**2017 Curriculum Framework for Mathematics**

**Detailed Revisions of 2010 Standards for PK-12**

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| Pre-Kindergarten and Kindergarten |
| **2011 Standard** | **Proposed 2017 Standard****Revisions are in red text** | **Rationale for Revision** |
| **PK.CC.4** Count many kinds of concrete objects and actions up to ten, using one-to-one correspondence, and accurately count as many asseven things in a scattered configuration. | Count many kinds of concrete objects and actions upto ten, using one-to-one correspondence, and accurately count as many as seven things in a scattered configuration. Recognize the “one more”, “one less” patterns.  | Edited to highlight that recognizing patterns in numbers is key to mathematics and fundamental for algebraic thinking.  |
| **K.CC.4.c** Understand the relationship between numbers and quantities; connect counting to cardinality. c. Understand that each successive number name refers to a quantity that is one larger.  | Understand the relationship between numbers and quantities; connect counting to cardinality. c. Understand that each successive number name refers to a quantity that is one larger. Recognize the one more pattern of counting using objects. | Edited to highlight that recognizing patterns in numbers is key to mathematics and fundamental for algebraic thinking.  |
| **K.CC.6** Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. | Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group for groups with up to ten objects, e.g., by using matching and counting strategies. | Edited to incorporate an accompanying footnote into the original standard to clarify student learning expectations for comparing number of objects in kindergarten. |
| **K.OA.5** Fluently add and subtract within 5. | Fluently add and subtract within 5 including zero. | Edited to include zero as a student learning expectations for fluency in kindergarten. |
| **K.MD.3** Classify objects into given categories; count the numbers of objects in each category and sort the categories by count. | Classify objects into given categories; count the numbers of objects in each category (up to and including 10) and sort the categories by count. | Edited to incorporate an accompanying footnote into the original standard to clarify student learning expectations for sorting by count in kindergarten. |

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| Grade I |
| **2011 Standard** | **Proposed 2017 Standard****Revisions are in red text** | **Rationale for revision** |
| **1.OA.1** Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. | Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations (number sentences) with a symbol for the unknown number to represent the problem. | Edited to include number sentences.  |
| **1.OA.3** Apply properties of operations as strategies to add and subtract. *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known.* *(Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)* | Apply properties of operations ~~as strategies~~ to add ~~and subtract~~. *Examples: When adding numbers, order does not matter. If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known.* *(Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12.(Associative property of addition.) When adding zero to a number, the result is the same number (Identity property of zero for addition).* | Edited to include an explanation of the properties of operations to clarify expectation of the standard. Students are not required to use the formal terms of these properties in grade 1 but they are required to apply them when adding. The properties of operations do not apply to subtraction of whole numbers. The identity property of zero is added to formalize understanding about adding zero to a number.  |
| **1.OA.MA.9** Write and solve number sentences from problem situations that express relationships involving addition and subtraction within 20. | ~~Write and solve number sentences from problem situations that express relationships involving addition and subtraction within 20.~~ | The content of this standard have been incorporated into the revised standard 1.OA.1.  |
| **1.NBT.5**  Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.  | Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used. Identify arithmetic patterns of 10 more and 10 less than using strategies based on place value. | Added the expectation to identify number patterns using strategies to enhance the development of number sense in grade 1 students as they look for and make use of mathematical structures. |
| **1.MD.MA.5** Identify the values of all U.S. coins and know their comparative values (e.g., a dime is of greater value than a nickel). Find equivalent values (e.g., a nickel is equivalent to 5 pennies). Use appropriate notation (e.g., 69¢). Use the values of coins in the solutions of problems. | **1.MD.~~MA.~~5** Identify the values of all U.S. coins and know their comparative values (e.g., a dime is of greater value than a nickel). Find equivalent values (e.g., a nickel is equivalent to 5 pennies). Use appropriate notation (e.g., 69¢). Use the values of coins in the solutions of problems (up to 100¢). | Edited to remove “MA” (Massachusetts) designation.Edited to provide parameters that clarify the student learning expectation of the standard. |

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| Grade 2 |
| **2011 Standard** | **Proposed 2017 Standard****Revisions are in red text** | **Rationale for revision** |
| **2.OA.2** Fluently add and subtract within 20 using mental strategies. By end of grade 2, know from memory all sums of two one-digit numbers.  | Fluently add and subtract within 20 using mental strategies. By end of grade 2, know from memory all sums of two ~~one~~ single-digit numbers and related differences. *For example, the sum 6 + 5 = 11 has related differences of 11 – 5 = 6 and 11 – 6 = 5.* | Edits made to clarify wording of the standard and to incorporate the learning expectation of deleted standard 2.OA.2.MA.2.a. (See below.) |
| **2.OA.MA.2.a**.By the end of grade 2, know from memory related subtraction facts of sums of two one-digit numbers. | ~~By the end of grade 2, know from memory related subtraction facts of sums of two one-digit numbers.~~ | The content of this standard have been incorporated into the revised standard 2.OA.2. |
| **2.NBT.2**  Count within 1000; skip-count by 5s, 10s, and 100s. | Count within 1000; skip-count by 5s, 10s, and 100s. Identify patterns in skip counting starting at any number. | Edited to include patterns to enhance the conceptual development of numbers in grade 2 students as they look for and make use of mathematical structures. |
| **2.MD.MA.7.a**  Know the relationships of time, including seconds in a minute, minutes in an hour, hours in a day, days in a week, a month, and a year; and weeks in a month and a year. | 2.MD.~~MA~~.7.a. Know the relationships of time, including seconds in a minute, minutes in an hour, hours in a day, days in a week; days in a month and a year; approximate number of weeks in a month and weeks in a year. | Edited to remove “MA” (Massachusetts) designation and to clarify the expectation for knowing the number of weeks in a month. |
| **2.MD.8** Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?*  | Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies (up to $10), using $ and ¢ symbols appropriately and whole dollar amounts. *Examples: If you have 2 dimes and 3 pennies, how many cents do you have?*  *If you have $3 and 4 quarters, how many dollars do you have?* | Edited to provide parameters that clarify the student learning expectation of the standard. |
| **2.MD.9**  Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units. | Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Organize and record data on a line plot (dot plot) where the horizontal scale is marked off in whole-number units. | Edited to include the words dot plot because it is a more commonly used term and to help clarify the standard about the expectation for using this method for representing data and to maintain the mathematical rigor. The term “dot plot” was added to glossary. |
| **2.G.1** Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. | Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, ~~quadrilaterals,~~ squares, rectangles, rhombuses, trapezoids, pentagons, hexagons, and cubes.  | Edited to provide clarity about types of quadrilaterals.  |

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| Grade 3 |
| **2011 Standard** | **Proposed 2017 Standard****Revisions are in red text** | **Rationale for revision** |
| **3.OA.5**  Apply properties of operations as strategies to multiply and divide. *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15 then 15 × 2 = 30, or by 5 × 2 = 10 then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property).* | Apply properties of operations ~~as strategies~~ to multiply. ~~and divide.~~ *Examples:* When multiplying two numbers order does not matter*. If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication)*. The product *3 × 5 × 2 can be found by 3 × 5 = 15 then 15 × 2 = 30, or by 5 × 2 = 10 then 3 × 10 = 30. (Associative property of multiplication.)* When multiplying two numbers **either** number can be decomposed and multiplied*. One can find 8 x 7 by knowing that 7= 5 + 2 and 8 × 5 = 40 and 8 × 2 = 16, resulting in 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)* When a number is multiplied by 1 the result is the same number. (Identity Property of 1 for multiplication.) | Edited to clarify the properties of operations for multiplication (not for division) to retain the mathematical rigor of the standard. Students are not required to use the formal terms of these properties in grade 3 but they are required to apply them when multiplying. The identity property for multiplication is added to formalize understanding about multiplying by the number 1. Division is introduced in standard 3.OA.6. |
| **3.OA.7**  Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of grade 3, know from memory all products of two one-digit numbers**.** | Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of grade 3, know from memory all products of two ~~one~~ single-digit numbers and the related division facts. *For example, the product 4 x 7 = 28 has related division facts 28 ÷ 7 = 4 and 28 ÷ 4 = 7.* | Edits made to clarify wording of the standard and increase student understanding of the relationship between multiplication and division for single digit numbers. |
| **3.OA.8** Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies, including rounding.[**Keep as Footnote**] Students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).] | Solve two-step word problems using the four operations for problems posed with whole numbers and having whole number answers. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies, including rounding. | Edited to incorporate accompanying footnotes into the standard itself to clarify the focus in this grade and student expectations. |
| **3.NF Develop understanding of fractions as numbers.** | Develop understanding of fractions as numbers for fractions with denominators 2, 3, 4, 6, and 8. | Edited to incorporate a footnote to clarify the expectation of the standards in this cluster. |
| **3.NF.1**  Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*. | Understand a fraction **1**/***b*** as the quantity formed by 1 part when a whole (a single unit) is partitioned into *b* equal parts; understand a fraction ***a***/***b*** as the quantity formed by *a* parts of size **1**/***b***. | Edit made to clarify the term whole as it relates to fraction expectations in grade 3. |
| **3.NF.2a**  Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line. | a. Represent a unit fraction **1**/***b,*** on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size **1**/***b*** and that ~~unit~~ the fraction, **1**/***b,*** is located **1**/***b*** of a whole unit from 0 ~~the endpoint of the part based at 0 locates the number 1/~~*~~b~~*on the number line. | Edit made to clarify the concept of a unit fraction and its location on a number line.  |
| **3.MD.2** Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. | Measure and estimate liquid volumes and masses of objects using standard metric units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same metric units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. | Edited to clarify the focus of this standard on the metric system.  |
| **3.MD.4** Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters. | Generate measurement data by measuring lengths of objects using rulers marked with halves and fourths of an inch. Record and show the data by making a line plot (dot plot), where the horizontal scale is marked off in appropriate units—whole numbers, halves, or ~~quarters~~ fourths. (See glossary for example.) | Edited to clarify student expectations for recording data and for representing data. A context example was provided to maintain the mathematical rigor of the standard. Dot plot added to glossary. |

