structure
MP.8 Look for and express regularity in repeated reasoning

These mathematical practices describe the habits of mind that students need to develop and are based on the National Council of Teachers of Mathematics process standards (2000, p. 31) and the five strands of mathematical proficiency from the National Research Council (2001, pp. 116-117).

INDICATOR	**K2.1 MA Professional Standards for Teachers of Adult Education** Demonstrates comprehensive knowledge and understanding of the state standards for teachers of adult learners through engagement in an educator growth and effectiveness process. Uses the Proficiency Guides to support effective instructional practice and to implement learning experiences that facilitate learners' achievement.	
Sample Math Applications		
What Effective Math Teachers KNOW	**What Effective Math Teachers DO**	✓ Focus
a. Having a set of standards to aspire to and to put into practice helps with continuous improvement efforts.	• Use the *MA ABE Professional Standards* and the *Mathematics Proficiency Guide* to guide teaching and learning	
b. Vignettes from the *Mathematics Proficiency Guide* illustrate the professional standards for teachers in action.	• Refer to vignettes and other guidance to reflect on present andragogical strategies used in the classroom	

INDICATOR	**K2.2 College and Career Readiness Standards for Adult Education (CCRSAE)** Demonstrates comprehensive knowledge and understanding of the CCRSAE anchor and level-specific standards and the Standards for Mathematical Practice by aligning instruction to the appropriate standards at Levels A through D-E. Instruction regularly reflects the instructional shifts for English language arts/literacy or mathematics, depending on the content being taught.

Sample Math Applications

What Effective Math Teachers KNOW	What Effective Math Teachers DO	✓ Focus
a. The CCRSAE, including the instructional shifts, the Standards for Mathematical Practice and each program's scope and sequence provide a framework for planning and designing instruction and assessment.	• Become proficient with the CCRSAE for Math and all its components and the program's scope and sequence through participating in professional development, reviewing and using the documents, and meeting regularly with colleagues to discuss alignment	
b. The CCRSAE provides a developmental continuum that helps students use their conceptual understanding in various applications.	• Choose one topic (such as algebra or data or fractions) and look level by level to see what changes from each level • Review the Overview Grid for the CCRSAE for Math and reference as needed for a quick view of the progressions for the CCRSAE • Download and explore the various developmental continuums for topics such as fractions and number lines that the SABES Math C&I PD Center has developed, then use them to guide teaching	
c. There are three key CCRSAE instructional shifts necessary to help students become adept at understanding and applying mathematical ideas.	• Consider depth over breadth to teach in a more focused manner • Use the CCRSAE to design lessons and activities that offer a coherent progression from level to level • Focus on conceptual understanding, not just procedures • Integrate opportunities for students to apply their understanding and procedural fluency and skill	
d. The Mathematical Practices (MP) are the heart of the CCRSAE. Without them, students will not become proficient math problem-solvers.	• Incorporate specific MPs into each unit of instruction • Provide explicit opportunities for students to apply the MPs as they address challenging problems	

Vignette:

Tamika has been focused on fractions, decimals, and percents for much of the semester, and she and her students are both tired of what seems like the same problems over and over again. Finally, she reflects back on something she heard one of the SABES Math C&I PD Center team members mention in a PD session. She thinks it was something about starting to teach algebra early on.

So, she grabs a copy of the CCRSAE for Math and begins looking at Level C. She sees a subheading: *Apply and extend previous understandings of arithmetic to algebraic expressions*. She thinks, "The verb *extends* suggests that it is introduced at a previous level. I never thought about teaching anything about algebra at an earlier level, especially if students are still struggling with topics like fractions and percents."

She backs up to Level B and discovers this heading: *Understand properties of multiplication and the relationship between multiplication and division.* Again, she ponders, "Properties? Aren't those taught in higher math classes, like in algebra? Wow! Let me back up even more."

So, she does. In Level A she finds this heading: *Use place value understanding and the properties of operations to add and subtract.* As she continues to explore, she finds this standard: *Understand the meaning of the equal sign and determine if equations involving addition and subtraction are true or false.* (1.OA.7) and this one: *Determine the unknown whole number in an addition or subtraction equation relating three whole numbers* (1.OA.8). She realizes that she needs to step back to the earliest level in order to teach big ideas like equality so that students can handle higher level ideas such as expressions. It seems that she should be introducing some basic properties even with whole numbers, rather than introducing the properties when students start to work with variables.

She is now curious about how the standards are organized so very differently than the old math workbook that she has been using. She notices that many of the standards use verbs such as 'build' and 'apply and extend'. For example, she finds a heading titled: *Build fractions from unit fractions by applying and extending previous understanding of operations on whole numbers.* "Wow!" she thinks. "Now I am beginning to see how the math is all integrated. I can teach algebra concepts even if my students don't know how to do fractions. And, I should be teaching fractions, not as its own topic, but by building on the properties that they need to learn for whole numbers."

Research/Resources for the Professional Knowledge Domain

Ball, D., Thames, M., & Phelps, G. (2008). Content Knowledge for Teaching: What Makes It Special? *Journal of Teacher Education (59*)5, 389-407.

Massachusetts Department of Elementary and Secondary Education, Adult and Community Learning Services Unit. (2019). *Massachusetts English Language Proficiency Standards for Adult Education*. Available at http://www.doe.mass.edu/acls/frameworks/elps.pdf.

National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics*. Reston, VA: Author.

National Research Council. (2001). *Adding It Up: Helping Children Learn Mathematics*. Washington, DC: National Academies Press.

U.S. Department of Education, Office of Vocational and Adult Education. (2013) *College and Career Readiness Standards for Adult Education*. Washington, DC. Available at https://lincs.ed.gov/publications/pdf/CCRStandardsAdultEd.pdf.

INSTRUCTIONAL PRACTICE DOMAIN (P)

Effective teachers operate effectively at all stages of the teaching and learning cycle, including planning learning experiences, delivering effective instruction, conducting formative assessments, providing feedback on student learning, and reflecting on and modifying practices as needed. Teachers have a repertoire of effective instructional strategies and use them to implement engaging, well-designed lessons with defined outcomes. They value diversity in the classroom so that all students are honored and respected, and all students feel a sense of belonging. They use technology to facilitate learning (e.g., providing extended practice, collaboration, differentiation) and expand their students' abilities with technology. Teachers design and implement engaging lessons based on well-defined learning objectives and use evidence-based instructional strategies that promote deep learning, problem solving, cultural identity, and the transfer of learning across authentic contexts.

Teachers regularly evaluate student progress to measure the effectiveness of their instruction and to ensure they are meeting the professional standards and the learning needs of their students. They interpret and use student data to assess progress, diagnose barriers to learning, and challenge students to improve their performance.

STANDARD P1: Design and Instruction
Uses knowledge of the adult education state standards to guide the design of academically rigorous instruction that makes knowledge and skills accessible to all students and facilitates mastery of adult learning standards. Incorporates a variety of differentiated instructional methods that engage adult learners in challenging but accessible tasks, support the language and academic development of English learners where applicable, the development of critical thinking, and accommodate diverse needs.

Supporting Explanation for Standard P1

Instructional design should be structured on the concept of backwards design – beginning with student understanding and transferrable skills in mind – based on *Understanding by Design*, the framework laid out by Grant Wiggins and Jay McTighe (2011). In Stage 1, the teacher considers the goals along with essential questions to guide learning, important understandings, and the knowledge and skills students will acquire. In Stage 2, she creates a plan for collecting evidence of the desired results such as a culminating project or open-ended problems where students demonstrate their learning.

Stage 3 is the learning plan—the activities, experiences, and lessons for students that lead them to the goal. Well-structured units and lessons draw on adults' experiences and support students in addressing misconceptions and developing mathematical reasoning in order to apply their knowledge to relevant tasks. Some teachers may limit their planning to a variety of engaging activities yet not consider whether they create coherent learning of the core content of mathematics. According to Wiggins and McTighe, "Such activities are like cotton candy—pleasant enough in the moment but lacking long-term substance." (p. 9).

When the teacher designs instruction, she needs to make choices that optimize the learning of all students. This is where Universal Design for Learning (UDL) comes in. UDL is a framework for proactively designing instruction where all learners can meet the goal of the unit. With the activities of the unit in mind, the teacher considers what barriers in the activity as currently designed might interfere with a student's achievement of the goal. For each barrier, she plans options that provide additional means for

students to reach the goal. Options include providing multiple means to take in information (e.g., visually, with manipulatives, by animation, etc.) and multiple means to express understanding (e.g., written, oral expression, drawing).

In order to create well-structured units and lessons, DESE/ACLS and the SABES C&I PD Centers have developed templates to help guide programs so that the math content and adult learning theory are taken into account.

In addition, the SABES Math C&I PD Center, with support from ACLS, has developed the CALM (Curriculum for Adults Learning Math). CALM addresses the full range of adult learners from the most basic to advanced and it stresses conceptual understanding as a means of changing habits of mind and facilitating a growth mindset.

Adult learners come with a range of diverse backgrounds (varied cultural backgrounds, school history, life experiences, language proficiency). Effective instructors respect and leverage diversity to support learning. Students tend to make greater gains when learning in a safe, inclusive, and respectful learning space that promotes justice and equity for all students, whether it is an online or face-to-face learning environment.

To ensure equity for all, effective teachers are sensitive to and recognize that adult education students are culturally and linguistically diverse. To support this diversity, teachers tap into the students' funds of knowledge and value what these bring to the classroom. Students learn better when teachers focus on the important math by making mathematical and context connections and incorporating students' identities (Van de Walle, 2014, pp. 58-64).

Scaffolding instruction refers to instructional techniques designed to move students to greater understanding and the capacity to work independently so that they become self-regulating problem-solvers. The teacher supports students to work on the level just beyond what they could do on their own. As students develop more competence, the support is slowly removed until students can work without assistance (Vygotsky, 1978). Scaffolding helps students build on prior knowledge and internalize new concepts and information. It includes modeling and clearly defining the expectations of the activity to be demonstrated.

In math classrooms, scaffolding does not mean demonstrating procedures on the board for students to first replicate and then do on their own. Rather, scaffolding supports learners to access productive struggle. Scaffolding for access is "temporary support provided prior to task engagement that affords students access to productive struggle that otherwise might not be accessible" (Barlow et al., 2018, p. 204). Strategies for math scaffolding include eliciting prior knowledge, delaying the question, and introducing a simpler problem. All are designed to help students reason about the mathematics and problem context and develop conceptual understanding.

According to Van de Walle et al. (2014), the problem-based approach [4]to teaching math is the best way to address the range of students in a class. In a traditional, highly directed lesson, it is often assumed that all students will understand and use the same approach and the same ideas as determined by the teachers. Students not ready to understand the ideas presented by the teacher must focus their attention on following rules or directions without developing conceptual understanding.

[4] See also problem-based learning in the Teaching Skills That Matter framework

However, in a problem-based class, students are expected to approach problems in a variety of ways that make sense to them. Therefore, differentiation is already built into some degree (p. 43).

