Proposed Massachusetts Additions

to the

Common Core State Standards for Mathematics

October 2010

I. Proposed Massachusetts Pre-Kindergarten Standards for Mathematics

The educator panels that reviewed the *K-12 Common Core State Standards for Mathematics* unanimously supported the addition of pre-Kindergarten standards, which have been an element of the Massachusetts Curriculum Frameworks since the very first editions were published in 1995. Because the *K-12 Common Core State Standards for Mathematics* contains no pre-Kindergarten standards at all, the pre-K standards are not counted as part of the potential 15 percent that Massachusetts is able to add to the *Common Core* standards.

An advisory panel composed of ESE staff with kindergarten expertise, staff of the Department of Early Education and Care (EEC) and early childhood educators from public and independent preschools (including Head Start), universities, and organizations such as Strategies for Children drafted the pre-Kindergarten standards. The draft pre-K standards are based on the EEC's *Preschool Learning Experiences* (2003), the ESE's *Kindergarten Learning Experiences* (2008), the existing *Massachusetts Mathematics Curriculum Framework* and working drafts, and the *Common Core* Kindergarten standards. The pre-K standards are intended for students who are in the final year of pre-Kindergarten, generally late four to five years of age.

The panel recommended 13 pre-K mathematics standards in the areas of counting and cardinality, operations and algebraic thinking, measurement and data, and geometry, and serve as the foundation for the 24 kindergarten mathematics standards in the *Common Core.*

Pre-Kindergarten Standards for Mathematics

Introduction

The preschool/pre-Kindergarten population includes children between at least 2 years, 9 months until they are kindergarten eligible. A majority attend programs in diverse settings—community-based early care and education centers, family child care, Head Start, and public preschools. Some children do not attend any formal program. These standards apply to children who are at the end of that age group, meaning older four- and younger-five-year olds.

In this age group, foundations of mathematical understanding are formed out of children's experiences with real objects and materials. The standards can be promoted through play and exploration activities, and embedded in almost all daily activities. They should not be limited to "math time." These mathematics standards correspond with the learning activities in the *Massachusetts Guidelines for Preschool Learning Experiences (2003).* The standards should be considered guideposts to facilitate young children's underlying mathematical understanding.

In preschool or pre-Kindergarten, activity time should focus on two critical areas: (1) developing an understanding of whole numbers to 10, including concepts of one-to-one correspondence, counting, cardinality (the number of items in a set), and comparison; (2) recognizing two-dimensional shapes, describing spatial relationships, and sorting and classifying objects by one or more attributes. Relatively more learning time should be devoted to developing children's sense of number as quantity than to other mathematics topics.

(1) These young children begin counting and quantifying numbers up to 10. Children begin with oral counting and recognition of numerals and word names for numbers. Experience with counting naturally leads to quantification. Children count objects and learn that the sizes, shapes, positions, or purposes of objects do not affect the total number of objects in the group. One-to-one correspondence with its matching of elements between the sets, provides the foundation for the comparison of groups and the development of comparative language such as, *more than*, *less than*, and *equal to*.

(2) Young children explore shapes and the relationships among them. They identify the attributes of different shapes including the length, area, weight by using vocabulary such as: long, short, tall, small, heavy, light and big. They compare objects using comparative language such as: longer/shorter, same length, heavier/lighter. They explore and create 2- and 3-dimensional shapes by using various manipulative and play materials such as: popsicle sticks, blocks, pipe cleaners, and pattern blocks. They sort, categorize, and classify objects and identify basic 2-dimensional shapes using the appropriate language.

Counting and Cardinality **PK.CC**

Know number names and the counting sequence.

MA.PK.CC.1 Listen to and say the names of numbers in meaningful contexts.

MA.PK.CC.2 Recognize and name the numerals 0 – 10.

Count to tell the number of objects.

MA.PK.CC.3 Understand the relationship between numerals and quantities.

Compare numbers.