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| **3. MD .6** Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units.) | Measure areas by counting unit squares (square cm, square m, square in, square ft, and non-standard units).  | Edited to clarify the term ‘improvised’ units. |
| **3.G.1** Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. | Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Compare and classify shapes by their sides and angles (right angle/non-right angle).Recognize rhombuses, rectangles, squares, and trapezoids as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. | Edited to clarify student expectations for comparing and classifying shapes and to clarify a focus on right and non-right angles.  |

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| Grade 4 |
| **2011 Standard** | **Proposed 2017 Standard****Revisions are in red text** | **Rationale for revision** |
| **4.OA.3.a (** See 4.NBT.MA. 5.a below) | Know multiplication facts and related division facts through 12 x 12. | Moved to the Operations and Algebraic Thinking domain as this standard extends the expectation of the standard 3.OA.7 |
| **4.NBT Generalize place value understanding for multi-digit whole numbers.** | Generalize place value understanding for multi-digit whole numbers less than or equal to 1,000,000. | Edited to incorporate a footnote to clarify the expectation of the standards in this cluster. |
| **4.NBT.1** Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.* | Recognize that in a multi-digit whole number, a digit in ~~one~~ any place represents 10 times as much as it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.* | Edited to improve the wording of the standard and understanding of place values. |
| **4.NBT Use place value understanding and properties of operations to perform multi-digit arithmetic.** | Use place value understanding and properties of operations to perform multi-digit arithmetic of whole numbers less than or equal to 1,000,000. | Edited to incorporate a footnote to clarify the expectation of the standards in this cluster. |
| **4.NBT.MA.5.a**  Know multiplication facts and related division facts through 12 x 12. | **~~4.NBT.MA.5.a~~ 4.OA.3.a**~~Know multiplication facts and related division facts through 12 x 12.~~ | Standard 4.NBT.MA.5.a moved to the Operations and Algebraic Thinking Domain because it extends an expectation of the standard 3.OA.7 - to know facts up to 10 x 10 from memory and fluently multiply and divide within 100.  |
| **4.NF Extend understanding of fraction equivalence and ordering.** | Extend understanding of fraction equivalence and ordering to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100. | Edited to incorporate a footnote to clarify the expectation of the standards in this cluster. |
| **4.NF.1**  Explain why a fraction *a*/*b* is equivalent to a fraction (*n*×*a*)/(*n*×*b*) by using visual fraction models, with attention to how the numbers and sizes of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. | Explain why a fraction ***a***/***b*** is equivalent to a fraction **(*n* × *a*)**/**(*n* × *b*)** by using visual fraction models, with attention to how the numbers and sizes of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions, including fractions greater than 1. | Edited to clarify the expectation of the standard for Grade 4 includes generating equivalent fractions for fractions greater than 1.  |
| **4.NF Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.** | Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers for fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100. | Edited to incorporate a footnote to clarify the expectation of the standards in this cluster. |
| **4.NF.3.a** Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. | Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. (The whole can be a set of objects). | Edited to clarify the meaning of the term ‘whole’ in Grade 4 to address the development of student understanding of the concept of ‘whole’ as it progresses through Grades 3-5.  |
| **4.NF.3.b** Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ;*  *2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.* | Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using drawings or visual fraction models. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ;*  *2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.* | Edited to add coherence to 4.NF.3.a. When ‘wholes’ are a set of objects, drawings are an appropriate model. |
| **4.NF.3.d** Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.  | Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using drawings or visual fraction models and equations to represent the problem.   | Edited to add drawings as an appropriate model to represent problems involving fractions. |
| **4. NF.6** Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as* ***62****/****100*** *; describe a length as 0.62 meters; locate 0.62 on a number line diagram.* | Use decimal notation to represent fractions with denominators 10 or 100. *For example, rewrite 0.62 as* ***62****/****100*** *; describe a length as 0.62 meters; locate 0.62 on a number line diagram.* | Edited the wording to clarify the expectation of the standard. |
| **4.MD.3**  Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.* | Apply the area and perimeter formulas for rectangles in real-world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*  *Note: When finding areas of rectangular regions answers will be in square units. For example, the area of a 1 cm x 1 cm rectangular region will be 1 square centimeter (1 cm2, students are not expected to use this notation.) When finding the perimeter of a rectangular region answers will be in linear units. For example, the perimeter of the region is: 1cm + 1cm + 1cm +1cm = 4 cm or 2(1cm) + 2(1cm) = 4 cm.* | Note added to clarify expectation for operations with quantities in this standard. |
| **4.MD.4** Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a* line plot *find and interpret the difference in length between the longest and shortest specimens in an insect collection.*  | Make a *line plot (dot plot)* representation to display a data set of measurements in fractions of a unit (**1**/**2**, **1**/**4**, **1**/**8**). Solve problems involving addition and subtraction of fractions by using information presented in line plots (dot plots). *For example, from a line plot (dot plot) find and interpret the difference in length between the longest and shortest specimens in an insect collection.*  | Edited to include the words dot plot because it is a more commonly used term and to help clarify the standard about the expectation for using this method for representing data and to maintain the mathematical rigor. Dot plot added to glossary. |

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| Grade 5 |
| **2011 Standard** | **Proposed 2017 Standard****Revisions are in red text** | **Rationale for Revision** |
| **5.OA.1**  Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. | Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols, *e.g., (6 x 30) + (6 x 1/2).* | Edited to include an example to clarify the expectation in the standard for grade 5. |
| **5.NBT.1**  Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. | Recognize that in a multi-digit number, including decimals, a digit in any ~~one~~ place represents 10 times as much as it represents in the place to its right and **1**/**10** of what it represents in the place to its left.  | Edited to include the term decimal. |
| **5.NBT.5**  Fluently multiply multi-digit whole numbers using the standard algorithm. | Fluently multiply multi-digit whole numbers (include 2 digit x 4 digit numbers and 3 digit x 3 digit numbers) using the standard algorithm.  | Edited to provide parameters that clarify the student learning expectation of the standard. |
| **5.NBT.7** Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. | Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction and multiplication and division; relate the strategy to a written method and explain the reasoning used. | Edited to retain focus in grade 5 on developing fluency in multi-digit multiplication and division.  |
| **5.NF.2**  Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem.Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example,* *recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2* ***.***  | Solve word problems involving addition and subtraction of fractions referring to the same whole (the whole can be a set of objects), including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem.Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example,* *recognize an incorrect result* ***2****/****5*** *+* ***1****/****2*** *=* ***3****/****7****, by observing that* ***3****/****7*** *<* ***1****/****2*** *.*  | Edited to clarify the concept of whole as being a single unit or a set of objects. |
| **5.NF.4.a**  Interpret the product (*a*/*b*) × *q* as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a* × *q* ÷ *b*. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with* *(2/3) × (4/5) = 8/15.* (In general, (*a*/*b*) × (*c*/*d*) = *ac*/*bd*.)  | Interpret the product (***a***/***b***) × *q* as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a* × *q* ÷ *b*. *For example, use a visual fraction model and/or area model to show (****2****/****3****) × 4 =****8****/****3****, and create a story context for this equation. Do the same with* *(****2****/****3****) × (****4****/****5****) =* ***8****/****15*** *.* (In general, (***a***/***b***) × (***c***/***d***) = ***ac***/***bd***.)  | Edited to maintain mathematical rigor by providing options for ways of showing conceptual understanding when multiplying fractions by whole numbers. |
| **5.NF.5.a**  Interpret multiplication as scaling (resizing), by:a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. | Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. *For example, which number is greater without multiplying: 225 or ¾ x 225; 11/50 or 3/2 x 11/50?* | Edited to provide an example to clarify the standard and to maintain the focus of the standard. |
| **5.NS.MA.1**  Use positive and negative integers to describe quantities such as temperature above/below zero, elevation above/below sea level, or credit/debit. | ~~Use positive and negative integers to describe quantities such as temperature above/below zero, elevation above/below sea level, or credit/debit.~~~~.~~ | Deleted to maintain focus in Grade 5 on fluency in multi-digit whole numbers and multiplying fractions. Integers are introduced in grade 6. |
| **5.MD.2**  Make a line plot to display a data set of measurements in fractions of a unit. Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.* | Make a line plot (dot plot) to display a data set of measurements in fractions of a unit. Use operations on fractions for this grade to solve problems involving information presented in a line plot (dot plot). *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.* | Edited to include the words dot plot because it is a more commonly used term and to help clarify the standard about the expectation for using this method for representing data and to maintain the mathematical rigor. Dot plot added to glossary. |
| **5.MD.4**  Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and non-standard units. | Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and ~~improvised~~ non-standard units. | Edited since ‘non-standard’ is the more frequently used term in measurement. |
| **5.MD.5.a**  Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. | Find the volume of a right rectangular prism with whole-number ~~side~~ edge lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. | Edited to clarify the focus of this standard to use edge length of a three dimensional prism when finding volume. |
| **5.MD.5.b**  Apply the formula *V* = *l* × *w* × *h* and *V* = *b* × *h* for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems.  | Apply the formula *V* = *l* × *w* × *h* and *V* = ~~b~~ *B* × *h* (where B stands for the area of the base) for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems.  | Edited to clarify the meaning of symbols used in formulas for volumes. |
| **5.G.4**  Classify two-dimensional figures in a hierarchy based on properties. | Classify two-dimensional figures in a hierarchy based on properties. For example, all rectangles are parallelograms, because they are all quadrilaterals with two pairs of oppositesides parallel. | Edited to include an example to clarify the expectation for student learning and to retain the mathematical rigor. |