When differentiating instruction, teachers need to ask themselves questions about the challenge levels of the math tasks:

1. What makes a math task easy or hard, and for whom?
2. How can we best provide support for students who are struggling while challenging those who are more advanced?
3. How can teachers integrate effective differentiation despite limited planning time? (Little, Hauser, & Corbishley, 2009, p. 36)

An overarching strategy for differentiation is tiering, which "involves preparing multiple tasks or versions of tasks that respond to common objectives while providing variety in their levels of complexity and challenge, the learning styles they address, or the interests to which they appeal." (p. 36).

Tiering begins with choosing or developing a worthwhile learning task that will help students achieve meaningful objectives, followed by developing adjusted versions of the task, with the same or similar objectives. There are multiple ways to adjust tasks including increasing or decreasing the number of facets, changing the level of abstraction, and expanding problems to stretch students' understanding. (pp. 36-42). Along with tiered lessons as a strategy for differentiated instruction, other strategies include parallel tasks, open questions, and flexible grouping.

See Appendix for more ideas related to what Standard P1: Design and Instruction might look like in an adult education classroom.

INDICATOR	**P1.1. Standards-based Units** Designs and/or uses instructional units that align to the CCRSAE and the program's scope and sequence. Academically rigorous units define clear evidence of outcomes and include differentiated learning experiences that enable all students to learn the knowledge and skills defined in state standards.

Sample Math Applications

What Effective Math Teachers KNOW	What Effective Math Teachers DO	✓ Focus
a. Effective units are planned backward, beginning by considering the result of student understanding and transferable skills.	• Write clear goals for student learning • Create essential questions to guide deep thinking and transfer ideas • Identify the knowledge and skills students will acquire that reflect the CCRSAE for Math and are aligned with the program's scope and sequence • Determine what evidence of success looks like and how it will be assessed • Plan learning experiences that take into consideration the range of learners. • Plan to monitor progress	
b. Knowledge is built upon previous knowledge. Core mathematical concepts are connected in a coherent manner.	• Begin with a hook and move into a genuine problem or challenge • Consider the concepts that come before the lesson and anticipate the standards that will come afterwards; plan a path that will connect ideas and build coherence • Create a concrete experience before addressing abstract concepts • Draw connections between "the basics" and other math topics typically only taught once students have "mastered" the fundamentals. • Plan to differentiate instruction, as needed, offering students a choice between simpler and more challenging problems	
c. Students learn best when ideas are connected to other learning.	• Create learning activities and lessons that connect mathematical concepts (across domains and subjects) and build coherence	
d. Students are more willing to learn math when it is connected to their own lives and communities.	• Ask students often how they use math (or wish they could) in their lives • Build lessons and units on issues and concerns that are important to students	
e. A math unit includes more than computation and procedures	• Embed computation within the context of rigorous problem solving	

	• Use different models and representations to connect conceptual understanding to procedures.	
f. Lessons and units follow coherent progressions based on the CCRSAE for Math	• Refer to the CCRSAE for Math when developing lessons and units • Keep the CCRSAE Standards for Mathematical Practices in mind when designing lessons to ensure that students are becoming proficient math problem-solvers	
g. Learning experiences allow for exploration of essential questions.	• Serve as the facilitator and support students through exploration of the core concepts • Provide opportunity for genuine application of the core concepts • Make a choice of relevant resources available to students	
h. Assessment tasks must align to the standards and goals of the unit.	• Clarify expectations for assessment with students' support • Create performance tasks that require students to apply their knowledge and skills in varied contexts • Provide on-going and descriptive feedback	

Vignette:

During the first few class sessions, Hector spends time getting to know his students' interests and goals. He knows about their activities outside of class (job, family responsibilities, etc.). It is important to Hector that he connects the work of the units he teaches to the lives of his adult students.

Hector considers relevant topics in which to integrate the math for this particular class. Early diagnosis reveals that his students have minimal knowledge of the value of data analysis. Since more than half of his students are parents with young children, his primary goal is that his *students will be able to represent and interpret data in order to make an informed decision about childcare choices in the area.* They will demonstrate their learning by presenting their reasoning to the class, supported by a visual representation. Hector considers the standards for data (from the CCRSAE) which students will demonstrate:

> Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple "put-together, take-apart, and compare" problems using information presented in a bar graph. (2.MD.10)
>
> Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.* (3.MD.3)

In all units, he recognizes the need to reinforce basic computation skills and he knows adult students acquire a deeper understanding when they apply their knowledge to a real task. He again references the standards, and includes a standard from *Number and Operations: Base Ten* that is appropriate to the money-related decisions students will be making:

> Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. [Note from panel: Applications involving financial literacy should be used.] (5.NBT.7)

Hector considers the essential questions and chooses activities and tasks that will challenge students' thinking and transfer to situations in their lives. He frames the performance task where students need to consider the relative value of different day care centers based on factors such as cost, accessibility, transportation costs, hours of operation, and educational opportunities. To guide student work, he formulates a series of questions which require data collection, computation with decimals (money), and data analysis. He plans to have students work in groups to create a visual representation to support their suggestions and present their findings to the class.

Hector plans to conclude the activity with students discussing how this type of thinking connects to other aspects of their lives and how generating and interpreting data can assist them in making decisions. Throughout, he will assess their ability to use and explain strategies for operations with decimals to the hundredths, as well as to create graphs to represent their collected data and then use them to solve problems.

After he has laid out the unit's activities, Hector revisits his goal: *Students will represent and interpret data in order to make an informed decision about childcare choices in the area.* He considers the barriers within the activities that a student might experience in reaching the goal because he wants to plan for all students to succeed. He knows that one of his English language learners is reluctant to speak up during cooperative learning, so he decides to create specific roles to guide the group work. He also knows presenting to the whole class may worsen the anxiety already felt by math-anxious students, so he decides to offer the option for groups to record themselves and play their presentations for the class rather than give them live. He remembers his students need more opportunities to work with technology tools, so he creates a handout with instructions for creating a graph within a spreadsheet as an optional way for students to express their understanding.

Lastly, Hector recognizes the need to respond to the different levels of math knowledge and experience among his students. The cooperative nature of the activities will help, but he decides to offer groups the option to use just whole numbers (no decimals) or a more advanced data representation.

INDICATOR	**P1.2 Well-structured Lessons** Develops well-structured lessons as part of instructional units that include clear learning objectives, meaningful formative assessments, appropriate pace and sequence, relevant resources, and the use of technology. Lessons are designed to optimize learner interaction.

Sample Math Applications

What Effective Math Teachers KNOW	What Effective Math Teachers DO	✓ Focus
a. The backbone of a good lesson is clear goals with measurable objectives.	• Refer to the CCRSAE for Math and Standards for Mathematical Practice to aid goal development; focus strongly where the standards focus • Use the goals of the unit to drive the choice of activities and resources in a lesson • Use appropriate assessments to measure those goals	
b. Student goals, attitudes, and interests influence their learning of math.	• Design instruction with student involvement • Build a positive classroom culture consciously	
c. Students differ in their approaches to learning, strengths and needs, and pace of mastering new material. There are many paths to the same academic goal.	• Use a variety of hands-on and visual experiences to accompany any math unit • Choose technology-based activities that both strengthen digital literacy and support the development of conceptual understanding. Examples include: use of virtual manipulatives to model understanding, online graphing tools or spreadsheets to present information visually, or interactive/ collaborative pages and slides (e.g., Jamboard, Pear Deck, or Padlet) that allow students to provide written feedback to the teacher and peers • Provide a variety of approaches to new material so that students can work from their strengths • Structure lessons so that all students can learn, including students with learning disabilities	
d. The topic of a unit and lessons should be chosen from the big ideas, or core content, of mathematics.	• Consider coherence and set up a progression of ideas • Include activities that balance conceptual understanding, procedural fluency, and application	
e. Well-structured lessons include opportunities for students at different levels to work together to develop conceptual understanding.	• Structure some group activities that bring students at different levels together • Provide problem solving activities with multiple levels of complexity while developing shared conceptual understanding	

Vignette:

At a recent conference, Michelle worked with other participants in a number of engaging math activities. At a session on representing data, she worked with her group to make sense of the "mean" of a set of data. Snap cubes (a math manipulative consisting of small plastic cubes that link together) were used to represent the number of people in each person's family. The task required them to use the cubes to represent the mean of the data. The activity clarified *why* data values are first added and then divided by the number to find the mean. It was an eye-opening experience to see how these visual and hands on activities could be used with her adult learners.

Michelle returned home energized to use similar activities with her students. She eagerly prepared three activities, each intended to deepen the understanding of the measure of central tendency (mean, mode, median). She used the conference activity for the mean and similar activities for the mode and median. The class was engaged, and Michelle thought the class a huge success. At the end of the class when she asked students to complete some practice problems from the test prep workbook, she was discouraged to see that students were struggling and getting the answers incorrect.

She wondered what had gone wrong. Upon reflection, she realized that the engaging activities were present but that she had not thought through the objectives of the lesson and the pacing, nor had she given students a range of difficulty levels. This illustrated to Michelle the importance of planning all aspects of a lesson (objectives, activities, pacing, sequencing, assessing). Michelle also realized that she had expected the students to master the three measures of central tendency too quickly; she needed to provide them with further opportunities within the lessons to explore and understand the material conceptually.

INDICATOR	**P1.3 Student Engagement** Uses a variety of student-centered instructional methods. Provides opportunities for all students to communicate in meaningful ways, interact within authentic contexts, and develop critical thinking and problem-solving skills.

Sample Math Applications

What Effective Math Teachers KNOW	What Effective Math Teachers DO	✓ Focus
a. Mathematics does not equal calculation. Mathematics is logical reasoning, problem solving, the study of patterns, and a set of connected ideas that help us understand our world.	• Present students with a variety of logic puzzles • Use math examples from students' workplaces • Provide flexible classroom groupings of students in problem-based learning • Practice calculation through routine activities which allow students opportunities to investigate number properties and develop number sense • Provide meaningful practice while minimizing the use of repetitive worksheets, flash cards, and other memory-only activities	
b. Manipulatives and diagrams are important tools for students to use in problem solving.	• Help students become familiar with a variety of tools so they can make informed choices about what would be useful for solving a particular problem • Stress independence in problem solving and encourage students to use tools that are meaningful to them • Plan math activities which require the use of particular tools (e.g., modeling equivalent fractions on a virtual geoboard) • If students are learning remotely, help them learn how to use virtual manipulatives • Have tools readily available in the classroom for students to use as needed. If students are learning remotely, either provide them with tools such as graph paper and cardboard or virtual or encourage them to use available materials such as dry beans or pennies. • Post examples of diagrams and tables that would be helpful in problem solving	
c. Adult learners must have a concrete experience before moving to abstract concepts.	• Have students explore new ideas first using concrete models when possible • When students are struggling with a topic they have been exposed to in their past education, back up by providing concrete experiences in order to build conceptual understanding • Begin new topics with an activity that requires students to show or explain their understanding with concrete	

	materials, pictures, or other representations (tables, diagrams, graphs) before moving to a symbolic notation	
d. The selection of meaningful math problems and activities is the key to student engagement and learning.	• Choose problems involving significant math ideas • Select problem contexts meaningful to adult learners; draw from the work and cultural background of your students or from those in your community • Select problems that allow for various entry points, multiple ways to solve, and multiple solutions • Facilitate lesson discussion focusing on conceptual understanding so that math and learning are explicit • Invite students to bring in their own materials, ideas, and real-life situations and experiences that required math	

Vignette A:

Freda begins her class asking students about their experience in math class and confirms her suspicion that most have experienced direct teaching with minimal student engagement in learning. With this in mind, she resolves to structure classes with minimal direct instruction and maximum time involving students in exploration. This can be challenging with remote instruction, but Freda knows some great online tools to make this work.

