**Summary of major themes in response to the public comment draft of**

**the *Massachusetts Curriculum Framework for Mathematics***

**December 1, 2016-February 17, 2017**

Sources:

* Online public comment survey (997 full or partial responses, of which 542 responded to questions about Mathematics; 78% pre-K-12 teachers, 10% pre-k-12 administrators)
* Additional correspondence related to the proposed mathematics framework
* Notes from 10 public regional meetings hosted by the Department (473 participants)
* Notes from Student Advisory Council meeting (60 participants)
* Notes from 4 public conference calls hosted by the Department (162 participants)

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| **Responses to Online Public Comment Survey Questions about the Overall Impact of the Mathematics Revisions** | | |
| **Question** | **% Agree or**  **Somewhat Agree** | **% Disagree or**  **Somewhat Disagree** |
| Do the revisions improve or maintain the clarity of the Framework? | 90.6% | 9.4% |
| Do the revisions improve or maintain the coherence of the Framework? | 85.7% | 14.3% |
| Do the revisions improve or maintain the rigor of the Framework? | 89.7% | 10.4% |

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| **Summary of major themes from all public comment sources** | |
| **Major Themes** | **Department Response and Rationale** |
| **Recognizing Patterns: Learning Progression**  Extend pattern recognition into the early grades, Pre-K-3.  *Page 21, PK.CC.4; page 24, K.CC.4.C; page 29, 1.NBT.5; page 32, 2.NBT.2.* | * **Specific references to number patterns were added to several standards at the early grades, Pre-K-3**. Several counting standards already embedded pattern/counting sequence language such as “count by 10s.” * Recognizing patterns in numbers is key to mathematics and fundamental for algebraic thinking. |
| **Clarity of Language and Expectations in the Standards**  Clearer expectations for which math facts students need to know from memory.  *Page 32,* *2.OA.2; page 36, 3.OA.7 and the Glossary.*  Clearer expectations for telling time and learning about money.  *Page 33, 2.MD.7a & 2.MD.8* | * **Clarifying language was added to standards in several ways.** To help clarify mathematical terms or learning expectations, the wording of the standard was revised, examples were added, a note was included, a table was created, or an entry was added to or revised in the glossary. Examples:   + Definitions of *know from memory* and *fluency* have been added to the glossary.   + Clarifying language was added to the grade 2 standard 2.MD.8 to specify that students are expected to solve word problems with money (up to $10) using the $ sign and the cents symbol for whole dollar amounts *(not decimals).* An additional example was also added to further clarify this standard. * Respondents asked for clarification of wording in the standards, definitions of terms, and examples to help clarify student expectations. |
| **Line Plots/Dot Plots in the Standards**  *Page 33, 2.MD.9; page 37, 3.MD.4; page 43, 4.MD.4; and the*  *Glossary.* | * **The standards that expect students to learn about line plots/dot plots have been maintained. Standard 2.MD.9 was revised to clarify that grade 2 students will show measurements on a line plot but will not have to create a line plot.** A visual example of a line plot/dot plot has been added to the glossary that shows “*the lengths of each pencil measured.”* * Respondents noted that line plot/dot plots were not useful for students in grades 2 and 3. A review of the learning progressions show that a key way to connect the measurement standards in the early grades with organizing and representing data is by students measuring objects and then representing the data (by count) on a line plot. * This progression continues as students advance through the grades and continue to collect and represent data and solve problems by creating and using a variety of types of displays, including histograms, a common graph used in statistics. |
| **Units of Measurement in Mathematics**  *Page 43, 4.MD.3.* | * **Revisions were made at grade 4 to provide further explanation about adding and multiplying square units.** Notes and examples have been added. |
| **Concept of Rate in grades 6 and 7**  *Page 52, 6.RP Cluster Heading & 6.RP.1; page 58, 7.RP.3.* | * Respondents were positive about strengthening the focus on rate concepts. This was accomplished by **adding rate to the grade 6 cluster heading and standards and by adding clarifying language and examples to select standards.** |