MA.PK.CC.4 Count many kinds of concrete objects and actions up to ten, using one-to-one correspondence, and accurately count as many as seven things in a scattered configuration.

MA.PK.CC.5 Use comparative language such as more/less than, equal to, to compare and describe collections of objects.

Operations and Algebraic Thinking **PK.OA**

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

MA.PK.OA.1 Use concrete objects to model real-world addition and subtraction problems up through five.

Measurement and Data **PK.MD**

Describe and compare measurable attributes.

MA.PK.MD.1 Recognize the attributes of length, area, weight, and capacity of everyday objects using appropriate vocabulary (e.g., long, short, tall, small, heavy, light, big).

MA.PK.MD.2 Compare the attributes of length and weight for two objects, including longer/shorter, same length; heavier/lighter, same weight; holds more/less, holds the same amount.

Classify objects and count the number of objects in each category.

MA.PK.MD.3 Sort, categorize, and classify objects by more than one attribute.

Work with money.

MA.PK.MD.4 Recognize that certain objects are coins.

Geometry PK.G

Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

MA.PK.G.1 Identify relative position of objects in space, and use appropriate language (e.g., beside, inside, next to, close to, above, below, apart).

MA.PK.G.2 Identify various two-dimensional shapes using appropriate language.

Analyze, compare, create, and compose shapes.

MA.PK.G.3 Create and represent three-dimensional shapes using various manipulative materials, such as popsicle sticks, blocks, pipe cleaners, pattern blocks, and so on.

II. Proposed Additional K-12 Standards in Mathematics and Model High School Courses

A. Proposed Additional K-12 Standards in Mathematics

The educator panels that reviewed the *Common Core State Standards for Mathematics* and the *Mathematics Curriculum Framework* Review Panel recommended a small percentage of additional standards. The committees' considerations included suggestions based on the results of several comparative studies of the Massachusetts standards and the *Common Core* standards conducted in June/July 2010, the Common Core Review Panels' reports, content experts' reviews, and the *Common Core* additions recommended in other states, including California.

General Rationale: Massachusetts current and working draft standards for mathematics addressed key skills and knowledge that students should master. While the Common Core *addresses mastery of many of these skills, there were some additional key concepts and skills that were missing. There were some instances when the progression of an important mathematical concept was developed but there was a gap. Suggested additions fill these gaps and will help students by presenting mathematics concepts in a more coherent progression through the grades.*

The additions below are organized by grade level K-8 and high school. They are designed to align with the existing structure of the *Common Core State Standards for Mathematics.* All page numbers below refer to the current pagination of the *Common Core State Standards for Mathematics.* The standards are coded beginning with **MA** (indicating Massachusetts); followed by the number of the grade level (**K, 1, 2, 3, 4, 5, 6, 7, 8, 9-12**); domain (K-8) or conceptual category (9-12); and number of standard. For example, **MA.1.OA.9** is read: **MA** (Massachusetts additional standard); **1** (grade 1); **OA** (Operations and Algebraic Thinking domain); **9** (standard number).

Note: Additional standards are presented in Calibri font, rationale for adding the standard is indicated by *Times New Roman italics*.

Kindergarten (0 additions)

No additions.

Grade 1 (2 additions)

MA.1.OA.9 Write and solve number sentences from problem situations that express relationships involving addition and subtraction within 20. (page 15)

Rationale: Recognizing that problem solving is important, this standard expects students to write and solve equations in problem solving situations.

MA.1.MD.5 Identify the values of all U.S. coins, know their comparative values (e.g., a dime is of greater value than a nickel), find equivalent values (e.g., a nickel is equivalent to 5 pennies), and use appropriate notation (e.g., 69¢). (page 16)

Rationale: Students need experience with the value of coins earlier than when it is introduced in the Common Core *(grade 2).*

Grade 2 (2 additions)

MA.2.OA.2a By the end of Grade 2, know from memory related subtraction facts of sums for two one-digit numbers. (page 19)

Rationale: Knowing subtraction facts is not explicitly referred to in the CCSS.