She carefully chooses the initial activity to activate students' number sense. She presents a new Jamboard that she has previously setup and reviews how to use this digital tool before starting the task. She knows students often need to revisit directions, so she puts a summary of that information on the first page. She presents a sample task on the next page that allows her to demonstrate how students can respond to the task using the pen, text box, sticky note, shapes, and images available in Jamboard.

She gives time for students to try out the tools themselves and then asks, "What questions do you have?" She provides time for students to think and respond aloud before moving on. On the following page, she presents the activity and asks her students to write five equations whose answers are 27 and gives an example, 54/2 = 27. Students share equations on the page using whatever Jamboard tools they like. Freda then invites students to use sticky notes to make observations about the mathematical ideas that their various equations suggest. Based on the students' responses, she then asks students to elaborate on some of the big ideas that may have been named, such as:

- The relation between multiplication and addition
- Use of exponents
- Value of parentheses in clarifying the order of operations
- Inverse operations

On a new page, Freda has setup another task on a sticky note. This one challenges students to model their equations with small objects, square tiles, a drawing, or an app. She provides a link to online virtual manipulatives and demonstrates how to take a screenshot or snipping of a visual made online. She then reviews how to copy/paste or upload a new image to the Jamboard page. As before, Freda asks, "What questions do you have?" and gives appropriate wait time before having students start the activity. As she observes students' work, Freda takes advantage of their own example to push their thinking by restating

what she sees ("Your model shows me that you are multiplying 3 x 9.") and asking probing questions ("How can you show that you need to square 5 before adding 2?").

At the end of the class, Freda moves to the final page of the Jam where she has created an exit ticket that asks students to identify three things they learned and three questions they have. Students can use sticky notes or the text box to respond and can fill additional pages as needed. She uses this information to inform future classes. She reflects on the class and is even more convinced that students learn best when they are starting from what they know, discussing their ideas, and engaging in hands on activities.

Vignette B:

After her first math class, Tam wrote in her reflection notes that students appeared comfortable answering questions she posed but were quite uncomfortable talking with one another and sharing more than short one-word answers with the group. Recognizing the importance of developing a safe environment where students talk with one another, share ideas, and are willing to defend their thinking, she decides on a simple task that will necessitate interaction with one another and the larger group.

Tam gives students 20 shape cards with an assortment of various types of triangles, quadrilaterals, pentagons, and hexagons. She divides the class into groups of three and explains the game, "Guess My Rule." One participant will group a few shapes together that he/she has chosen to put together since they share a particular characteristic. (For example, one set contains all the shapes that have one or more right angles). The other group members talk to one another and guess what the "rule" is. Students take turns formulating the rule. Tam circulates the room and notes the language students are using; she occasionally poses a question for students to consider (e.g., "Is there another rule that would include the same shapes but is a different rule?").

Students play for 15-20 minutes and then gather as a class to share and review what they noticed. Tam calls on students to share some of the rules that were used. She lists them: shapes with parallel lines, shapes with more than three sides, shapes with both acute and obtuse angles, etc. She also asks them to think of the academic language used in the activity and share the meaning of the words.

Finally, she switches the conversation to emphasize the significance of working together. She asks, "What did you learn from working with your group?"

Using the ideas students presented in answer to this question, Tam concludes the class by setting norms with the group for working collaboratively in the classroom. She posts the following list which represents their ideas of what would make for a comfortable, safe classroom:

- Listen and learn from one another
- Watch your airtime; don't dominate the group
- Ask questions if you don't understand
- Disagree respectfully

Tam will post this list and add to it as the class term continues. She will also reference this when she sees students exemplifying the behavior (e.g., “Soledad, that’s a great question. Do you agree with Miguel’s answer? If not, how are you thinking about it?”).

INDICATOR	**P1.4 Meeting Diverse Needs** Uses an understanding of students' diverse needs to differentiate instruction according to learning preferences and abilities, needs, interests, prior education, cultural beliefs and values, first languages, and life experiences.

Sample Math Applications

What Effective Math Teachers KNOW	What Effective Math Teachers DO	✓ Focus

a. Students need to know why something works and have realistic contexts for the math they do in order to stay engaged.	• Consider relevant contexts when presenting math problems
	• Present problems which will allow application to students' daily lives. For example:
	  ○ Invite students to contribute examples of where they have used math in their work, family, or home
	  ○ Build from those experiences to demonstrate how often math is needed and used beyond the classroom	

b. Each class is a unique group of students with varying needs, strengths, and challenges.	• Plan initial class activities and discussions which allow the teacher to learn what students know and help them understand where they need to be and how they can be successful – initial activities may focus on the Standards for Mathematical Practice rather than specific content • Include open math tasks, such as *Number of the Day* or *Number Talks*, that invite participation by all students and provide opportunities for students to share the many ways to write math symbols and operations, like decimal vs. comma, division, or multiplication • Consider students' culture in designing lessons and activities • Invite English learners to share how they understand a math solution based on their educational experiences from their home country. Spend time as a class or in small groups discussing how that person's method compares to other ways the problem might be solved, seeking common math connections to demonstrate why math can be solved in more than one way.	

c. Teachers recognize the importance of knowing all students and the impact of class, culture, and language in mathematics learning.	• Spend time getting to know students and their cultural backgrounds through a variety of initial activities (interviews, surveys, discussions, ice breakers)
	• Draw on students' knowledge (interests, needs, strengths) of mathematical ideas when planning lessons, selecting mathematical tasks, and designing assessments
	• Provide multiple ways for students to share information, providing appropriate time for students to think and respond. For example:
	  ○ ELs may feel more comfortable thinking through their response in their first language before answering in English
	  ○ They also might benefit from being allowed to illustrate or demonstrate their understanding in support of the verbal or written response. This practice could be useful for all learners regardless of their English skills	

d. The Universal Design Principles, when applied consistently during the planning and delivery of lessons, allow more students the chance to engage in and master the work.	• Make materials accessible in a variety of formats, especially if instruction is remote and not all students have access to computer • Design lesson plans that allow for various approaches in presenting material (video, hands-on activities, text explanations, modeling, online resources such as Desmos classroom activities) • Design classroom activities for engagement by having students explore the content by sight, touch, movement, listening, and creating opportunities for understanding • Design a variety of assessment activities (e.g., project work, oral presentations, power points, teacher-developed quizzes) which allow choice so that students can best show what they know	
e. Teachers recognize the importance of a safe learning community.	• Collaborate with students to develop class norms which assure safety and respect	
f. Teachers know that persons from many cultures have contributed to the study of mathematics.	• Acquaint students with persons from various cultures who have contributed to the body of mathematical knowledge • Use a variety of instructional materials to emphasize the contributions of BIPOC, women, and others to the field of mathematics.	
g. Teachers recognize that math is a social justice issue and that "educational inequities are	• Critically examine materials for racial, class, gender, cultural, and language bias and to ensure cultural relevance	

historical and systemic" (Aguirre, 2016).	• Critically reflect on their own backgrounds, biases, cultures, and beliefs when developing new materials • Do self-reflection to try to eliminate implicit bias in instruction	

Vignette:

After the first class session, Charlotte reflects on the students in her beginner level ABE math class. There are 11 students, all of whom were placed in the class based on their MAPT-CCR for Math scores. Five of these students are English learners with solid social language. She recognizes the need for a strong language focus and the importance of planning scaffolded lessons with multiple opportunities for math conversation. She knows that two of her students had less than a 6th grade education before coming to the U.S. and struggled with the content of the secondary classrooms into which they were placed based on their ages. Neither student graduated from high school.

Charlotte wants to know how her students have previously experienced math instruction to be more sensitive to the need for possible additional supports. With one exception, Charlotte notes that the students' past experience in mathematics class have been very teacher centered. Students spoke of teacher explanation followed by practice with little opportunity for student discussion or collaboration. Charlotte knows that engaging students in meaningful math tasks will be challenging. She makes a note to find problems that will necessitate group work and will challenge students to use what they know to solve the problems. She reminds herself to keep lecture/explanation times at a minimum.

Charlotte reviews what she has learned from her own experience and from recent professional development sessions about meeting the needs of diverse populations. Primarily she reminds herself about the importance of designing instruction to allow access for all students and reviews practices she has used and knows about. She writes down a list before she begins her planning. Though not every practice is applicable to every lesson, she knows this list will help her focus on the various needs of her students:

- *Collect and chart information about individual students (family, interests, employment, strengths, activities) to use to develop relationships and include in class lessons.*
- *Consider relevant contexts which connect to lives of students when presenting problems by beginning class with informal discussions; this also serves as a way to check on how students are doing.*
- *Consider problems that are relevant to students and allow students various entry points and methods of solution.*
- *Provide time for students to share their solutions to problems.*
- *Encourage the use of hands-on materials to explore new ideas and represent understanding.*
- *Encourage the use of graphic organizers and model, over time, how they can be used.*
- *Provide for social learning in the classroom through regular partner and group work either face-to-face or using virtual breakout rooms.*
- *When possible, offer students choices in the ways in which they take in information, process it, and present what they know.*
- *Provide time and opportunities for students and the teacher to bring explicit attention to ways students can use language to clarify understanding, reasoning, and explanation about math concepts.*

STANDARD P2: Assessment
Uses a variety of formative and summative assessments to measure student learning and understanding, evaluate the effectiveness of instruction, develop differentiated and advanced learning experiences, and inform future instruction.

Supporting Explanation for Standard P2

According to Stiggins (2006), we assess for two reasons:

1. To gather evidence to inform instructional decisions
2. To encourage students to try to learn (p.1).

Teachers need to keep both of these reasons in mind as they assess and consider which one they are focusing on at any given time.

According to the National Council of Teachers of Mathematics' *Principles to Action*, "an excellent mathematics program ensures that assessment is an integral part of instruction, provides evidence of proficiency with important mathematics content and practices, includes a variety of strategies and data sources, and informs feedback to students, instructional decisions, and program improvement" (2014, p. 89). Assessment is not separate from instruction but drives what gets taught.

Assessments generally fall into one of two main categories: formative or summative. A formative assessment is used to assess students' understandings (and misunderstandings) in order to inform what gets taught. A summative assessment is a cumulative test such as an end of unit assessment task or a placement test such as the TABE or MAPT-CCR. A summative assessment can also be a high-stakes assessment such as the high school equivalency or college placement tests. These assessments are considered 'high-stakes' since they serve as gatekeepers to higher education.

Formative assessments should have the greatest influence on the plan for instruction. In fact, formative assessment has been touted as assessment *for* learning, compared to summative assessment, or assessment *of* learning (Stiggins, 2008). Piaget (1976) suggested three broad approaches to formative assessments: observations, interviews, and tasks.