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| Grade 6 |
| **2011 Standard** | **Proposed 2017 Standard****Revisions are in red text** | **Rationale for Revision** |
| **6.RP Understand ratio concepts and use ratio reasoning to solve problems.** | Understand ratio and rate concepts and use ratio and rate reasoning to solve problems. | Edited to emphasize rate in the ratio and proportional reasoning cluster since it is a critical area at this grade level. |
| **6.RP.1**  Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For e*xample, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”*  | Understand the concept of a ratio including the distinctions between part**:**part and part**:**whole and the value of a ratio; part/part and part/whole. Use ratio language to describe a ratio relationship between two quantities. *Examples: “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak”; “For every vote candidate A received, candidate C received ~~nearly~~ three votes”, meaning that candidate C received 3 out of every 4 votes or ¾ of all votes.*  | Edited to add coherence for developing student understanding of ratio relationships and to clarify the expectation for student learning is and to retain the mathematical rigor.  |
| **6.RP.2**  Understand the concept of a unit rate ***a***/***b*** associated with a ratio *a*:*b* with *b* ≠ 0, and use rate language in the context of a ratio relationship. *For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is ¾ cup of flour for each cup of sugar.” “We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger.”*  | Understand the concept of a unit rate ***a***/***b*** associated with a ratio *a*:*b* with *b* ≠ 0, and use rate language in the context of a ratio relationship, *including the use of units*.*Examples:“This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is ¾ cup of flour for each cup of sugar.” “We paid $75 for 15 hamburgers, which is a rate of ~~$~~5 dollars per hamburger.”*  | Edited to clarify the focus of this standard on the relationship between ratios and rates and units of rates.  |
| **6.RP.3.d**  Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. | Use ratio reasoning to convert measurement units within and between measurement systems; manipulate and transform units appropriately when multiplying or dividing quantities. *For example, Malik is making a recipe, but he cannot find his measuring cups! He has, however, found a tablespoon. His cookbook says that 1 cup = 16 tablespoons. Explain how he could use the tablespoon to measure out the following ingredients: 2 cups of flour, ½ cup sunflower seed, and 1¼ cup of oatmeal.(Illustrative Mathematics Project.)* | Edited to clarify the expectation of conversion between and within measurement systems. |
| **6.RP.MA.3.e** | **6.RP.~~MA~~.3.e** | Edited to remove “MA” (Massachusetts) designation. |
| **6.NS.4.** *Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4(9 + 2).*  | **6.NS.4.**  *Use prime factorization to find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two relatively prime ~~whole~~ numbers. ~~with no common factor.~~ For example, express 36 + 8 as 4(9 + 2).*  | Edited to incorporate a related number theory standard (6.NS.MA.4.a.) to clarify the expectations of this standard for Grade 6. |
| **6.NS.MA.4.a.** Apply number theory concepts, including prime factorization and relatively prime numbers, to the solution of problems. | **~~6.NS.MA.4.a.~~** ~~Apply number theory concepts, including prime factorization and relatively prime numbers, to the solution of problems.~~ | This standard was deleted and incorporated into standard 6.NS.4.  |
| **6.NS.5** Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge). Use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. | Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge). Use positive and negative numbers (**whole numbers, fractions, decimals**) to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. | Edited to clarify student expectations for positive and negative numbers in this standard include whole numbers, fractions, and decimals. |
| **6.G.MA.1.a**  Use the relationships among radius, diameter, and center of a circle to find its circumference and area. | ~~Use the relationships among radius, diameter, and center of a circle to find its circumference and area.~~ | The content of this standard has been incorporated into the revised standard 7.G.4 which is focused on the development of measurement concepts of circles. |
| **6.G.MA.1.b.** Solve real-world and mathematical problems involving the measurements of circles. | ~~Solve real-world and mathematical problems involving the measurements of circles.~~ | The content of this standard has been incorporated into the revised standard 7.G.4 which is focused on the development of measurement concepts of circles. |
| **6.SP.2** Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. | Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center (median, mean, and/or mode), spread (range interquartile range), and overall shape. | Edited to clarify the expectation for student learning about statistical measures in this standard. Mode is included to introduce comparisons of measures of center and their uses. The focus for summarizing and describing distributions does not include using mode. |
| **6.SP.MA.4.a** | **6.SP.~~MA.~~4.a** | Edited to remove “MA” (Massachusetts) designation. |
| **6.SP.5.c**  Summarize numerical data sets in relation to their context, such as by: c. Giving quantitative measures of center (median, and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. | Summarize numerical data sets in relation to their context, such as by:c. Giving quantitative measures of center (median, and/or mean), and variability(range and/or interquartile range ~~and/or mean absolute deviation~~), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. | Edited to create a coherent learning progression from grade 6 to grade 7 in relation to summarizing and describing distributions. In grade 6 the focus is on developing understanding of measures of center and spread. Mean absolute deviation is studied in grade 7. |

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| ­Grade 7 |
| **2011 Standard** | **Proposed 2017 Standard****Revisions are in red text** | **Rationale for Revision** |
| **7.RP.3** Use proportional relationships to solve multi-step ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.* | Use proportional relationships to solve multi-step ratio, rate, and percent problems. *Examples: simple interest, tax, ~~markups~~ price increases and ~~markdowns~~ discounts, gratuities and commissions, fees, percent increase and decrease, percent error.* | Edited to maintain coherence and focus in analyzing proportional relationships and ratio/rate learning progression. |
| **7.NS.1**  Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. | Apply and extend previous understandings of addition and subtraction to add and subtract integers and other rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. | Edited to specify the integer numbers as part of the system of rational numbers used in this standard. |
| **7.NS.1.a** Describe situations in which opposite quantities combine to make 0. *Examples: a hydrogen atom has 0 charge because its two constituents are oppositely charged.* | Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged; if you open a new bank account with a deposit of $30 and then withdraw $30, you are left with a $0 balance.* | Edited to provide an example connecting a financial real world context to the standard and providing for improved conceptual understanding.  |
| **7.NS.2**  Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. | Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide integers and other rational numbers. | Edited to specify the integer numbers as part of the system of rational numbers used in this standard. |
| **7.NS.3** Solve real-world and mathematical problems involving the four operations with rational numbers. | Solve real-world and mathematical problems involving the four operations with integers and other rational numbers. | Edited to clarify the distinction about different sets of numbers in the set of rational numbers that are an expectation in this standard. |
| **7.EE.1**  Apply properties of operations as strategies to add, subtract, factor, and expanding linear expressions with rational coefficients.  | Apply properties of operations ~~as strategies~~ to add, subtract, factor and expand linear expressions with rational coefficients. *For example, 4x + 2 = 2(2x +1) and -3(x – 5/3) = -3x + 5.* | Edited to clarify the expectation of this standard for factoring and expanding linear expressions. |
| **7.EE.2**  Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that “increase by 5%” is the same as “multiply by 1.05.”* | Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *Examples: a + 0.05a = 1.05a means that “increase by 5%” is the same as “multiply by 1.05.” A shirt at a clothing store is on sale for 20% off the regular price, “p”. The discount can be expressed as 0.2p. The new price for the shirt can be expressed as p – 0.2p or 0.8p.*  | Edited to give a context example to clarify the expectation of the standard and mathematical rigor of the standard. |
| **7.NS.MA.4.c** | **7.NS.~~MA.~~4.c** | Edited to remove “MA” (Massachusetts) designation. |
| **7.G.2** Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. | Draw (freehand, with ruler and protractor, and with technology) two-dimensional geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. | Edited to clarify the expectation for types of shapes to be drawn. |
| **7.G.3**  Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.  | Describe the shape of the two-dimensional face of the figure that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. | Edited to improve the clarity of the standard.  |
| **7.G.4** Know the formulas for the area and circumference of a circle and solve problems; give an informal derivation of the relationship between the circumference and area of a circle. | ~~Know the formulas for the area and circumference of a circle and solve problems; give an informal derivation of the relationship between the circumference and area of a circle.~~Circles and measurement:1. Know that a circle is **a two-dimensional shape** created by connecting all of the points equidistant from a fixed point called the center of the circle.
2. ~~Explore~~ Understand and describe the relationships among the radius, diameter, and circumference of a circle.
3. ~~Explore~~ Understand and describe the relationship among the radius, diameter, and area of a circle.
4. Know the formulas for the area and circumference of a circle and solve problems.
5. Give an informal derivation of the relationship between the circumference and area of a circle.
 | Edited to improve the coherence and focus of the learning progression and expectations for students to know, calculate, and solve problems related to circumferences and areas of circles. |
| **7.G.5**  Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and use them to solve simple equations for an unknown angle in a figure.  | Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write simple equations and use them to solve ~~simple equations~~ for an unknown angle in a figure.   | Edited to improve the clarity of the standard. |
| **7.G.MA.7**  Solve real-world and mathematical problems involving the surface area of spheres. | ~~Solve real-world and mathematical problems involving the surface area of spheres.~~ | Deleted to maintain the focus in grade 7 on finding the surface area of objects that can be decomposed into two-and-three dimensional polygonal shapes.  |
| **7.SP.3**  Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.* | Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team and both distributions have similar variability (mean absolute deviation) of about 5cm. The difference between the mean heights of the two teams (10 cm) is about twice the variability (5 cm) on either team. On a dot plot, the separation between the two distributions of heights is noticeable.*  | Edited to improve the clarity of the standard. The wording of the example was edited to reinforce the intent of drawing informal comparative inferences about two populations. |