MA.2.MD.7a Know relationships of time, including seconds in a minute, minutes in an hour, hours in a day, days in a week or month, weeks in a year. (page 20)

Rationale: Although knowing the names of the months and days of the week are in the Massachusetts History and Social Science Curriculum Framework, students will need to know the number of days in a week, month, year, etc. to calculate in mathematical applications.

Grade 3 (0 additions)

No additions

Grade 4 (1 addition)

MA.4.NBT.5a Know multiplication facts and related division facts through 12 x 12. (page 29)

Rationale: Common Core *requires students to know multiplication facts to 10 x 10 in grade 3. This standard extends students' mastery of multiplication and related division facts to 12 x 12 (as is currently required in the Massachusetts Mathematics Curriculum Framework in grade 4).*

Grade 5 (1 addition)

MA.5.NS.1 Use positive and negative integers to describe quantities such as temperature above/below zero, elevation above/below sea level, or credit/debit. (page 35)

Rationale: This standard introduces the concept of negative numbers using real world contexts. Common Core *introduces positive and negative axes for graphing, but does not include negative numbers until grade 6.*

Grade 6 (3 additions)

MA.6.NS.4a Apply number theory concepts, including prime factorization and relatively prime numbers, to the solution of problems. (page 35)

Rationale: This standard introduces arithmetic and geometric sequences, which will be further extended in high school coursework.

MA.6.G.1a Use the relationship between radius, diameter, and center of a circle to find the circumference and area; solve real world and mathematical problems involving the measurements of circles. (page 44)

Rationale: Students need experience with measurements of circles earlier than when it is introduced in the Common Core *(grade 7).*

MA.6.SP.4a Read and interpret circle graphs. (page 45)

Rationale: Students' work with circles would be enhanced by including circle graphs as a type of data display.

Grade 7 (1 addition)

MA.7.EE.4c Extend analysis of patterns to include analyzing, extending, and determining an expression for simple arithmetic and geometric sequences (e.g., compounding, increasing area), using tables, graphs, words, and expressions. (page 49)

Rationale: This standard introduces arithmetic and geometric sequences, which will be further extended in high school coursework.

Grade 8 (0 additions)
No additions

Grades 9-12 (9 additions)
MA.9-12.N.Q.3a Describe the effects of approximate error in measurement and rounding on measurements and on computed values from measures. (page 60)

Rationale: Critical to science and understanding the consequence of errors in measurement.

MA.CC.9-12A.REI.3a. Solve linear equations and inequalities in one variable involving absolute value. (page 65)

Rationale: The absolute value function is essential to understanding piecewise functions. The concept of absolute value is important to understand the limits of domain and is not adequately addressed in the Common Core.

MA.9-12.A.REI.4c Demonstrate an understanding of the equivalence of factoring, completing the square, or using the quadratic formula to solve quadratic equations. (page 65)

Rationale: Understanding that the methods are equivalent is essential in order to understand that one method may be more efficient in a specific case.

MA.9-12.F.IF.8c Translate between different representations of functions and relations: graphs, equations, point sets, and tables. (page 69)

Rationale: Translating between representations provides a comprehensive understanding of the equivalence of functions.

MA.9-12.F.IF.10 Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, exponential, or trigonometric. (page 70)

Rationale: Recognizing type of function as presented in a variety of ways assists in assessing how to approach solving a problem.

MA.9-12.G.CO.11a Prove theorems about polygons. Theorems include: measures of interior and exterior angles, properties of inscribed polygons. (page 76)

Rationale: Proof of properties for shapes other than triangles and quadrilaterals should be included. (Related to MA.9-12.G.C.8a)

MA.9-12.G.C.8a Derive the formula for the relationship between the number of sides and sums of the interior and sums of exterior angles of polygons and apply to the solutions of mathematical and contextual problems. (page 76)

Rationale: Deriving the formulas for the sum of exterior angles is fundamental for understanding the relationship between circles and multi-sided polygons.