Students should be active partners in the process of assessing their own understanding. For example, they can identify the elements for checklists or be active partners in the development of rubrics to assess their skills and conceptual understanding. Students can self-assess prior to meeting with teachers to discuss their progress and gaps or needs that still need to be addressed. They can use journals to explain what they have learned and where they are still having difficulty.

INDICATOR	**P2.1 Assessment Methods** Uses a variety of methods to equitably assess a broad range of skills, accommodate diverse needs, and motivate students. Designs and administers a variety of formative and summative assessments to inform instruction, identify learning needs, and monitor students' progress toward achieving both personal goals and state standards. Understands and shares with students the importance of performing well on NRS-approved assessments.

Sample Math Applications

What Effective Math Teachers KNOW	What Effective Math Teachers DO	✓ Focus

a. There is a clear purpose for assessing.	• Help students understand the difference between assessment of learning and high stakes assessments. For example, after reviewing student responses to an exit ticket (provided on paper or through a Google Doc, Padlet, or Jamboard), share with students how that information can be useful to the teacher (i.e., recognizing more practice or concrete understanding is needed, seeing which students might need more time or more challenging tasks, or noting how much students learned from that experience to determine next steps). Use this experience to note how this assessment tool differs from a pre- or post-test that measures completions or mastery of skills
	• Have a clear plan for integrating assessment of learning into daily lessons. For example:
	  ○ Use a warm-up and a closing activity
	  ○ Offer deep questioning vs. providing answers to students' questions
	  ○ Provide manipulatives and tools (virtual or in-person) to observe how students are thinking and which materials are most useful to them
	• Use assessment strategies to motivate, not penalize, students. For example, invite students to work in small groups (breakout rooms if online) to collaborate on a math task that asks them to evaluate if a given response is correct and if not, what could be done differently to make it correct. The idea is for students to see mistakes as a way to demonstrate deeper understanding and for group work to emphasize problem-solving as a team.	

b. The CCRSAE for Math provide language that is measurable and observable.	• Use language from the CCRSAE for Math to help craft measurable and observable objectives and determine what will be assessed	
c. Assessing students' math competence involves much	• Assess strengths and gaps in student conceptual understanding of major math topics (such as interpreting data, thinking proportionately)	

	more than computation and procedures.	• Assess student's competence with the Standards for Mathematical Practice • Plan lessons that build on students' strengths while addressing their gaps • Use a variety of assessment tools (e.g., interview, analysis of student work, oral explanations, exit tickets) to determine next steps for individual or group learning	
d.	Students who are actively involved in their own assessment are more likely to make progress toward their personal goals and to reach state standards.	• Engage students in the development of assessment strategies • Ask students to self-assess in a variety of ways	

Vignette:

After several lessons on measures of central tendency, students have been tasked to design and conduct a survey, then analyze the resulting data and represent it. They will then be expected to do an oral presentation about their findings. Before setting them off to do the task, the teacher, Shondra, asks them to think about what their presentation should look like when it's done well.

Shondra has been focusing on three math standards related to developing an understanding of statistical variability:

> Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.* (6.SP.1)
>
> Understand that a set of data collected to answer a statistical question has a distribution that can be described by its center, spread, and overall shape. (6.SP.2)
>
> Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. (6.SP.3)

Because these standards have been the focus of lessons, she nudges students by asking, "What are you going to be listening for when the data are presented?"

They then pipe in with ideas:

> *The data collected must match the question asked.*
>
> *Have to state which "average" we're using.*
>
> *The graph needs labels.*
>
> *The information has to be organized properly.*
>
> *Whatever measures we use, they have to be correct.*

Now that the students have a set of criteria to work from, together they build a rubric with Shondra's support. They begin with a blank table and insert their criteria in the left column.

Criteria			
The question posed is a statistical one.			
Data collected is organized in a way that makes sense and can be explained.			
The presenter uses a measure of central tendency for a particular purpose.			
The data tallied and measure of central tendency used are both accurate.			
The graph must be based only on the data collected.			

Once they establish their list of criteria that everyone agrees with, they begin to decide what evidence they will look for to determine whether each criterion has been met.

Criteria	☺	😐	☹
The question posed is a statistical one.	The question is one that will produce different data.		The question is not statistical; there is only one possible answer to the question.
Data collected is organized in a way that makes sense and can be explained.	The data are well organized and are reasonably explained.	The data are complete but there is some question about how they are organized.	Some data are missing, or the data are randomly organized.
The presentation uses a measure of central tendency for a particular purpose.	The presenter uses mean, median, or mode in describing the data and explains why.	The presenter uses mean, median, or mode in describing the data but doesn't explain why.	The presenter forgets to use mean, median, or mode in describing the data.
The data tallied and measure of central tendency used are both accurate.	All the calculations are accurate.	Either the total number in the survey does not align with the total data reported, or the measure of central tendency used is not correct.	The total number in the survey does not align with the total data reported, and the measure of central tendency used is not correct.
The graph must be based only on the data collected.	The graph is based on the data.	It is unclear how the data and graph are connected.	The graph is not based on the actual data collected.

Now that the students clearly know the expectations, they begin the task. Shondra makes sure that the rubric is posted so that they can refer to it as they work through their task

INDICATOR	**P2.2 Modifying Instruction** Analyzes results from a variety of assessments and other data (e.g., attendance, engagement) to measure student learning, inform instruction, and determine differentiated interventions. Evaluates the effectiveness of instruction and modifies it based on formative assessment results and feedback from students and colleagues.

Sample Math Applications		
What Effective Math Teachers KNOW	**What Effective Math Teachers DO**	✓ Focus
a. Learning targets should be broader than just aimed at enabling students to pass a high stakes test.	• Articulate to students what the targets are and how they can be attained • Include both teacher and students in defining the targets and strategies for assessing	
b. Lessons are tweaked based on ongoing formative assessment activities.	• Question students to get at their understanding • Re-adjust lessons when students' thinking is off and/or when more information on students' thinking is needed	
c. Formative assessment is an integral part of instructional lessons.	• Use a variety of strategies (e.g., authentic tasks, explicit questions that probe for reasoning, observations) and build them into each lesson • Informally assess students' understanding of the material throughout each lesson	
d. Lessons must be differentiated when a portion of the students miss the mark.	• Provide further exposure to content for those students who are struggling • Ensure struggling students have access to concrete experiences to further their understanding • Provide more complex, abstract problem-solving situations for students who demonstrate basic mastery	

Vignette:

Keisha has been working with her students on comparing fractions. She asked her students, "Which is larger: 3/4 or 3/5?" All students answered correctly. But, when she asked, "How do you know?" several of the students responded with, "Because there is only a difference of one between 3 and 4, but a difference of two between 3 and 5."

Keisha wants to know more about their thinking. In order to do so, she has to ask for more than the right answer. While looking at the relationship between the denominator and numerator is very important, Keisha thinks that the students may not have as firm a grasp on fractions as she first thought. By asking for the reasoning behind the answer, Keisha can better assess students' understanding/ misunderstanding.

She poses a similar question with another pair of fractions, "Which is larger: 4/5 or 5/6?" She intentionally keeps the difference between the denominator and numerator the same so she can push students' thinking.

For those students who struggle to answer her question, Keisha will provide further opportunities for students to explore what happens when wholes are cut into equal pieces. She wants them to discover for themselves that the greater the number of slices, the smaller each slice will be.

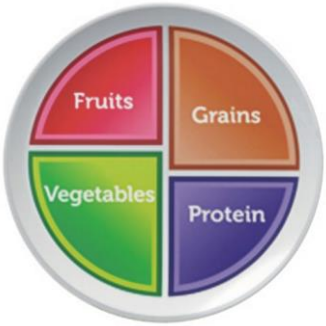
INDICATOR	**P2.3 Student Progress** Shares assessment results with students to acknowledge progress, identify gaps, and determine next steps. Seeks and implements feedback from students and colleagues to improve learning. Understands and shares with learners the role and importance of formative, summative, and NRS assessments as tools that allow students to demonstrate their learning. Encourages students to do their best on all assessments and helps students use results.

Sample Math Applications		
What Effective Math Teachers KNOW	**What Effective Math Teachers DO**	✓ Focus
a. Students need to access and understand the results of both formative and summative assessments.	• Share and discuss the results of both formative and summative assessments with students on a regular basis, including Math Score Reports from the MAPT-CCR, completed Workforce Application Assessments from CALM, and end of unit test-like questions in CALM • Meet individually with students to discuss their progress, identify gaps, and plan next steps • Provide positive, honest feedback to students on their progress; contradict student's misconceptions (e.g., under-estimation) of their progress with concrete examples	
b. Student self-assessment of progress is as important as teacher assessment of progress.	• Provide students with regular opportunities to self-assess using a variety of approaches (e.g., math journals or other writing, checklists, discussion) • Model providing clear, thoughtful, honest feedback, especially feedback about strengths	
c. Building a community of practice gives practitioners a bank of resources for help.	• Seek advice from peers • Subscribe to blogs or other sources of reading for ideas for improving student learning	
d. Understanding different ways to assess provides teachers with a variety of strategies to use in the classroom.	• Share assessment tools and strategies. For example, use the Teacher Dashboard in a Desmos Classroom Activity to monitor student progress and responses, pause activity, take snapshots of (anonymized) student work to do focused classroom instruction when most needed, and partner students based on observed skill level (when appropriate) • Try new assessment strategies and reflect on their effectiveness with students and colleagues. For example, use My Favorite No with students and	

	consider the impact of this assessment activity upon student learning and a teacher's instructional approach	
e. Student errors are shared and discussed with colleagues in order to understand students' thinking and learn how to redirect student learning.	• Share student work and listen to peers to gain insights into students' thinking • Use suggestions from colleagues to inform strategies for giving students feedback	

Vignette:

Kai works at a MassSTEP program and teaches a math class for immigrant and refugee adults seeking to pass the Certified Nursing Assistant (CNA) state licensure exam. He has 15 students who participate in math classes for 1 ½ hours per week over a 17-week program. His curriculum and lessons are health-contextualized, and, given the few total hours he has for math instruction, focus on essential numeracy skills that CNAs will need on the job. The class has spent several weeks working with fractions, decimals, and percents. Kai gives the class an exit ticket which uses the illustration on the right (from myplate.gov, excluding the dairy) and asks the students to write a fraction, decimal or percent to represent each food group (fruits, vegetables, grains, protein). He reminds students to work individually on the exit ticket.



Normally, Kai would go over and return the completed exit tickets in the following week's class. He's surprised by some of the student's answers, however, so he decides to do a follow-up activity the next week. He makes a handout with a table containing samples of different responses to the exit ticket:

Food Group	Response #1	Response #2	Response #3	Response #4
Fruits	1/8	1/4	.2	20%
Vegetables	1/4	1/4	.25	30%
Grains	1/2	1/4	.3	30%
Protein	1/4	1/4	.1	20%

Kai tells the class that some of the answers are reasonable and some of them are not. After distributing the handout, Kai asks the students how they can tell a reasonable answer from a not-so-reasonable one. After some discussion, the class comes up with the following list:

1. The answer should make sense
2. The answer should add up to 1
3. Fruits and vegetables are half the picture and grains and protein are half the picture.