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| Grade 8 |
| **2011 Standard** | **Proposed 2017 Standard****Revisions are in red text** | **Rationale for Revision** |
| **8.EE.8.b**  Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6.* | Solve systems of two linear equations in two variables algebraically (using substitution and elimination strategies), and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6.* | Edited to clarify student expectations for solving systems of equations in Grade 8 and maintain the mathematical rigor of the standard. |
| **8.G.1.a.b.c**  Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length.b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.  | Verify experimentally the properties of rotations, reflections, and translations: a. Lines are ~~taken~~ transformed to lines, and line segments to line segments of the same length. b. Angles are ~~taken~~ transformed to angles of the same measure. c. Parallel lines are ~~taken~~ transformed to parallel lines.  | Edited the wording to clarify the focus and expectation of the standard. |
| **8.G.6**  Explain a proof of the Pythagorean Theorem and its converse.  | a. Understand the relationship among the sides of a right triangle. b. Analyze and justify ~~Explain a proof of~~ the Pythagorean Theorem and its converse using pictures, diagrams, narratives or models. | Edited to clarify student expectations for understanding and explaining the Pythagorean Theorem in Grade 8 and to maintain the mathematical rigor of the standard. Proving theorems are introduced in high school mathematics. In grade 8 students will demonstrate their understanding of the Pythagorean Theorem by analyzing and justifying the relation among the sides of a right triangle and applying the Pythagorean Theorem to solve problems in different contexts.  |

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| **High School Model Course Standards: Several standards are repeated in two or more model courses within a pathway (Traditional and Integrated). Footnotes in the 2011 Framework clarified aspects of the duplicated standard relevant to each course. In the 2017 Framework, most model course footnotes have been removed and incorporated into the standard itself**. |
| High School Model Algebra I: |
| **2011 Standard** | **Proposed 2017 Standard****Revisions are in red text** | **Rationale for Revision** |
| **N-Q.MA.3.a.**Describe the effects of approximate error in measurement and rounding on measurements and on computed values from measurements. Identify significant figures in recorded measures and computed values based on the context given and the precision of the tools used to measure. | ~~N-Q.MA.3.a Describe the effects of approximate error in measurement and rounding on measurements and on computed values from measurements. Identify significant figures in recorded measures and computed values based on the context given and the precision of the tools used to measure~~.  | Standard deleted from Algebra I. This standard is an expectation for the Model Geometry Course. |
| **A-SSE Interpret the structure of expressions.** | Interpret the structure of linear, quadratic, and exponential expressions with integer exponents. | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this cluster for Algebra I. |
| **A-SSE.2** Use the structure of an expression to identify ways to rewrite it. *For example, see x4 – y4 as (x2)2 – (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 – y2)(x2+ y2).* | Use the structure of an expression to identify ways to rewrite it. *For example, see ~~x~~~~4~~ ~~– y~~~~4~~ ~~as (x~~~~2~~~~)~~~~2~~~~– (y~~~~2~~~~)~~~~2~~ ~~thus recognizing it~~* *(x + 2)2 – 9as a difference of squares that can be factored as ((x + 2) + 3)((x + 2) – 3). ~~(x~~~~2~~~~– y~~~~2~~~~)(x~~~~2~~~~+ y~~~~2~~~~).~~* | Edited the example in the standard to align with the focus and expectations for Algebra I. |
| **A-APR.1** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. | Understand that polynomials form a system analogous to the integers, namely, they are closed under certain operations. a. Perform operations on polynomial expressions (addition, subtraction, multiplication), and compare the system of polynomials to the system of integers. b. Factor and/or expand polynomial expressions, identify and combine like terms, and apply the Distributive Property.  | Edited to emphasize the similarities in the systems of polynomials and the integers and to incorporate a footnote that clarifies the expectation for operating with polynomial expressions of this standard for Algebra I.  |
| **A-CED.1**  Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. | Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear, ~~and~~ quadratic, ~~functions and simple rational~~ and exponential functions with integer exponents. | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this cluster for Algebra I. |
| **A-CED.3**  Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* | Represent constraints by linear equations or inequalities, and by systems of linear equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* | Edited to incorporate a footnote that clarifies the focus and expectation of this standard for Algebra I. |
| **A-CED.4**  Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm’s lawV = IR to highlight resistance R.* | Rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations (Properties of equality). *For example, rearrange Ohm’s lawto solve ~~to highlight~~ for voltage V.* Manipulate variables in formulas used in financial contexts such as for simple interest,.★ | Edited to add a financial application to the standard and to clarify the expectations of this standard for Algebra I. |
| **A-REI.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. | Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify or refute a solution method. | Edited to be coherent with Standard for Mathematical Practice 3; *Construct viable arguments and critique the reasoning of others.*  |
| **A-REI.MA.3.a**  | A-REI.~~MA~~.3.a  | Edited to remove “MA” (Massachusetts) designation. |
| **A-REI.MA.4.c**  Demonstrate an understanding of the equivalence of factoring, completing the square, or using the quadratic formula to solve quadratic equations. | ~~Demonstrate an understanding of the equivalence of factoring, completing the square, or using the quadratic formula to solve quadratic equations.~~ | The content of this standard is contained in the standard A-REI.4.b.  |
| **A-REI.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).  | Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Show that any point on the graph of an equation in two variables is a solution to the equation. | Edited to include an expectation of applying the understanding between graphs, equations, and solutions of equations. |
| **A-REI.11** Explain why the *x*-coordinates of the points where the graphs of the equations *y* = *f*(*x*) and *y* = *g*(*x*) intersect are the solutions of the equation *f*(*x*) = *g*(*x*); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f*(*x*) and/or *g*(*x*) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. | Explain why the *x*-coordinates of the points where the graphs of the equations *y* = *f*(*x*) and *y* = *g*(*x*) intersect are the solutions of the equation *f*(*x*) = *g*(*x*); find the solutions approximately, e.g., using technology to graph the functions and make tables of values. ~~or find successive approximations.~~ Include caseswhere *f*(*x*) and/or *g*(*x*) are linear ~~polynomial, rational, absolute value,~~ and exponential~~, and logarithmic~~ functions. | Edited to incorporate a footnote that clarifies the focus and expectation of this standard for Algebra I. This standard will be further explored in Algebra II. |
| **A-REI.12** Graph the solutions to a linear inequality in two variables as a half-plane, and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. | Graph the solutions ~~to~~ of a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set ~~to~~ of a system of linear inequalities in two variables as the intersection of the corresponding half-planes. | Edited to clarify wording of the standard to be precise and avoid misconceptions. |
| **F-IF.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. | Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. *For example, given a function representing a car loan, determine the balance of the loan at different points in time.* | Edited to provide a financial context for this Model Algebra I standard. |
| **F-IF Interpret functionsthat arise in applications in terms of the context.** | Interpret linear, quadratic, and exponential functions with integer exponents that arise in applications in terms of the context. | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this cluster for Algebra I. |
| **F-IF.4**  For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior. ~~and periodicity~~.* | Edited to be consistent with the focus for Algebra I expectations for functions (linear, quadratic, and exponential functions) as noted in the cluster footnote. Periodic functions are studied in Algebra II.  |