MA.9-12.G.GPE.3a (+) Use equations and graphs of conic sections to model real-world problems. (page 78)

Rationale: Conic sections, such as parabolas and ellipses, model common real-life phenomena.

MA.9-12.G.MG.4 Use dimensional analysis for unit conversions to confirm that expressions and equations make sense. (page 78)

Rationale: Dimensional analysis is not currently included in measurement. It is an essential part of modeling.

B. Model High School Courses

The high school *Common Core State Standards for Mathematics* are presented by conceptual category (Number and Quantity, Algebra, Functions, Geometry, Modeling, and Statistics and Probability) and specify the mathematics that all students should study in order to be career and college ready, with advanced mathematics that students should learn indicated with a "+". The *Common Core* document brought before the Board in July 2010 did not indicate how the high school standards were to be organized into courses.

Achieve (in partnership with the *Common Core* writing team) convened a group of state mathematics experts, high school teachers, mathematics faculty from two and four year institutions, mathematics teacher educators, and workforce representatives to develop model courses. In August 2010 *Appendix A: Designing High School Mathematics Course Based on the Common Core State Standards* was published. *Appendix A* outlined possible approaches to organizing the all of the standards without a "+" into coherent and rigorous courses in two course-taking pathways: 1) Traditional (Algebra I, Geometry, Algebra II) and 2) Integrated (Mathematics I, Mathematics II, and Mathematics III).

During September 2010 the high school Common Core Committee:

- reviewed the *Common Core* high school standards by conceptual category and recommended that 9 standards be added to the high school standards (see grades 9-12 additions));
- reviewed the model high school mathematics courses presented in *Appendix A* and made suggestions for revisions and additions to the three traditional courses, Algebra I, Geometry, Algebra II, and three Integrated courses, Mathematics I, Mathematics II, and Mathematics III; and
- designed two additional courses, Precalculus and Advanced Quantitative Reasoning, for students to study after completing one of the Pathways. These advanced courses are comprised of the "+" standards in the high school conceptual categories.

The model courses include, where appropriate, the recommended additional high school standards. All of these high school model courses are presented in a separate document (see Attachment 1).

III. Other Features to the *Common Core Standards for Mathematics*

Features to be added by December 2010

The educator panels that reviewed the *Common Core State Standards for Mathematics* and the Mathematics Curriculum Framework Review Panel recommended that the following features be added to the *Common Core*.

1. The Guiding Principles from the 2000 edition of the Massachusetts Mathematics Curriculum Framework, updated to provide coherent connections to the 2010 Standards.
2. The Glossary from the 2000 edition of the Massachusetts Mathematics Curriculum Framework, updated to include key mathematical terms used in the *Common Core* Standards.

Rationale: These distinctive features have helped guide program development and create a common language about mathematics for all teachers.

Features to be added between January 2011 and December 2015

The educator panels that reviewed the *Common Core State Standards for Mathematics* and the Curriculum Framework Review Panel recommended that the following features be added to the *Common Core* over time.

Over the course of the next four years, add to an interactive web version:

- relevant curriculum maps, curriculum units, and formative assessments developed through the Race to the Top and Partnership for the Assessment of Readiness for College and Career grants;
- links to resources developed by partner states or in collaboration with them;
- mathematical problems that illustrate expected depth of understanding of the standards and that show applications of mathematics in a variety of real world contexts including careers and financial decision-making; and
- guidelines for connecting literacy and mathematics developed jointly by mathematics and literacy educators.

Rationale: The first three elements mirror the examples and guidance in the current Mathematics Framework and that educators have found useful. The fourth is recommended to parallel the Common Core's Standards for Literacy in History/Social Studies, Science, and Technical Subjects. *At present neither the* Mathematics *nor the* ELA *Common Core* Standards *address the topic of math and literacy, a resource which the panels recommended be developed jointly by literacy and math educators.*