Kai breaks students into pairs; the students then rate each response as correct or incorrect, giving a reason in each case for their rating. Kai then asks volunteer pairs report to the group how they rated the response and their reasoning. Any pair that has a different rating, can also give their rating and rationale. Most of the pairs rated each response accurately, and after the discussion, there is general agreement across the pairs.

Kai then asks the students to write down three things:

1. Whether they feel they understood the activity or are confused and need more work
2. One thing they learned
3. One question they have about fractions, decimals, or percents.

Research/Resources for the Instructional Practice Domain

ACLS Templates and Helpful Resources: http://www.doe.mass.edu/acls/frameworks/resources.html

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CONTINUOUS IMPROVEMENT DOMAIN (C)

Effective teachers value continuous learning, both for their students and for themselves. They cultivate a learning environment that nurtures a growth mindset for their students and support them as they claim ownership of their learning. Teachers guide students to persevere and put forth effort as they engage in productive struggle, challenge misconceptions about their abilities, identify their own strengths and learning needs, set goals for themselves, and monitor their own progress as they become independent, lifelong learners.

Effective teachers are reflective practitioners. They identify their own learning needs to best support their students, set student learning and professional practice goals, and participate in a variety of high-quality professional development activities to refine and expand their practices (e.g., coaching, workshops, courses, research, professional learning communities). They seek and integrate constructive feedback from students and colleagues. They demonstrate respect and professionalism in all interactions with their students and colleagues.

STANDARD C1: Growth Mindset
Cultivates a welcoming and judgement-free learning environment that motivates students and challenges them to believe that their abilities can be developed through persistence and hard work, both now and in the future. Promotes learning outside the classroom and over the lifespan.

Supporting Explanation for Standard C1

The environment that fosters lifelong learning is one where the teacher is a facilitator of learning, rather than the transmitter of knowledge, in which adult students are active participants in choosing, planning, and evaluating their own learning. Adult students understand why lessons are important and how they connect to their goals. Activities are perceived as a valuable use of time because of their relevancy and practical application. Problem-centered learning rather than subject-centered learning is primary.

Learning is a social activity. According to Vygotsky (1978), we learn through our interactions and communication with others. Learning together creates an environment conducive to learning. Cooperative group work is especially beneficial for urban, racial, and ethnic minority students, as well as low-income adult students. Individual students benefit significantly from working in cooperative groups to solve a problem or to produce something (a product, presentation, etc.). In *Powerful Learning: What We Know About Teaching for Understanding,* Darling-Hammond et al. (2008), discuss how these benefits apply to both academic performance and workplace performance and are increased by using inquiry approaches to learning.

One persistent challenge, especially significant for adult educators, is changing students' beliefs about their ability to succeed in math. Researchers have begun to consider the idea of math identity as having a significant impact in how students learn math content (or not). According to Grootenboer & Zevenbergen (2008), a student's math identity incorporates his/her knowledge, abilities, skills, beliefs, disposition, attitudes, and emotions (p. 224).

The learning environment must acknowledge all of these facets and begin to address them in order to help students develop a healthy attitude regarding math learning. In *Adding It Up*, the National Research Council (2001) refers to the idea of productive disposition as the tendency to see sense in math, to perceive it as both useful and worthwhile, to believe that steady effort pays off, and to see oneself as an effective learner and doer of math (p. 131).

Mathematics is about concepts that are all linked together. However, many people think of mathematics as a bunch of memorized rules. Learning environments with an emphasis on memorizing facts and steps have led many of us to think that being successful with math means that we must recall facts quickly. The thinking becomes, "If I'm not fast with calculation, I'm not good at math."

On the other hand, students who see math concepts and think math can be understood seek to understand more. They believe that through persistent problem solving, they will continue to learn. This phenomenon is the growth mindset mentioned in the work of Carol Dweck (2006) and written about in the form of mathematical mindsets by Jo Boaler who says, "A mathematical mindset reflects an active approach to mathematics knowledge, in which students see their role as understanding and sense making. Number sense reflects a deep understanding of mathematics, but it comes through a mathematical mindset that is focused on making sense of numbers and quantities…. and learning ways to develop one helps the development of the other." (2016, p. 36).

Unfortunately, fostering an environment that encourages growth mindset is important, but it's not the only thing teachers must consider. Lifelong learning for many adult learners means recognizing that there are systems in place and traumatic events from their past that can hinder educational progress. It is the responsibility of adult educators to offer opportunities for students to learn about systemic inequality and what can be done by students and teachers to dismantle or at least minimize these structures and policies. In the math classroom, this can be a matter of analyzing statistical data for a community or studying how legislative maps are drawn in a region and how the percentage of a population's demographics might contribute to those maps.

Steven Goodman in his 2018 book, *It's Not About Grit: Trauma, Inequity, and the Power of Transformative Teaching*, shares the following:

> *"…educators have a power of a different kind to make a lasting difference in the lives of students. Ours is a power of transformative teaching that works to counter these dehumanizing experiences with lessons of compassion, dignity, empowerment, and 'critical hope' (Duncan-Andrade, 2009) that help students critically analyze the systems that oppress them and create new possibilities for social justice. Engaging students in personal and community inquiry, they not only learn the history behind these unjust systems but also develop the resilience, agency, and collective voice needed to bring about more equitable and just futures for themselves and their local communities." (Goodman, p. 5)*

An effective learning environment facilitates the building of a respectful mathematical learning community where students who have experienced failure and ridicule are comfortable asking questions, sharing their reasoning and ideas, and presenting their work. Respectful learning means that teachers value who their students are as individuals and as members of their community. Teachers see students' experiences as valuable resources to help connect math to their lives.

This is probably one of the most effective strategies for a teacher – to convince her students that they can learn math, that math is a relevant part of their lives, and that they have the power within and the support from their teachers to research, learn about, and affect change on the root causes of inequity in their lives and communities.

See Appendix for more ideas of what Standard C1: Growth Mindset might look like in an adult education classroom.

INDICATOR	**C1.1 High Expectations** Establishes high expectations for the quality of student work and the effort required to produce it. Within a safe classroom environment, encourages risk taking and productive struggle. Models and reinforces ways that students can master challenging material through persistence, focused effort, and critical thinking.

Sample Math Applications		
What Effective Math Teachers KNOW	**What Effective Math Teachers DO**	✓ Focus
a. Students need clear expectations for acceptable work.	• Engage students in the development of rubrics for assignments. • Present students with examples of model work. • Involve students in evaluation of student work (their own and that of others).	
b. Effort and perseverance are primary factors in mathematical success.	• Reward effort, not correct answers. • Allow students the time to struggle with difficult problems. • Teach students to recognize what is correct in their work; encourage students to build on what they know and not be impeded by errors; stress the importance of CCRSAE Math Practice 1 and refer to it during times when students are struggling.	
c. Making mistakes leads students to make connections, clear up misconceptions, and fully understand mathematics.	• Change student mindset to viewing mistakes as legitimate steps in the learning process. • Set up safe learning environment where students are comfortable participating without fear of ridicule or anxiety about making mistakes. • Arrange activities so that common student misconceptions come to light. • Involve students in examining common errors with a goal toward strengthening critical thinking and increasing conceptual understanding. • Use small group activities and partnered discussions. • Expect students to regularly explain their reasoning and critique the reasoning of others. • Embrace mistakes so that everyone sees mistakes as a learning opportunity.	
d. Students need to struggle with complex mathematical problems.	• Guide students when they are stuck by posing purposeful questions rather than showing students how to proceed or taking away the challenge of the task. • Provide the time needed to work on, discuss, and solve complex problems.	

e. The Standards for Mathematical Practice (CCRSAE) are important in establishing the mathematical community and engaging students in the work of mathematics.	• Present the Standards for Mathematical Practice and their importance in the initial class with an example experience to refer to. • Refer to practices as they are evident in the classroom (e.g., "You grabbed strips of paper to show equivalent fractions. That's a great tool for the job." and "What a great discussion where we heard a variety of strategies, and I even heard some challenges to them. Will they work every time?)	

Vignette:

As Sophia prepares for class with a new group of students, she notes the wide range of diversity in the class. Students have reached various levels of math competency, have differing levels of formal education from grade 6 to grade 12, are from three different language backgrounds (Chinese, Spanish, English), and have different levels of tech skills and accessibility. She knows that many students will approach the math class with anxiety based on past experiences and failure. Others will think that math is not for them because they see very few scientists or mathematicians that look like them. She also knows that many of her students don't believe that there is any purpose for math other than to pass a test.

Sophia is determined to design a learning environment where the focus is on the students and the belief that they can learn, where math is meaningful, and where students see mistakes as learning opportunities. She knows that this is something that develops over time as students learn to trust her and as she learns more about each of her students.

Sophia begins each session with a routine that gets students to think critically. One routine she uses is *Which One Doesn't Belong?* to help students begin to understand that what is important in math class is being able to reason and to explain that reasoning to others. She first uses simple visuals such as pictures of fruits or vegetables so that students are not concerned about the 'math' but rather their reason for choosing which of the pictures does not belong. Over time she extends this to mathematical visuals.

As she chooses routines to use with her students, she continues to be cognizant of the fact that many of her students are only able to access the class's Zoom lesson via smart phone. She has to be aware of how students can access the materials and offers different strategies for students to share their thinking. This is true not just of the routines that Sophia chooses but also all other class activities that she has the students engage with. She expects all students to participate, all students to make an effort and to be willing to take a risk, but she realizes that high expectations does not mean everyone accessing the information the same way, nor does it mean that everyone has to share their reasoning and solutions using the same strategies.

After a few lessons where Sophia and students have opportunities to get to know each other through routines and other activities. Sophia presents the students with challenging problems, always with different access points. She provides only a short explanation and very little direction. She sometimes has students work together in breakout rooms to solve the problem and show how they reached their answer. As she moves between breakout rooms to observe students' work, she makes positive comments where appropriate and responds to requests for help with a question to encourage students to proceed on their own. Sophia then asks students to present their work.

She follows these presentations with a discussion of the process students used in reaching a solution stressing that, despite some anxiety and initial confusion, students were able to rely on each other, activate prior knowledge, and successfully solve the problems. She has the class brainstorm a list of norms for cooperative class work and a list of suggestions for persisting when the work is difficult. She discusses Mathematical Practice 1 (Make sense of problems and persevere in solving them) and continually reminds students throughout the semester.
(http://lincs.ed.gov/publications/pdf/CCRStandardsAdultEd.pdf)

INDICATOR	**C1.2 Student Ownership** Provides learning experiences that enable students to claim ownership of their learning by identifying their own strengths, interests, and needs; setting meaningful and challenging learning goals for themselves; asking for support when needed; and monitoring their own progress.