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| **F-IF.7.b.** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.  | Graph ~~square root, cube root, and~~ piecewise-defined functions, including step functions and absolute value functions.  | Edited to incorporate a footnote that clarifies the focus and expectation of this standard for Algebra I. This standard will be further explored in Algebra II. |
| **F-IF.7.e**. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. | Graph exponential ~~and logarithmic~~ functions, showing intercepts and end behavior~~., and trigonometric functions, showing period, midline, and amplitude~~ | Edited to incorporate a footnote that clarifies the focus and expectation of this standard for Algebra I. This standard will be further explored in Algebra II. |
| **F-IF.8.a**  Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.  | Use the process of factoring and completing the square in a quadratic function to show zeros, maximum/minimum ~~extreme~~ values, and symmetry of the graph, and interpret these in terms of a context.  | Edited to clarify the wording of the standard to be consistent with the key features of linear, exponential and quadratic functions. |
| **F-IF.8.b**  Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, and y = (1.2)t/10, and classify them as representing exponential growth or decay.* | Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, and y = (1.2)t/10, and classify them as representing exponential growth or decay. Apply to financial situations such as identifying appreciation and depreciation rate for the value of a house or car some time after its initial purchase:.* | Edited to provide an example that connects real world financial application with mathematical expressions. |
| **F-IF.8.MA.8c**. Translate among different representations of functions and relations: graphs, equations, point sets, and tables. | ~~Translate among different representations of functions and relations: graphs, equations, point sets, and tables.~~ | The contents of this standard were incorporated into standard F-IF.9 that has a similar focus on functions represented in different ways.  |
| **F-IF.9**  Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* | Translate among different representations of functions ~~and relations: graphs, equations, point sets, and tables.~~ (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way. *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* | Edited to incorporate standard F-IF.MA.8.c which has a related expectation for representing functions in different ways. |
| **F-IF.MA.10** Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, exponential, or trigonometric. | ~~Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational,~~ ~~logarithmic, exponential, or trigonometric.~~ | Moved from Algebra I and added to Algebra II. The student expectations for learning this content; rational, trigonometric, and logarithmic functions, is the focus in Algebra II. The focus in Algebra I is linear, exponential and quadratic functions. |
| **F-BF.1** Write a functions that describes a relationship between two quantities. | Write linear, quadratic, and exponential functions that describe a relationship between two quantities. | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this Algebra I cluster.  |
| **F-BF.3**  Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *kf*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* | Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *kf*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Include linear, quadratic, exponential, and absolute value functions. Utilize ~~using~~ technology to experiment with cases and illustrate an explanation of the effects on the graph. ~~using~~. *~~Include recognizing even and odd functions from their graphs and algebraic expressions for them.~~* | Edited to clarify expectation of the standard and to reinforce the use technology to explore effects on the graph when varying k and the function. |
| **F-BF.4.a** Find inverse functions. Solve an equation of the form *f*(*x*) = *c* for a simple function *f* that has an inverse and write an expression for the inverse. *For example, f(x) =2x3 or f(x) = (x + 1)/(x − 1) for x ≠ 1.* | Find inverse functions algebraically and graphically.a. Solve an equation of the form *f*(*x*) = *c* for a ~~simple~~ linear function *f* that has an inverse and write an expression for the inverse. *~~For example, f(x) =2x~~~~3~~ ~~or f(x) = (x + 1)/(x − 1) for x ≠ 1.~~* | Edited to incorporate a footnote that clarifies the focus and expectations of this standard for Algebra I.  |
| **F-LE.5** Interpret the parameters in a linear or exponential function in terms of a context. | Interpret the parameters in a linear or exponential function (of the form f*(x) = bx + k)* in terms of a context. | Edited to incorporate a footnote that clarifies the focus and expectations of this standard for Algebra I. |
| **S-ID Summarize, represent, and interpret data on a single count or measurement variable.**  | Summarize, represent, and interpret data on a single count or measurement variable. Use calculators, spreadsheets, and other technology as appropriate. | Edited to emphasize appropriate technology use in this cluster to support understandings for interpreting the data.  |
| **S-ID.4** Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.  | ~~Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.~~ | This standard is an expectation of the Model Algebra II course. In Algebra I students are introduced to summarizing, representing and interpreting data. |
| **S-ID.6.** a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Uses given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* Footnote: Linear focus; discuss as general principal in Algebra I. | Fit a linear function to the data and use the fitted function~~s~~ to solve problems in the context of the data. Use given functions fitted to data or choose a function suggested by the context (emphasize linear~~, quadratic,~~ and exponential models)*.*~~Footnote: Linear focus; discuss as general principal in Algebra I.~~ | Edited to clarify the focus and expectations of this standard for Algebra I. Functions already fitted to the data may be linear, quadratic, and/or exponential. |

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| High School Model Geometry |
| **2011 Standard** |  **Proposed 2017 Standard****Revisions are in red text** | **Rationale for Revision** |
| **N-Q.2.** Define appropriate quantities for the purpose of descriptive modeling. | ~~Define appropriate quantities for the purpose of descriptive modeling.~~ | Standard deleted from the Geometry Model Course. This standard is an expectation of the Model Algebra I Course. |
| **N-Q.3MA.a** | **N-Q.3~~MA~~.a** | Edited to remove “MA” (Massachusetts) designation. |
| **G-CO Prove geometric theorems.** | Prove geometric theorems and, when appropriate, the converse of theorems. | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this Geometry cluster. |
| **G-CO.9** Prove theorems about lines and angles. *Theorems include: vertical angles are* *congruent; when a transversal crosses parallel* *lines, alternate interior angles are congruent* *and corresponding angles are congruent;* *points on a perpendicular bisector of a line* *segment are exactly those equidistant from the* *segment’s endpoints.* | Prove theorems about lines and angles. *Theorems* *include: vertical angles are congruent; when a transversal* *crosses parallel lines, alternate interior angles are* *congruent and corresponding angles are congruent, and* *conversely prove lines are parallel; points on a* *perpendicular bisector of a line segment are exactly those equidistant from the segment’s endpoints.* | Edited to include the expectation for proving converses of theorems in Geometry and to improve the mathematical rigor. |
| **G-CO.10** Prove theorems about triangles. *Theorems include: measures of interior angles* *of a triangle sum to 180°; base angles of* *isosceles triangles are congruent; the* *segment joining midpoints of two sides of a* *triangle is parallel to the third side and half the* *length; the medians of a triangle meet at a* *point.*  | Prove theorems about triangles. *Theorems include:* *measures of interior angles of a triangle sum to 180°;* *base angles of isosceles triangles are congruent, and* *conversely prove a triangle is isosceles; the segment* *joining midpoints of two sides of a triangle is parallel to* *the third side and half the length; the medians of a* *triangle meet at a point.*  | Edited to include the expectation for proving converses of theorems in Geometry and to improve the mathematical rigor. |

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| **G-CO**.~~MA~~.**11.a**  Prove theorems about polygons. *Theorems include: measures of interior and exterior angles, properties of inscribed polygons.* | **G-CO.11.a** Prove theorems about polygons. *Theorems include the measures of interior and exterior angles.* *~~properties of inscribed polygons.~~* Apply properties of polygons to the solutions of mathematical and contextual problems. | Edited to provide focus and coherence within the standard and preserve the mathematical rigor. Inscribed polygons are deleted in this standard and are included in standard G-C.3. |
| **G-C.3**  Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle, | Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateraland other polygons inscribed in a circle.  | Edited to clarify the focus and expectations for this standard to polygons more generally. |
| **G-C.MA.3.a**  Derive the formula for the relationship between the number of sides and sums of the interior and sums of the exterior angles of polygons and apply to the solutions of mathematical and contextual problems. | ~~Derive the formula for the relationship between the number of sides and sums of the interior and sums of the exterior angles of polygons and apply to the solutions of mathematical and contextual problems.~~ | This standard was deleted as the expectations of this standard, knowing the relationship between the sides and angles of polygons is contained in standard G-CO.11.a. |
| **G-GPE.4**  Use coordinates to prove simple geometric theorems algebraically*. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, ) lies on the circle centered at the origin and containing the point (0, 2).* | Use coordinates to prove simple geometric theorems algebraically including the distance formula and its relationship to the Pythagorean Theorem*. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, ) lies on the circle centered at the origin and containing the point (0, 2).* | Edited to provide deeper focus of important and specific connections with other standards within the Model Geometry course and preserve mathematical rigor. |
| **G-MG.MA.4** | **G-MG.~~MA~~.4** | Edited to remove MA designation |
| **S-CP Understand independence and conditional probability and use them to interpret data.** | Understand independence and conditional probability and use them to interpret data from simulations or experiments. | Edited to incorporate a footnote that clarifies the focus and expectations of the standard in this Geometry cluster. |
| **S-MD.6.** (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).  | (~~+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).~~  | This (+) standard is now included in the Model Algebra II course and is optional.  |
| **S-MD.7 (+)** Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). | ~~Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game and replacing with an extra skater).~~ | This (+) standard is now included in the Model Algebra II course and is optional.  |

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| High School Model Algebra II |
| **2011 Standard** | **Proposed 2017 Standard****Revisions are in red text** | **Rationale for Revision** |
| **A-SSE Interpret the structure of expressions.** | Interpret the structure of exponential, polynomial and rational expressions. | Edited to define the student learning expectations of this standard for the Model Algebra II course. |
| **A-SSE.2**  Use the structure of an expression to identify ways to rewrite it. *For example, see x4 – y4 as (x2)2 – (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 – y2)(x2 + y2).* | Use the structure of an expression to identify ways to rewrite it. *For example, see x4 – y4 as (x2)2 – (y2)2, thus recognizing it as a difference of squares that can be factored (x2 – y2)(x2 + y2) and further factored (x - y) x + y)(x - yi)(x + yi).* | Edited the example in the standard to align with the focus and expectations for Algebra II. |
| **A-APR.1** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. | Understand that polynomials form a system analogous to the integers, namely, they are closed under certain operations. a. Perform operations on polynomial expressions (addition, subtraction, multiplication, and division), and compare the system of polynomials to the system of integers when performing operations.  | Edited to incorporate standard A-APR.MA.1.a and to clarify that the content of Algebra II includes division of polynomials and the property of closure.  |
| **A-APR.MA.1.a** Divide polynomials. | ~~Divide polynomials.~~ | The content of this standard (dividing polynomials) has been incorporated into the revised Algebra II standard A-APR.1. |
| **A-APR.5** (+) Know and apply the Binomial Theorem for the expansion of (*x* + *y*)*n* in powers of *x* and *y* for a positive integer *n*, where *x* and *y* are any numbers, with coefficients determined for example by Pascal’s Triangle. | (+) Know and apply the Binomial Theorem for the expansion of (*x* + *y*)*n* in powers of *x* and *y* for a positive integer *n*, where *x* and *y* are any numbers, with coefficients determined for example by Pascal’s Triangle. | Edited to remove a footnote about proving the Binomial Theorem. Proof of this theorem is an expectation of the Model Precalculus course |

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| **A-CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions | Create equations and inequalities in one variable and use them to solve problems. Include equations arising from ~~linear and quadratic functions, and~~ simple root and rational functions and exponential functions. | Edited to clarify the focus and expectations of the standard for Algebra II. |
| **A-CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.  *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* | Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent equations describing satellites orbiting earth and constraints on earth’s size and atmosphere.*  | Edited the example in this standard to match the expectation of this standard in Algebra II as it progresses from Algebra I.  |
| **A-CED.4**  Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm’s lawV = IR to highlight resistance R.* | ~~Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.~~ *~~For example, rearrange Ohm’s law~~~~V = IR to highlight resistance R.~~* | Deleted the standard in Model Algebra II. Solving equations is a focus in the model Algebra I course. See revised standard A-CED.4 in Algebra I. |
| **A-REI.11** Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. | Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are ~~linear,~~ polynomial, rational, ~~absolute value, exponential,~~ and logarithmic functions. | Edited to clarify the focus for the types of functions for this standard in Algebra II. |
| **F-IF Interpret functions that arise in applications in terms of the context**. | Interpret functions that arise in applications in terms of the context. (Include polynomial, rational, square and cube root, trigonometric, and logarithmic functions.) |  Edited to clarify the focus for the types of additional functions studied in Algebra II. |