| **Understand and Apply the Pythagorean Theorem**  *Page 65*  Public Comment Version:  8. G.6.  a. Explore the relationship among the sides of a right triangle.  b. Analyze and justify ~~Explain a proof of~~ the Pythagorean Theorem and its converse using pictures, diagrams, narratives or models.  8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.  8.G.8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.  (*Black text is the 2010 standard; red text shows proposed revisions in the Public Comment Draft*.) | * **Standard 8.G.6 has been further revised based on public comment feedback. The standard now reads:** * **8.G.6.**   a. ~~Explore~~ Understand the relationship among the sides of a right triangle.  b. Analyze and justify ~~Explain a proof of~~ the Pythagorean Theorem and its converse~~,~~ using pictures, diagrams, narratives or models.   * There are 3 standards (8.G.6; 8.G.7; and 8.G.8.) in this cluster: *Understand and Apply the Pythagorean Theorem*. Standards 8.G.7 and 8 have not been revised. * Respondents expressed concern that in standard 8.G.6. grade 8 students are expected to explain a “proof” and they have not yet been introduced to proofs (proving theorems is an expectation at the high school). Feedback varied regarding the revisions. Some readers thought that the public comment version was an improvement and others thought that the proposed wording represented a loss of rigor. This was not the intent. After reviewing all feedback and further discussions with math specialists, an additional revision was made to 8.G.6. (See wording above in red.) * Using the terms “analyze” and “justify” maintains the level of rigor of the original standard and at the same time addresses the concern about using the term “proof” in grade 8. The two related standards that follow require students to apply their understanding of this theorem. |
| **Alignment of High School Standards**  Alignment of the Content Covered (by the end of the three Model Courses) in the Traditional and the Integrated Pathway  *Page 105, Traditional Pathway; Page 124, Integrated Pathway;*  *Page 144, Advanced Courses.*  Alignment of the Model Courses and the Conceptual Category Standards  *Page 71+ High School Section*  Role of the (+) Plus Standards in the High School Courses  *Page 68, High School Introduction* | * **The model courses were edited** **and aligned to ensure that the same content is expected and covered by the end of the three model courses in both pathways.** * The footnotes in the previous framework were incorporated into the language of the model course standards, in order to clarify student expectations for each of the model courses. * Based on readers’ feedback, a review of the alignment of the Model Course standards was conducted and revisions were made. These reviews resulted in clear, specific expectations for each model course that progresses student learning across the three courses in each pathway. * **The Conceptual Category Standards were edited to specifically describe the full range of content expected in the revised model courses.** The content of the model courses is aligned with the content in the High School Conceptual Category Standards. * **Revisions were made to the text in the High School Introduction to clarify that the (+) plus standards are optional** and are presented as options for differentiation when they are included in the model courses in the two pathways. * **Revisions were made to the High School Introduction to clarify that the two Model Advanced Courses in the framework (Advanced Quantitative Reasoning and Precalculus) are optional.** These Advanced Model Courses were designed using the (+) plus standards that indicate content that is beyond the college and career-ready benchmark. Students completing the standards in either of the two pathways can proceed to these advanced model courses or to other advanced courses offered by their districts. |
| **High School Pathways Options, including an option to take Algebra I in grade 8 leading to Calculus in grade 12.**  *Page 155, High School Section* | * **Detailed explanations about various pathways were added to the framework.** * Respondents were positive about including an updated version of the 2011 *Making Decisions about High School Sequences and Algebra I in Grade 8* document in the framework. * Public comment requested a variety of pathways and course-taking sequences that include: the Integrated Pathway Model Courses, the Traditional Pathway Model Courses; and the Advanced Model Courses, as well as other advanced courses, such as AP courses, Statistics, etc. |
| **Grade-span Descriptions of Standards for Mathematical Practice**  *Page 164, Appendix II* | * **Feedback for the new version of the Grade-Span Descriptions of the Standards for Mathematical Practice was positive and minor revisions were made to add more examples of technological tools.** * The 2010 Standards for Mathematical Practice presented one set of descriptions for grades Pre-K-12. * Readers requested more customized descriptions in narrower grade spans: Pre-K-5; 6-8; and 9-12 that include examples of how the content and practices can be connected in classrooms in each grade span. This more customized version is included as an Appendix. * In the online survey, 88.6% of those responding to the Mathematics questions “agreed” or “somewhat agreed” that “these descriptions help clarify how math content can be connected to the mathematics practices by providing more specific examples. |