Sample Math Applications

What Effective Math Teachers KNOW	What Effective Math Teachers DO	✓ Focus
a. Students benefit from active involvement in their learning goals.	• Gather student input. • Offer choice of tools and of how to demonstrate understanding. • Ask students to describe what success with a task would look like.	
b. Students who feel connected to staff and peers and feel valued are more likely to take academic risks such as sharing what they don't know, venturing conjectures, and tackling challenging problems with persistence.	• Acknowledge and value students' prior experiences and culture. • Respect students as peers and adults. • Generate respectful dialogue and interaction among students. • Value the diversity of students rather than consider differences as deficiencies.	
c. In order to continue to grow and to genuinely model a mindset of continual growth, teachers need to actively seek feedback on their own practice.	• Seek formal feedback thorough the EGE Cycle (see Standard C2: Reflective Practice). • Seek informal feedback from colleagues. • Seek feedback from students (both formal and informal).	
d. Relevant materials and lessons are engaging for students and take the emphasis off memorization, which leads to a fixed mindset.	• Use math tasks that allow multiple entry points and various strategies to get to a solution. • Build on examples students come across in their lives or that they have explicitly talked about. • Use real-life math tasks that have multiple solutions.	
e. Using concrete materials is valuable in gaining conceptual understanding, which leads to a growth mindset.	• Offer a variety of culturally relevant materials, such as contextualized stories and lessons from *The Change Agent* or activities and tasks from *Mathematical Modeling with Cultural and Community Contexts*. • Model use of concrete objects (e.g., tiles, strips of paper) or virtual manipulatives to show conceptual understanding.	

f. Flexibility with numbers is important for conceptual understanding and procedural fluency.	• Stay away from decontextualized, timed math fact checks. • Use visuals such as number lines, arrays, and area models. • Make their mental math explicit and expect the same of students. • Ask for several ways to solve the same problem. • Expect students to articulate their reasoning, whether right or wrong.	
g. Heterogeneous cooperative groups give all students the opportunity to learn.	• Plan group roles. • Provide open-ended tasks. • Offer a choice of tasks, such as gathering data from other groups, organizing and presenting the information, or summarizing findings.	

h. There are pervasive and damaging stereotypes related to math abilities that must be actively and continually addressed.	• Model and talk about:
	  ○ Math is more than just calculation.
	  ○ Math can be understood by everyone.
	  ○ Race, gender, and ethnicity do not predispose people to success in math and math-focused careers.
	  ○ We are all lifelong learners of math.	

Vignette A:

Maria is working with a group of 12 students, most of whom are English language learners. Maria observed that they didn't know their times tables, were uncomfortable asking questions, and generally were unsuccessful in testing situations. She noticed that when they got stuck, they just stopped working on the problem. He is determined to change their attitude toward math and help them to gain confidence in their ability to *do* mathematics.

She has been reading a lot about diversity, equity, and inclusion lately and feels that he probably hasn't done enough in the past to connect to students' lives. Typically, she is always in a rush to help them prepare for the high-stakes assessment, although she has done so while trying to get students to begin to have a more positive attitude about being math learners. She now realizes that it is not enough just to get students to think about themselves as math learners; they also need to see role models – with similar backgrounds – to further help them see themselves as math learners.

Maria's class is run remotely, so she decides that she would like to integrate math class with stories of mathematicians from different cultures – something other than white men which is all that he ever learned about in school. Maris herself has to do some learning on her own, and over time has had her students help in the research.

However, at the beginning of the semester, she provides many examples of distinguished mathematicians of color. She usually shares a brief biography of an important mathematician once a week during one of

two weekly Zoom meetings. She first posts the mathematician's name and then asks what they know about the individual. It's usually very little or nothing.

The first name Maria offers up is Lonnie Johnson, a famous African American. She wants students to understand that not all mathematicians are white men and that they do sometimes engage in some exciting research and inventions. Once Maria explained that Lonnie not only worked on the stealth bomber but also invented the Super Soaker, students became interested in learning more about people who looked more like themselves or who came from the same countries they did. Maria realizes that including these kinds of conversations can help students improve their math growth mindset and belief in their own ability to do math.

Vignette B:

Ibrahim is working with a group of 12 students whose past math experiences have, for the most part, been negative. Students reported that they didn't know their times tables, were uncomfortable asking questions, and generally were unsuccessful in testing situations. He noticed that when they got stuck, they just stopped working on the problem. He is determined to change their attitude toward math and help them to gain confidence in their ability to *do* mathematics.

Ibrahim begins each class with mental math examples such as "Is the answer to the following example (4 x 27) less than, equal to, or greater than 100?" At first students are hesitant since mental math is new to them, but he encourages and supports many different ways to think about this problem.

One student says it's more because she knows that four quarters are $1.00 and 27 is more than 25. Another student uses her fingers and writes the example in the standard form in the air and multiplies, using the algorithm. A third student guesses and says it's less than 100. Another student breaks apart the number 27 into 4 x 20 + 4 x 7 and calculates the answer as 108. Skillfully, Ibrahim leads the discussion, asking the second student how his method is like the first student's and how it's different. The short discussion that follows leads the third student to exclaim, "Oh, I never thought about math like that. I thought there was only one way to do it."

With repeated opening activities like this, Ibrahim sees that students are gaining a better sense of number relationships. He also notices that students are more and more willing to share and defend their thinking, ask questions of others, and participate in group discussions.

Ibrahim explains early on that there are concrete materials (such as manipulatives) available in the classroom and urges students to use them, stressing that good mathematicians "model" their thinking. (He is identifying these materials as helpful for all students and not just crutches for struggling students.) Sometimes he provides a specific material to assist in solving a particular problem; other times he simply urges students to use what is helpful. The use of concrete materials (tiles, counters, shape cards) and visual representations (arrays, number lines, tape diagrams) become regular classroom tools.

Ibrahim carefully chooses the class problems he presents so that the students can engage with interesting material. He also chooses problems that connect various mathematical ideas so that review and new learning are constantly intertwined. He sees a growing confidence in his students as they realize they

actually know more that they thought they did, are not afraid to make mistakes, and are encouraged to solve problems in the way that makes sense to them.

By the sixth week of class, Ibrahim sees that the students he faces are a very different group than the ones he met at the first class. He is glad that he paid attention to ways to do mathematics that were different than the more traditional way students had known. He is excited by the fact that students are spending far more time working on difficult problems than they had at the start of class. He sees this as a sign that they know that they can learn mathematics – they have changed their mindset.

INDICATOR	**C1.3 Lifelong Learning** Incorporates strategies that assist students in becoming self-reliant, independent learners who are motivated and adequately prepared for postsecondary education and careers.

Sample Math Applications

What Effective Math Teachers KNOW	What Effective Math Teachers DO	✓ Focus
a. Knowledge of and access to independent study resources for students can maximize the learning of motivated students.	• Seek mentors and colleagues with whom they can share ideas, ask questions, and receive feedback. • Know quality independent study resources. • Create a classroom or program website to compile resources for study and practice as well as additional materials.	
b. Having confidence in their ability is one of the things that will help students to continue learning.	• Give specific feedback statements, rather than empty phrases such as "good job". Examples include: o "You created a simpler version of the problem and used what you learned from that to solve this. That's a great problem-solving strategy." o "You identified a pattern in this table and you generalized it to fit examples not listed on the table. That's a skill for algebra.") • Acknowledge classroom examples of effort.	
c. Extrinsic rewards given for tasks that require perseverance hinder students' motivation.	• Focus on the inherent value of the task (e.g. developing understanding of a math concept, using math for personal decision making, the satisfaction of solving a math challenge).	
d. Embrace the multi-level classroom.	• Choose problems that can be accessed in multiple ways (using concrete objects and drawings, basic math, algebraically). • Allow time for conversation during struggle with challenging problems.	

	• Ask and encourage students to defend their reasoning using visual representations and in writing. • Challenge students to connect visual representations to procedures with which they are familiar. • Expect students to share and listen to others' strategies.	
e. Practicing the CCRSAE Standards for Mathematical Practices is not only what mathematically proficient people do, but the same skills prepare students for the workplace and lifelong learning.	• Create a classroom environment where struggle is expected, making mistakes is okay, and respectfully critiquing others' work is appreciated. • Help students learn to form an argument and dispute the claims of others. • Provide opportunities for students to examine solutions that are incorrect and explain what was wrong. • Ask students to make sense of a relationship between quantities in a problem.	
f. Learning is a continuum rather than defined by the beginning and end of a course.	• Teach students how to persevere with challenging problems, gather information, and use math to make decisions.	

Vignette:

Irlando has been teaching adult education for many years and knows that most of his students come to his ABE math class with anxieties about math. Since many have had negative experiences in math, they believe they cannot do math. They believe that math is for 'smart' people. They often don't even believe that math is supposed to make sense.

This school year, Irlando decided to focus more energy on students' beliefs and attitudes about math, even if that means taking some time from teaching math content. After all, students with low self-esteem regarding math don't seem to be very successful in learning the content. One of the first changes he committed to was having the students work in groups so he could work on being a classroom community where students can trust each other and him to be kind and respectful. In doing so, he would make use of various group roles, and assign group-worthy tasks whereby everyone participates and no one individual dominates interaction. He also decided that students should keep journals every day in class.

Irlando knew that, in order to help students build their math identity, he first had to really understand each student and their unique life circumstances, and to facilitate students' belief in their ability to learn important math. At the outset of his class, he asked each student to create a "math biography" or "mathography". He asked each student to represent herself as a mathematician, including strengths and challenges and ways she planned to be an active member of the class. These "mathographies" would then be shared and revised as students changed their beliefs about who they are as mathematicians.

Irlando also decided to rethink what it means to be successful in math. With the students' help, they created a list of what 'success' means in a math class. With Irlando's urging, students began to see that success was more than just getting the right answer. Success included using a different representation to explain an idea, sticking with a challenging problem and not giving up, asking important questions, and making connections to other math and to real-life situations. As students begin to feel more positive about

math, Irlando expects the list to grow. He anticipates his students will see themselves as successful math

STANDARD C2: Reflective Practice
Engages in a continuous improvement process that includes self-assessment, goal setting, high quality professional development, and ongoing reflection to gain greater expertise, develop new teaching approaches, and refine current instructional practices.

doers.

Supporting Explanation for Standard C2

"Teachers of mathematics continue to learn throughout their careers in the areas of mathematical knowledge for teaching, mathematical pedagogical knowledge, and knowledge of students (adult learners) as learners of mathematics." (Principles to Action, p. 102)

Reflective practice is one way that a practitioner can continue to learn. It requires taking an objective look at her own practice, then thinking about what is working (and what isn't) and determining strategies to making changes to improve. Reflective practice can reveal disconnects between what we believe and what we do in reality. For example, we may believe that adults are different from children and should therefore be taught differently, but in reality, we may still teach the way that we were taught as children.

The MA Department of Elementary and Secondary Education, Adult and Community Learning Services (ACLS) uses a 6-step Educator Growth and Effectiveness (EGE) Continuous Learning Cycle (see graphic below), part of the Educator Growth and Effectiveness Model. This proficiency guide is intended to be used in conjunction with the EGE model which is designed to support teachers and directors in reflecting upon and taking an active role in improving instructional practices. It is grounded in three key questions: *1) Are students learning? 2) What is the teacher doing that contributes to and supports that learning?* and *3) What else might the teacher do to enhance student learning?*

Six Step EGE Continuous Learning Cycle

The EGE model requires an inclusive planning process that creates opportunities for program teams to consider how the EGE can be adapted and implemented to meet individual staff needs (Step 1). Next, individual practitioners self-analyze and reflect on their teaching (Step 2), using the relevant proficiency guide(s) and recent feedback on their teaching by a director, supervisor, or other colleague. Based on their reflection, teachers then draft goals for enhancing their practice (Step 3). These goals may vary in the time it takes to complete them, depending on the type of goal and urgency. Teachers seek out professional learning opportunities related to their goals and implement specific approaches (Step 4), periodically assessing their progress (Step 5) to ensure they are on target. At the end of the academic year, or at another designated time, they assess and report on their progress, dialoguing with their colleagues about what they have learned (Step 6).