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| **F-IF.7.b.** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.  | Graph square root and cube root~~, and piecewise-defined functions, including step functions and absolute value~~ functions.  | Edited to clarify the focus and expectation of functions applied in this standard for Algebra II.  |
| **NEW TO ALGEBRA II****F-IF.8.a**  Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph and interpret these in terms of a context.  | Use the process of factoring ~~and completing the square in a quadratic~~  in ~~a~~ polynomial and rational functions to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. | Edited to clarify the focus and expectation of functions applied in this standard for Algebra II.  |
| **F-IF.8.MA.8c**. Translate among different representations of functions and relations: graphs, equations, point sets, and tables. | ~~Translate among different representations of functions and relations: graphs, equations, point sets, and tables.~~ | The contents of this standard were incorporated into standard F-IF.9 that has a similar focus on functions represented in different ways.  |
| **F-IF.9**  Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* | Translate among different representations of functions ~~and relations: graphs, equations, point sets, and tables.~~ (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way. *For example, given a graph of one ~~quadratic~~ polynomial function and an algebraic expression for another, say which has the larger relative maximum and/or smaller relative minimum.*  | Edited to clarify the focus and expectations for this standard In Algebra II. Also edited to incorporate standard F-IF.MA.8.c which has a related expectation for representing functions in multiple ways. |
| **F-IF.MA.10** **Standard new to Model Algebra II** | **F-IF.10** Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, exponential, or trigonometric. | This standard was moved from Model Math I and added to Math III. The expectation for student learning of rational, trigonometric, and logarithmic functions in this standard is a focus of the Math III model course. |
| **F-BF.1** Write a functions that describes a relationship between two quantities. | Write a function (simple rational, radical logarithmic, and trigonometric functions) that describes a relationship between two quantities. | Edited to reflect the focus and expectations of this standard for Algebra II. |
| **F-BF.3**  Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *kf*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* | Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *kf*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Include simple rational, radical, logarithmic, and trigonometric functions. Utilize ~~using~~ technology to experiment with cases and illustrate an explanation of the effects on the graph. ~~using~~. Include recognizing even and odd functions from their graphs and algebraic expressions for them. | Edited to clarify the focus and expectation of this standard for Algebra II and to reinforce the use of technology to explore effects on graphs of functions when varying k. |
| **F-BF.4.a** Find inverse functions. Solve an equation of the form *f*(*x*) = *c* for a simple function *f* that has an inverse and write an expression for the inverse. *For example, f(x) =2x3 or f(x) = (x + 1)/(x − 1) for x ≠ 1.* | Find inverse functions algebraically and graphically.a. Solve an equation of the form *f*(*x*) = *c* for a ~~simple~~ linear function *f* that has an inverse and write an expression for the inverse. *For example, f(x) =2x3 or f(x) = (x + 1)/(x − 1) for x ≠ 1.* | Edited to include finding inverse of functions both algebraically by solving equations and graphically. |
| **S-ID Summarize, represent, and interpret data on a single count or measurement variable.**  | Summarize, represent, and interpret data on a single count or measurement variable. Use calculators, spreadsheets, and other technology as appropriate. | Edited to emphasize appropriate technology use in this cluster to support understandings for interpreting the data.  |
| **S-MD.7** (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game | (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game and replacing the goalie with an extra skater) | Edited to incorporate a footnote to clarify the example in the standard. |

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| High School Model Integrated Math I |
| **2011 Standard** | **Proposed 2017 Standard****Revisions are in red text** | **Rationale for Revision** |
| **N-Q.MA.3.a.**Describe the effects of approximate error in measurement and rounding on measurements and on computed values from measurements. Identify significant figures in recorded measures and computed values based on the context given and the precision of the tools used to measure. | ~~Describe the effects of approximate error in measurement and rounding on measurements and on computed values from measurements. Identify significant figures in recorded measures and computed values based on the context given and the precision of the tools used to measure.~~ | Standard deleted from Model Math I course. This standard is an expectation for the Model Math II Course. |
| **A-SSE Interpret the structure of expressions.** | Interpret the structure of linear and exponential expressions with integer exponents. | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this Model Math I cluster. |
| **A-CED.1**  Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. | Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and ~~quadratic functions, and simple rational and~~ and exponential functions with integer exponents. | This standard was edited to incorporate a footnote that clarifies the expectation of this standard for Math I.  |
| **A-CED.3**  Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* | Represent constraints by linear equations or inequalities, and by systems of linear equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* | Edited to clarify the focus and to incorporate a footnote that clarifies the expectation of this standard for Math I. |

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| **A-CED.4**  Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm’s lawV = IR to highlight resistance R.* | Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations (properties of equality). *For example, rearrange Ohm’s lawV = IR ~~to highlight resistance~~ to solve for resistance R. Manipulate variables in formulas used in financial contexts, such as for simple interest.* | Edited to clarify the expectations of this standard for Math I and to add an application to the standard related to finance. |
| **A-REI.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify or refute a solution method. | Explain each step in solving a simple linear equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify or refute a solution method. | Edited to incorporate a footnote that clarified the focus and expectations for the types of equations in this standard for Math I. |
| **A-REI.MA.3.a**  | A-REI.~~MA~~.3.a  | Edited to remove “MA” (Massachusetts) designation. |
| **A-REI.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).  | Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Show that any point on the graph of an equation in two variables is a solution to the equation. | Edited to incorporate a footnote that clarified the focus and expectations for the types of equations in this standard for Math I. An expectation of applying that understanding between graphs, equations, and solutions was added. |
| **A-REI.11** Explain why the *x*-coordinates of the points where the graphs of the equations *y* = *f*(*x*) and *y* = *g*(*x*) intersect are the solutions of the equation *f*(*x*) = *g*(*x*); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f*(*x*) and/or *g*(*x*) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. | Explain why the *x*-coordinates of the points where the graphs of the equations *y* = *f*(*x*) and *y* = *g*(*x*) intersect are the solutions of the equation *f*(*x*) = *g*(*x*); find the solutions approximately, e.g., using technology to graph the functions and/or make tables of values. ~~or find successive approximations.~~ Include cases where *f*(*x*) and/or *g*(*x*) are linear polyn~~omial, rational,~~ ~~absolute value~~ and exponential ~~and logarithmic~~ functions. | Edited to incorporate a footnote that clarifies the focus and expectations in this standard for Model Math I. |
| **A-REI.12** Graph the solutions to a linear inequality in two variables as a half-plane, and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. | Graph the solutions ~~to~~ of a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set ~~to~~ of a system of linear inequalities in two variables as the intersection of the corresponding half-planes. | Edited to clarify wording of the standard be more precise.  |
| **F-IF.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. | Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. *For example, given a function representing a car loan, determine the balance of the loan at different points in time.* | Edited to provide a financial context for this Model Math I standard. |
| **F-IF Interpret functionsthat arise in applications in terms of the context.** | Interpret linear and exponential functionshaving integer exponents that arise in applications in terms of the context. | Edited to incorporate a footnote to clarify the types of functions being interpreted in Math I. |
| **F-IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior. ~~periodicity~~* | Edited to be consistent with the focus for Math I expectations for functions (linear and exponential functions). Periodic functions are studied in Math III. |
| **F-IF.7.a** Graph linear and quadratic functions and show intercepts, maxima, and minima. | Graph linear ~~and quadratic~~ functions and show intercepts~~. maxima, and minima~~. | Edited to address a footnote that clarifies the focus on linear and exponential functions in Math I. |
| **F-IF.7 e.** Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. | Graph exponential ~~and logarithmic~~ functions, showing intercepts and end behavior.  ~~and trigonometric functions, showing period, midline, and amplitude~~ | Edited to incorporate a footnote that clarifies the focus and expectations in this standard for Model Math I. |
| **F-IF.9**  Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. | Translate among different representations of functions ~~and relations: graphs, equations, point sets, and tables.~~ (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way. *For example, given a graph of one ~~quadratic~~ exponential function and an algebraic expression for another, say which has the larger ~~maximum.~~y~~-~~Intercept.* | Edited to incorporate a footnote that clarifies the focus and expectations in this standard for Model Math I. |
| **F-IF.MA.10** Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, exponential, or trigonometric. | ~~Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational,~~ ~~logarithmic, exponential, or trigonometric.~~ | Moved from Math I and added to Math III. The student expectations for learning this content; rational, trigonometric, and logarithmic functions, is the focus in Math III. The focus in Math I is linear and exponential functions. |
| **F-BF.1** Write functions that describes a relationship between two quantities. | Write linear and exponential functions that describe relationships between two quantities. | Edited to incorporate a footnote that clarifies the focus and expectations of this standard in Model Math I. |
| **F-BF.3**  Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *kf*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* | Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *kf*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Include linear and exponential models. (Focus on vertical translations for exponential functions). Utilize technology to experiment with cases and illustrate an explanation of the effects on the graph. *~~Include recognizing even and odd functions from their graphs and algebraic expressions for them.~~* | Edited to incorporate a footnote that clarifies the focus and expectations in this standard for Model Math I and to reinforce the use of technology. |
| **F-LE Construct and compare linear, quadratic, and exponential models and solve problems.** | Construct and compare linear~~, quadratic,~~ and exponential models and solve problems. | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this cluster for Math I. |
| **F-LE.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. | Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly. ~~quadratically, or (more generally) as a polynomial function~~ | Edited to incorporate a footnote that clarifies the focus and expectations of the standards for Math I. |
| **F-LE.5** Interpret the parameters in a linear or exponential function in terms of a context. | Interpret the parameters in a linear or exponential function (of the form f*(x) = bx + k)* in terms of a context. | Edited to incorporate a footnote that clarifies the focus and expectations in this standard for Model Math I. |
| **G-GPE.4**  Use coordinates to prove simple geometric theorems algebraically*. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, ) lies on the circle centered at the origin and containing the point (0, 2).* | ~~Use coordinates to prove simple geometric theorems algebraically including the distance formula and its relationship to the Pythagorean Theorem~~*~~. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, ) lies on the circle centered at the origin and containing the point (0, 2).~~* | ~~Edited to incorporate a footnote that connects specific content with other standards within the Math I course.~~This standard was moved to Math II from Math I where it continues the progression of expressing geometric properties with equations. |
| **S-ID Summarize, represent, and interpret data on a single count or measurement variable.**  | Summarize, represent, and interpret data on a single count or measurement variable. Use calculators, spreadsheets, and other technology as appropriate. | Edited to emphasize appropriate technology use in this cluster to support understandings for interpreting the data.  |
| **S-ID.6.** a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Uses given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* Footnote: Focus on linear applications; learn as a general principle to be expanded in Math II and III. | Fit a linear function to the data and use the fitted function~~s~~ to solve problems in the context of the data. Uses given functions fitted to data or choose *a function suggested by the context. Emphasize linear~~, quadratic,~~ and exponential models.**.*Footnote: ~~Focus on linear applications; learn as a general principle to be expanded in Math II and III.~~ | Edited to clarify the focus and expectations of this standard for Math I. Functions already fitted to the data may be linear, quadratic, and/or exponential. |