Teachers working with adult learners ideally are lifelong learners themselves. They are aware that research on using best practices for teaching mathematics with understanding is constantly evolving. They are aware that to effectively teach adult learners strong content knowledge in mathematics is important. They must be knowledgeable about the CCRSAE content standards and the Standards for Mathematical Practice and the best ways to make them visible in the classroom. They understand the instructional shifts in the CCRSAE and how they impact the planning and delivery of lessons. Teachers must know more than just "how to do it"; they must know how to engage students in developing understanding.

The needs of each teacher are different. Teachers can best plan for professional learning by reflecting on their own teaching practices to see how they align with required expectations (CCRSAE). They can assess their strengths and challenges and seek out opportunities to strengthen their perceived areas of weakness. They can make use of feedback provided by their supervisors, colleagues, and students. Developing a personal professional development plan is helpful in prioritizing learning needs.

In Massachusetts in particular, there are a variety of opportunities for teachers to strengthen their practice. Through the various offerings of the SABES Mathematics and Adult Numeracy Curriculum and Instruction PD Center (https://sabes.org/pd-center/math-numeracy), teachers can seek out a variety of professional growth activities, including both face-to-face, on-line, and blended professional development. Coaching is available for teachers, and teachers can attend onsite professional growth offerings and those sponsored by organizations for specifically learning about teaching mathematics to adult learners. They can benefit by reading in the field of adult numeracy. Working collegially with peers is also a good source for learning new and better classroom practices.

INDICATOR	**C2.1 Self-assessment** Engages in a self-assessment process using state professional standards, student data, and feedback from students and colleagues to reflect on the effectiveness of instruction, with the intention of improving practice and student learning. Considers how personal beliefs and cultural values influence instructional decisions.

Sample Math Applications		
What Effective Math Teachers KNOW	**What Effective Math Teachers DO**	✓ Focus
a. Teachers need to assess their own strengths and challenges in teaching adult learners.	• Reflect on the strengths they bring to the field of teaching mathematics. • Assess their knowledge of current research on adult learning. • Assess their needs for growth. • Seek to improve their knowledge about teaching adults. • Look for high quality professional growth opportunities. • Maintain a connection to the most relevant remote learning resources, including free online math curriculum, virtual manipulatives, and classroom activities. • Participate in professional development to learn how to incorporate new digital tools and resources into math instruction.	
b. Teachers need to assess their knowledge of the math content for the varying levels of the CCRSAE.	• Attend professional development sessions which clarify the math requirements for adult learner proficiency. • Study the requirements of all the levels of the curriculum to know what comes before their course and what comes after.	
c. Teachers must reflect on their own teaching vis-à-vis their students' grasp of the content.	• Push for students to explain their thinking to determine whether they truly understand what they are doing. • Pay attention to the types of errors students tend to make, especially if those errors are made in more than one class session. • Question why a particular lesson is taught the way that it is and consider how it could be presented differently.	
d. Practitioners building communities of practice can support each other as 'critical friends'.	• Reach out to peers to discuss lessons that didn't go as planned. • Ask peers to observe a lesson to offer advice on a specific aspect of teaching. • Keep an open mind and accept criticism.	

e. Insights gained through reflection and self-analysis should impact changes in practice.	• Ask themselves 'what if' and then try out the new idea. • Make a change to a lesson based on self-reflection. • Monitor the change to see if it makes a difference in student understanding or their own understanding of how students learn.	
f. One's personal beliefs and cultural values influence instructional decisions.	• Work to identify cultural predispositions and implicit bias. • Participate in a variety of high quality professional learning experiences. • Share instructional materials with colleagues who teach in a variety of settings. • Use a variety of instructional methods, techniques, and tools to facilitate learning.	

Vignette:

After an evening teaching operations with fractions to her students, Cheryl is lost in her thoughts, reflecting about her class. She realizes that she spends a great deal of time teaching fraction operations in all levels of math class, and it seems that students have to constantly be reminded of how to do the various procedures. She thinks her students must be very tired of trying to learn about fractions, and she knows that she's tired of teaching the same thing over and over again.

Cheryl believes that the reason students are not remembering the procedures is because they don't practice enough. This is especially true since evening classes are offered only two evenings a week and all content areas have to be taught, not just math. Cheryl begins to do some serious reflection, not just mulling over how well (or poorly) students did with the math lesson. Instead, she thinks that perhaps the reason students can't remember the procedures is not their fault but maybe hers.

The next morning, she grabs the CCRSAE for Math and looks at the various levels where the fractions standards are. She begins to wonder whether her students have some of the requisite conceptual understanding of fractions – not with operations but simply understanding what a fraction is and how it can be modeled in different ways. Do the students know how to use benchmark fractions to make sense of less familiar fractions? Can they show a fraction on a number line? Questions continue to gush forth and Cheryl admits to herself that she is taking the first big step to reflect on her own teaching.

INDICATOR	**C2.2 Goal Setting** Uses insights from self-assessments to identify meaningful student learning and professional practice goals that are clear, results-focused, and measurable or observable. Reviews goals, monitors progress, and makes revisions as needed.

Sample Math Applications

What Effective Math Teachers KNOW	What Effective Math Teachers DO	✓ Focus
a. Written goals with a plan to meet those goals helps lead to success in life and career.	• Assess needs and identify a goal. • Describe what success with that goal looks like. • Identify steps toward the goal.	
b. Goals that are clear, results focused, and measurable or observable are easier to meet.	• Ensure that goals can be accomplished within a realistic timeframe, especially for part-time staff. • Consider strategies (such as analyzing student data) for determining if the goal attainment has an impact on student learning.	
c. Goals are based on student and program needs.	• As a program team, discuss needs of the program to ensure sufficient capacity to meet those needs. • Analyze student achievement to determine what areas need improvement and consider the feasibility of learning new ways to teach to those areas.	

Vignette:
Carol Jones, director of the Lincoln Program, has observed her teacher, Drake, several times over the past six months. Drake is a retired engineer who has a strong understanding of math. While he is quite knowledgeable about the math that he needed for his work as an engineer, Carol feels that he does not know about the various ways to approach math with adult learners. Carol and Drake have several conversations about how the students seem to be frustrated with his teaching style. He says students need to do more homework and has suggested they watch more videos on how to do the various procedures that he is trying to teach.

Carol wants Drake to understand the benefit of exploring math with manipulatives and visuals long before procedures and suggests he observe a math class where these kinds of instructional approaches are common. She notes there are some local classes in their program, as well as some videos on the SABES Math Center site that could be viewed, such as the one on How to Use Singapore Strips that shows how students use this tool to make sense of word problems. She also suggests that he look at the CCRSAE for math beginning at level A and B so that he can get a sense for what he's expecting his students to already know by the time they get to his pre-ASE class. Carol anticipates review of those levels will help Drake understand that students need to progress from concrete to visual/representational before reaching the more abstract algorithms and procedures commonly found in math classes.

After a couple of weeks, Carol and Drake meet again. He acknowledges that he does not know about a lot of the things addressed in the CCRSAE. He notes that as an engineer math always came naturally for him, so he has never had to engage in simple visuals or concrete models. He decides that maybe he needs to better understand how to teach the core math concepts rather than suggesting that students just learn procedures. He now knows that even though he has the math content needed for his work, he does not have the math knowledge for teaching math.

INDICATOR	**C2.3 Professional Development** Engages in a variety of high-quality professional development activities. Seeks out and applies new ideas from professional development, supervisors, colleagues, and other resources to gain expertise and advance student learning.

Sample Math Applications

What Effective Math Teachers KNOW	What Effective Math Teachers DO	✓ Focus
a. Participating in high quality professional development that leads to changes in practice also leads to positive student outcomes.	• Put new ideas into practice in the classroom in order to make lasting change. • The most important thing to do is to try out something new to see how it affects student learning. • Systematically monitor new strategies to determine if they are making a difference.	
b. Professional development choices must be connected and build on one another for a useful learning experience.	• Choose sessions which connect and build on one another. • Use SABES Math PD offerings on content instructional strategies, and teaching tools. Several of these PD opportunities are designed as a series and qualify for Professional Development Points (PDPs).	
c. Connecting personal goals to program goals and collaborating with colleagues to achieve program improvement goals leads to student gains.	• Set professional goals that complement those of colleagues. For example, setting a professional goal of including more digital literacy into lesson planning can complement a colleague's professional goal of adding more project-based learning opportunities. The digital literacy skills can enrich the experience of the project-based work. • Share planning time with colleagues. • Share resources with colleagues. • Participate in national communities of practice such as the Adult Numeracy Network and LINCS • Observe each other and request and give specific, evidence-based feedback to peers.	

Vignette:

Lynne is a retired middle and secondary school math teacher with over 25 years of teaching experience and an excellent teaching record. Two years after her retirement she makes the decision to return to the classroom and takes a part-time job teaching adult learners at a local community education center. She is initially happy to be again "in the front of the room" and feels quite confident in her ability to do a good job. She is excited about her first class and prepares the lesson much as she had always done in a secondary classroom where she mainly taught algebra and geometry.

Within two weeks, however, Lynne is aware that, though she brings definite strengths (strong math content knowledge, knowledge of lesson planning and assessment, good relationships with students), there are in fact many new things to consider.

Lynne is aware that she knows very little about how to plan engaging lessons for adult learners and recognizes that they are in a very different place than her high school freshmen or sophomores. She realizes that adult learners have life experiences that should be valued, and she also is becoming more aware that she needs to value each adult's unique identity, not just as a math student. She talks with a colleague who understands her dilemma and suggests that Lynn do some reading about culturally responsive teaching and also about the characteristics of adult learners and how best to engage them in filling in mathematical gaps while still doing significant mathematics that connects to their lives.

Her colleague gives her some ideas for planning for more social interaction through meaningful adult-focused activities and suggests she plan opportunities for students to share their work, including how that math work relates to them as individuals. Lynn acts on her colleagues' suggestions and notices an improvement. She knows, however, that she needs more than this since her experience previously has been a direct teaching approach.

Lynn also discovers that she has to teach remotely rather than in an actual classroom. And even though Lynn knows a lot about technology, she soon finds that knowing technology and teaching through technology are two very different skill sets. She again turns to her colleague for suggestions whose main suggestion is to contact the SABES Math Center to learn how to teach math using technology.

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Appendix

WHAT TO LOOK FOR IN THE MATH CLASSROOM: A BRIEF OVERVIEW FOR DIRECTORS AND EVALUATORS

Introduction

Many teachers and directors are familiar with only one way to teach math – typically the traditional model that they were exposed to when they went to school. In this model, the teacher stands at the front of the class and shows students how to do procedural, decontextualized problems. The new College and Career Readiness Standards for Adult Education (CCRSAE) detail three key shifts for instruction in math. These shifts require teachers to modify how and what they teach. The shifts require teachers to:

- Narrow their focus in order to deepen the manner in which they teach mathematics, instead of racing to cover topics.
- Design learning around coherent progressions from level to level so that students can build new understanding onto previous foundations.
- Pursue conceptual understanding, procedural skill and fluency, and application – all with equal intensity (U.S. Department of Education, p. 44).