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| High School Model Integrated Math II |
| **2011 Standard** | **Proposed 2017 Standard****Revisions are in red text** | **Rationale for Revision** |
| **N-Q.MA.3.a** Standard new to Math II . | **N-Q.~~MA.~~3.a Describe** the effects of approximate error in measurement and rounding on measurements and on computed values from measurements. Identify significant figures in recorded measures and computed values based on the context given and the precision of the tools used to measure. | The expectation for student learning of approximate error in measurement in this standard is a focus of the Math II model course. |
| **N-CN.8**. (+) Extend polynomial identities to the complex numbers. For example, rewrite *x^2 + 4 as (x + 2i)(x – 2i).* | ~~(+) Extend polynomial identities to the complex numbers. For example, rewrite~~ *~~x^2 + 4 as (x + 2i)(x – 2i).~~* | This (+) standard is included as optional in the Model Math III course. |
| **N-CN.9.** (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. | ~~(+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.~~ | This (+) standard is included as optional in the Model Math III course. |
| **A-SSE Interpret the structure of expressions**. | Interpret the structure of quadratic and exponential expressions. | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this cluster for Math II. |
| **A-SSE.2**  Use the structure of an expression to identify ways to rewrite it. *For example, see x4 – y4 as (x2)2 – (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 – y2)(x2+ y2).* | Use the structure of an expression to identify ways to rewrite it. For example, see ~~x~~~~4~~ ~~– y~~~~4~~ ~~as (x~~~~2~~~~)~~~~2~~ ~~– (y~~~~2~~~~)~~~~2~~~~, thus recognizing it as a difference of squares that can be factored as (x~~~~2~~ ~~– y~~~~2~~~~)(x~~~~2~~ ~~+ y~~~~2~~~~)~~ (x + 2)2 – 9 as a difference of squares that can be factored as ((x + 2) + 3)((x + 2) – 3).  | Edited the example in the standard to clarify the focus and expectations of the standard for Math II students. |
| **A-SSE Write expressions in equivalent forms to solve problems.** | Write quadratic and exponential expressions in equivalent forms to solve problems. | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this cluster for Math II. |

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| **A-APR.1** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.*Note: See page 50 for a revision of this conceptual category standard.* | Understand that polynomials form a system analogous to the integers, namely, they are closed under certain operations. a. Perform operations on polynomial expressions (addition, subtraction, multiplication), and compare the system of polynomials to the system of integers. b. Factor and/or expand polynomial expressions, identify and combine like terms, and apply the Distributive Property.  | Edited to incorporate a footnote to clarify the expectation of the standard. In Math II, students are introduced to operations of addition, subtraction, and multiplication on polynomials. The study of polynomials is continued in Math III. |
| **A-CED.1**  Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. | Create equations and inequalities in one variable and use them to solve problems. Include equations arising from ~~linear and~~ quadratic functions ~~and simple rational~~ and exponential functions. | This standard was edited to incorporate a footnote that clarifies the expectation of this standard for Math II.  |
| **A-CED.4**  Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm’s lawV = IR to highlight resistance R.* | Rearrange formulas, including formulas with quadratic terms, to highlight a quantity of interest, using the same reasoning as in solving equations (properties of equality). *For example, rearrange Ohm’s law~~to highlight resistance R~~  to solve for the variable V. ~~Manipulate variables in formulas used in financial contexts, such as for simple interest ().~~*  | Edited to incorporate a footnote to clarify the expectations of the standard for Math II students add to provide a financial application.  |
| **A-REI.MA.4.c**  Demonstrate an understanding of the equivalence of factoring, completing the square, or using the quadratic formula to solve quadratic equations. | ~~Demonstrate an understanding of the equivalence of factoring, completing the square, or using the quadratic formula to solve quadratic equations.~~ | The content of this standard is contained in the standard A-REI.4.b.  |
| **F-IF Interpret functions that arise in applications in terms of the context.** | Interpret functions that arise in applications in terms of the context. (Include quadratic and exponential functions.) | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this cluster for Math II. |

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| **F-IF.4**  For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior; ~~and periodicity~~.* | Edited to be consistent with the focus for Math II expectations for functions (linear, quadratic, and exponential functions). Periodic functions are studied in Math II. |
| **F-IF.7a** Graph linear and quadratic functions and show intercepts, maxima, and minima. | Graph ~~linear and~~ quadratic functions and show intercepts, maxima, and minima. | Edited to incorporate a footnote that clarifies the focus of quadratic functions for this standard in Math II when analyzing key features of their graphs. |
| **F-IF.7.b** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.  | Graph ~~square root, cube root, and~~ piecewise-defined functions, including step functions and absolute value functions.  | Edited to incorporate a footnote that clarifies the focus of piece-wise, step, and absolute value functions for this standard in Math II when analyzing key features of their graphs.  |
| **F-IF.8.a**  Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.  | Use the process of factoring and completing the square in a quadratic function to show zeros, maximum/minimum ~~extreme~~ values, and symmetry of the graph and interpret these in terms of a context.  | Edited to clarify the wording of the standard to be consistent with the key features of quadratic functions. |
| **F-IF.8.b** Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, and y = (1.2)t/10, and classify them as representing exponential growth or decay.* | Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, and y = (1.2)t/10, and classify them as representing exponential growth or decay. Apply to financial situations such as identifying appreciation/depreciation rate for the value of a house or car some time after its initial purchase,.* | Edited to provide an example that connects real world financial application with mathematical expressions. |
| **F-IF.8.MA.c**. Translate among different representations of functions and relations: graphs, equations, point sets, and tables. | ~~Translate among different representations of functions and relations: graphs, equations, point sets, and tables.~~ | The content of his standard has been incorporated into the revised Model Math II standard F-IF.9. |

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| **F-IF.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* | Translate among different representations of functions ~~and relations: graphs, equations, point sets, and tables.~~ (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way. *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* | Edited to incorporate standard F-IF.MA.8.c which has a related expectation for representing functions in different ways. |
| **F-IF.MA.10** Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, exponential, or trigonometric. | ~~Given algebraic, numeric and/or graphical representations of~~ ~~functions, recognize the function as polynomial, rational,~~ ~~logarithmic, exponential, or trigonometric.~~ | Moved from Math II to Math III. The student expectation for learning rational, trigonometric, and logarithmic functions is the focus in Math III. The focus in Math II is on linear, quadratic, and exponential functions. |
| **F-BF.1 Write** a function that describes a relationship between two quantities. | Write linear, quadratic, and exponential functions that describe relationships between two quantities. | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this cluster for Math II. |
| **F-BF.1c** (+) Compose functions. *For example, if* *T(y)* *is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*  | ~~(+) Compose functions.~~ *~~For example, if~~**~~T(y)~~**~~is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.~~* ~~★~~ | This (+) standard was deleted as an optional (+) standard in Model Math II. This standard is now an expectation in the Advanced Model Precalculus Course. |
| **F-BF.3**  Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *kf*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* | Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *kf*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Include, quadratic, exponential and absolute value functions. Utilize ~~using~~ technology to experiment with cases and illustrate an explanation of the effects on the graph. *~~Include recognizing even and odd functions from their graphs and algebraic expressions for them.~~* | Edited to incorporate a footnote that clarifies the focus and expectations of this standard and to reinforce the use of technology.  |