This companion piece to the *MA Professional Standards for Teachers of Adult Education* (2017, revised 2019) and *Indicators of Proficiency Rubric* (2017, revised 2019) has been designed to support directors and other educators to evaluate math instruction, including these three key shifts, in an adult education classroom. The *MA Professional Standards for Teachers of Adult Education* and *Indicators of Proficiency Rubric* contain three domains: 1) Professional Knowledge; 2) Instructional Practice; and 3) Continuous Improvement. Of the three, only P1 of the second domain: Instructional Practice and C1 of the third domain: Continuous Improvement are used in this document since these domains focus on what effective classroom teaching should look like.

The Instructional Practice Domain includes the following standard and indicators:

Standard P1. Design and Instruction
P1.1 Standards-based Units
P1.2 Well-structured Lessons
P1.3 Student Engagement
P1.4 Meeting Diverse Needs

The Continuous Improvement Domain includes the following standard and indicators:

Standard C1. Growth Mindset
C1.1 High Expectations
C1.2 Student Ownership
C1.3 Lifelong Learning

Unlike the *Indicators of Proficiency Rubric*, this document does not include a rating scale. It offers examples of what a math classroom looks and sounds like when teaching to the key shifts by providing examples of what students might be doing, what questions teachers might be asking, and what artifacts an evaluator might be expected to find.

Although this document is organized around P1 and C1 of the Standards of the *Indicators of Proficiency Rubric*, it is also cross-referenced with the College and Career Readiness Standards for Adult Education Mathematical Practices and the Mathematical Proficiency Strands from the National Research Council (2001).

The eight Mathematical Practices describe what all students should be able to do in order to be expert math problem solvers. The Practices are the following:

MP1. Make sense of problems and persevere in solving them.
MP2. Reason abstractly and quantitatively.
MP3. Construct viable arguments and critique the reasoning of others.
MP4. Model with mathematics.
MP5. Use appropriate tools strategically.
MP6. Attend to precision.
MP7. Look for and make use of structure.
MP8. Look for and express regularity in repeated reasoning. (Pimentel, p. 48)

Even though the CCRSAE Mathematical Practices are based on the Mathematical Proficiency Strands from the National Research Council (NRC), this document calls out the strands separately since these are more easily observed in the classroom. Mathematical proficiency is what is considered necessary for anyone to learn math well. The National Research Council describes five components to being mathematically proficient:

- Conceptual understanding
- Procedural fluency
- Strategic competence
- Adaptive reasoning
- Productive disposition (p. 116)

Since the Mathematical Proficiency Strands, the CCRSAE Mathematical Practices, and the CCRSAE key instructional shifts are interrelated, you will see overlap in this document. Therefore, you may find this document most useful by reviewing it before you visit a math class, then use it as a reference when discussing the results with an instructor.

Research/Resources

National Research Council (2001). *Adding It Up: Helping Children Learn Mathematics*. Washington, DC: National Academy Press.

U.S. Department of Education, Office of Vocational and Adult Education. (2013) *College and Career Readiness Standards for Adult Education*. Washington, DC. Available at https://lincs.ed.gov/publications/pdf/CCRStandardsAdultEd.pdf

Instructional Practice Domain P1 Design and Instruction	National Research Council's Mathematical Proficiency Strands	CCRSAE Mathematical Practices	What this might look like in an adult education class	Questions you might hear a teacher ask	Collectable artifacts to review
P1.1 **Standards-based Units**	All, depending on what the teacher decides to focus on within a unit.	All, depending on what the teacher decides to focus on within a unit.	Students know what they are learning and can articulate why they are learning it. Students use precise language (as appropriate to their level, conceptual understanding, and English language skills) as they explain their reasoning.	What do you already know about this topic? How does this build on what we learned in the last unit?	Real-life project in which students were given a challenging situation to work through based on what they learned in the unit.
P1.2 **Well-structured Lessons**	All, depending on what the teacher decides to focus on within a lesson	All, depending on what the teacher decides to focus on within a lesson; she'll probably only want to focus on one Practice per lesson.	Students can articulate what they just learned during a lesson. Students, in a positive way, question the reasoning of their peers. Students make conjectures and build a logical process of reasoning to come to a conclusion. Students use different strategies and share them with their peers.	What do you already know about this topic? How does this build on what we learned in the last lesson? Can you prove it to your peers? Do you know how he reached that conclusion? Can you explain his reasoning in your own words? Does his answer make sense? How do you know? Do you agree with what you just heard? Why or why not?	Teacher lesson connects students' prior knowledge to new conceptual understanding. Formative assessment that asks for student understanding of the material learned in the lesson.

Instructional Practice Domain P1 Design and Instruction	National Research Council's Mathematical Proficiency Strands	CCRSAE Mathematical Practices	What this might look like in an adult education class	Questions you might hear a teacher ask	Collectable artifacts to review
				Do you have a different strategy for solving this problem? Explain your approach and reasoning.	
P1.3 **Student Engagement**	Productive disposition	MP1. Make sense of problems and persevere in solving them.	Students may be frustrated but keep trying new ways to tackle the problem. Students put into their own words what they think the problem or situation is about.	What are you thinking about this problem? What might be your first step in tackling this problem? What are some options about what you might try next? Why did you decide to use that method? Where are you getting "stuck"? How does this connect to what you do in real-life?	Student's self-assessment (pre- vs. post- on attitude, beliefs, etc.). Student journal jottings. Completion of a project in which student had to problem solve on his own.
P1.4 **Meeting Diverse Needs**	Conceptual understanding Procedural fluency Strategic competence	MP1. Make sense of problems and persevere in solving them.	Teacher draws new students into the conversation and activities.	Can you draw a picture to show it? Can you create a model? Why did you choose that strategy?	Student's written response or drawing to explain reasoning. Student picture or representation to illustrate why a

Instructional Practice Domain P1 Design and Instruction	National Research Council's Mathematical Proficiency Strands	CCRSAE Mathematical Practices	What this might look like in an adult education class	Questions you might hear a teacher ask	Collectable artifacts to review
	Productive disposition Adaptive reasoning	MP3. Construct viable arguments MP4. Use math models MP5. Use appropriate tools strategically. Diversity can mean many things: learning styles where someone is more vocal, or hands on, or visual so MP3 gets to verbal in a way and MP4 gets to hands on in a way	Teacher differentiates by asking questions or posing problems of varying difficulty. Students are encouraged to explain in words, draw a picture, or create a mathematical model to illustrate a concept, idea, or solution. Students mentally solve calculations. Students may be frustrated but keep trying new ways to tackle the problem. Students ask each other questions in order to clarify thinking. Students decide what tools to use to solve problems.	Does anyone have the same answer but a different way to explain it? Did you notice a pattern? What are you thinking about this problem? Where are you getting "stuck"? Can you think of a counter example? What assumptions are you making? Do you agree with what you just heard? Why or why not?	procedure works (such as a visual model to show why 2/3 x 4/5 = 8/15). Teacher lesson connects students' prior knowledge to new conceptual understanding. Student work shows that some false starts were made before altering strategy. Student's oral explanation about how he tackled a new type of situation involving a mathematical aspect.

Continuous Improvement Domain C1 Growth Mindset	National Research Council's Mathematical Proficiency Strands	CCRSAE Mathematical Practices	What this might look like in an adult education class	Questions you might hear a teacher ask	Collectable artifacts to review
C1.1 **High Expectations**	Conceptual understanding Procedural fluency Strategic competence Productive disposition Adaptive reasoning	MP2. Reason abstractly and quantitatively. MP4. Model with mathematics. MP5. Use appropriate tools strategically. MP6. Attend to precision. MP7. Look for and make use of structure. MP8. Look for and express regularity in repeated reasoning.	Teacher poses a challenging problem for students, then gives them time to work through it. Students are encouraged to explain in words, draw a picture, or create a mathematical model to illustrate a concept, idea, or solution. Student decontextualizes the problem for computational purposes, then contextualizes the results to make sure the solution makes sense. Students get frustrated but keep trying new ways to tackle the problem. Students decide what tools to use to solve problems.	Can you create a model to show it? Can anyone else build on x's explanation? Why did you choose that strategy? Why did you choose that tool? Does anyone have the same answer but a different way to explain it? Can you convince me that your answer makes sense? How did you reach that conclusion?	Student's written response or drawing to explain reasoning using level appropriate math vocabulary. Student picture or representation to illustrate why a procedure works (such as a visual model to show why 2/3 x 4/5 = 8/15). Project in which students are given a challenging situation to work through; students include explanations and reasoning in their final documentation. Student work showing that some false starts were made before altering strategy.

Continuous Improvement Domain Standard C1. Growth Mindset	National Research Council's Mathematical Proficiency Strands	CCRSAE Mathematical Practices	What this might look like in an adult education class	Questions you might hear a teacher ask	Collectable artifacts to review
C1.2 **Student Ownership**	Productive disposition Adaptive reasoning	MP1. Make sense of problems and persevere in solving them. MP3. Construct viable arguments and critique the reasoning of others.	Teacher ensures that all students are participating by having them do the hard thinking rather than simply telling students how to solve a problem. Students get frustrated but keep trying new ways to tackle the problem. Students ask each other questions in order to clarify thinking. Students go to the board and show how they solved the problem.	What are you thinking about this problem? What might be your first step in tackling this problem? What are some options about what you might try next? Why did you decide to use that method? What seems to be your "stuck" place? Can you prove it? How did you reach that conclusion? Is that true in all cases? Can you think of a counter example? What assumptions are you making? Can you convince me that your answer makes sense?	Student journal jottings. Project in which students are given a challenging situation to work through; students include explanations and reasoning in their final documentation. Student's oral explanation about how he tackled a new type of situation involving a mathematical aspect. Student's self-assessment (pre- vs. post- on attitude, beliefs, etc.).

Continuous Improvement Domain Standard C1. Growth Mindset	National Research Council's Mathematical Proficiency Strands	CCRSAE Mathematical Practices	What this might look like in an adult education class	Questions you might hear a teacher ask	Collectable artifacts to review
C1.3 **Lifelong Learning**	Conceptual understanding Strategic competence Productive disposition Adaptive reasoning	MP4. Model with mathematics. MP8. Look for and express regularity in repeated reasoning.	Students think about their own personal experiences to see how they might use those strategies to tackle the new situation at hand. Students connect their new learning to their own lives. Students get frustrated but keep trying new ways to tackle the problem. Students are using authentic materials to learn math concepts. Students simplify the situation to connect it to what they already know, then use estimation to first get a sense for the solution.	Have you used similar strategies in your own life? Did this problem remind you of any math situations from your own life? Can you think of a real-life example of this kind of problem?	Student's oral or written explanation about how a concept applies to his own situation. Real-life project in which students were given a challenging situation to work through; students include explanations and reasoning in their final documentation.