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| **F-BF.4.a** Find inverse functions. Solve an equation of the form *f*(*x*) = *c* for a simple function *f* that has an inverse and write an expression for the inverse. *For example, f(x) =2x3 or f(x) = (x + 1)/(x − 1) for x ≠ 1.* | Find inverse functions algebraically and graphically.Solve an equation of the form *f*(*x*) = *c* for a ~~simple~~ linear function *f* that has an inverse and write an expression for the inverse. *~~For example, f(x) =2x~~~~3~~ ~~or f(x) = (x + 1)/(x − 1) for x ≠ 1.~~* | Edited to clarify the focus and expectations of this standard for Math II.  |
| Prove and apply trigonometric identities.**F-TF.8** Prove the Pythagorean identity *sin2(θ) + cos2(θ) = 1* and use it to find *sin(θ), cos(θ),* or *tan(θ)* given *sin(θ), cos(θ),* or *tan(θ)* and the quadrant. | Prove and apply trigonometric identities.~~F-TF.8 Prove the Pythagorean identity~~ *~~sin~~~~2~~~~(θ) + cos~~~~2~~~~(θ) = 1~~* ~~and use it to find~~ *~~sin(θ), cos(θ),~~* ~~or~~ *~~tan(θ)~~* ~~given~~ *~~sin(θ), cos(θ),~~* ~~or~~ *~~tan(θ)~~* ~~and the quadrant.~~ | This standard was moved to Math III where it is an expectation for the Model Math III course. |
| **G-CO Prove geometric theorems.** | Prove geometric theorems and, when appropriate, the converse of theorems. | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this cluster for Math II. |
| **G-CO.9** Prove theorems about lines and angles. *Theorems include: vertical angles are* *congruent; when a transversal crosses parallel* *lines, alternate interior angles are congruent* *and corresponding angles are congruent;* *points on a perpendicular bisector of a line* *segment are exactly those equidistant from the* *segment’s endpoints.* | Prove theorems about lines and angles. *Theorems* *include: vertical angles are congruent; when a transversal* *crosses parallel lines, alternate interior angles are* *congruent and corresponding angles are congruent, and* *conversely prove lines are parallel; points on a* *perpendicular bisector of a line segment are exactly those equidistant from the segment’s endpoints.* | Edited to include the expectation for proving converses of theorems in Math II and to improve the mathematical rigor. |
| **G-CO.10** Prove theorems about triangles. *Theorems include: measures of interior angles* *of a triangle sum to 180°; base angles of* *isosceles triangles are congruent; the* *segment joining midpoints of two sides of a* *triangle is parallel to the third side and half the* *length; the medians of a triangle meet at a* *point.*  | Prove theorems about triangles. *Theorems include:* *measures of interior angles of a triangle sum to 180°;* *base angles of isosceles triangles are congruent, and* *conversely prove a triangle is isosceles; the segment* *joining midpoints of two sides of a triangle is parallel to* *the third side and half the length; the medians of a* *triangle meet at a point.*  | Edited to include the expectation for proving converses of theorems in Math II and to improve the mathematical rigor. |
| **G-CO.MA.11.a**  Prove theorems about polygons. *Theorems include: measures of interior and exterior angles, properties of inscribed polygons.* | **G-CO.11.a** Prove theorems about polygons *including theorems about the measures of interior and exterior angles. A*pply properties of polygons to the solutions of mathematical and contextual problems. *~~properties of inscribed polygons.~~* | Edited to incorporate related standard G-C.MA.3.a.to provide focus and coherence within the Model Math II course for proofs about polygons. |

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| **G-SRT Prove theorems involving similarity.** | Prove theorems involving similarity using a variety of ways of writing proofs, showing validity of underlyingreasoning. | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this cluster for Math II. |
| **G-C.3** Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. | Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral and other polygons inscribed in a circle. | Edited to add content from another standard in the course (G-CO.MA.11.a)  |
| **G-C.MA.3.a**. Derive the formula for the relationship between the number of sides, and sums of the interior and sums of the exterior angles of polygons, and apply to the solutions of mathematical and contextual problems. | ~~Derive the formula for the relationship between the~~ ~~number of sides, and sums of the interior and sums of the~~ ~~exterior angles of polygons, and apply to the solutions of~~ ~~mathematical and contextual problems.~~ | This standard is incorporated into G-CO.MA.11.a as it has related content and provides coherence withG-CO.MA.11.a. |
| **G-C.5.** Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. Footnote removed | ~~[Limit Mathematics II use of radian to unit of measure.]~~ | Edited to remove footnote that limited the expectation for Model Math II. |
| **G-GPE.4**  NEW TO MATH II. | Use coordinates to prove simple geometric theorems algebraically including the distance formula and its relationship to the Pythagorean Theorem*. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, ) lies on the circle centered at the origin and containing the point (0, 2).* | This standard was moved to Math II from Math I where it continues the progression of expressing geometric properties with equations. |
| **G-MD.2** NEW TO MATH II. | G-GMD.2. (+) Give an informal argument using Cavalieri’s principle for the formulas for the volume of a sphere and other solid figures. | This standard was added to Math II to align the Integrated Pathway expectations with the Traditional Pathway. |
| **G-GMD.4.** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. | ~~Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.~~ | This standard is an expectation in Model Math III where geometric measurement and dimension is a focus. It is deleted in Math II. |

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| S-CP Understand independence and conditional probability and use them to interpret data.  | Understand independence and conditional probability and use them to interpret data from simulations or experiments. | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this cluster for Math II. |
| **S-MD.6.** (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).  | (~~+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).~~  | This (+) standard is now included, and is optional, in the Model Math III course.  |
| **S-MD.7 (+)** Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). | ~~Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game .~~ | This (+) standard is now included, and is optional, in the Model Math III course. Deleted from Math II. |

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| High School Model Integrated Math III |
| **2011 Standard** | **Proposed 2017 Standard****Revisions are in red text** | **Rationale for Revision** |
| **N-VM.1. (+)****Standard new to Model Math III.** | N-VM.1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, |v|, ||v||, v). | This (+) standard is optional and was added to the Model Math III course to provide students the opportunity to learn the same content in the model integrated pathway as those in the model traditional pathway. |
| **N-VM.3. (+)** **Standard new to Model Math III.** | N-VM.3. (+) Solve problems involving velocity and other quantities that can be represented by vectors. | This (+) standard is optional and was added to the Model Math III course to provide students the opportunity to learn the same content in the model integrated pathway as those in the model traditional pathway. |
| **N-VM.6. (+)** **Standard new to Model Math III.** | N-VM.6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network | This (+) standard is optional and was added to the Model Math III course to provide students the opportunity to learn the same content in the model integrated pathway as those in the model traditional pathway. |
| **N-VM.8. (+)** **Standard new to Model Math III.** | N-VM.8. (+) Add, subtract, and multiply matrices of appropriate dimensions.  | This (+) standard is optional and was added to the Model Math III course to provide students the opportunity to learn the same content in the model integrated pathway as those in the model traditional pathway. |
| **N-VM.12. (+)** **Standard new to Model Math III.** | N-VM.12. (+) Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area. | This (+) standard is optional and was added to the Model Math III course to provide students the opportunity to learn the same content in the model integrated pathway as those in the model traditional pathway. |
| **A-SSE Interpret the structure of expressions** | Interpret the structure of polynomial and rational expressions. | Edited to incorporate a footnote that clarifies the focus and expectations of the standards in this cluster for Math III. |

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| **A-SSE.2** Use the structure of an expression to identify ways to rewrite it. *For example, see x4 – y4 as (x2)2 – (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 – y2)(x2 + y2).* | Use the structure of an expression to identify ways to rewrite it. *For example, see x4 – y4 as (x2)2 – (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 – y2)(x2 + y2) and as (x-y)(x+y)(x-y****i****)(x+y****i****).* | Edited the example in the standard to align with the focus and expectations for Algebra II. |
| **A-APR.1**  Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. | Understand that polynomials form a system analogous to the integers, namely, they are closed under certain operations. a. Perform operations on polynomial expressions (addition, subtraction, multiplication, and division), and compare the system of polynomials to the system of integers when performing operations.  | Edited to incorporate standard A-APR.MA.1.a and to clarify that the content of Math III includes division of polynomials and a focus on closure. Understand that polynomials form a system analogous to the integers, namely, they are closed under certain operations.  |
| **A-APR.** **MA.1.a** Divide polynomials.  | ~~Divide polynomials.~~ | The content of this standard (dividing polynomials) has been incorporated into the revised Math III standard A-APR.1. |
| **A-APR.5** (+) Know and apply the Binomial Theorem for the expansion of (*x* + *y*)*n* in powers of *x* and *y* for a positive integer *n*, where *x* and *y* are any numbers, with coefficients determined for example by Pascal’s Triangle. 1 | (+) Know and apply the Binomial Theorem for the expansion of (*x* + *y*)*n* in powers of *x* and *y* for a positive integer *n*, where *x* and *y* are any numbers, with coefficients determined for example by Pascal’s Triangle. ~~1~~ | Edited to remove a footnote about proving the Binomial Theorem. Proof of this theorem is an expectation of the Model Precalculus course |
| **A-CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.  | Create equations and inequalities in one variable and use them to solve problems. Include equations arising from ~~linear and quadratic functions~~, simple root and rational ~~and exponential~~ functions.  | Edited to clarify the focus and expectations of the standard for Math III. |
| **A-CED.4**  Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm’s lawV = IR to highlight resistance R.* | ~~Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.~~ *~~For example, rearrange Ohm’s law~~~~V = IR to highlight resistance R.~~* | Deleted the standard in Model Math III as the expectations for solving equations is a focus in Model Math I and Math II courses. See revised standard A-CED.4 in Model Math I and Math II. |

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| **A-REI.11** Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. | Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are ~~linear,~~ polynomial, rational, ~~absolute value, exponential,~~ and logarithmic functions. | Edited to clarify the focus for the types of functions for this standard in Math III. |
| **F-IF Interpret functions that arise in applications in terms of the context.**  | Interpret polynomial, rational, square and cube root, trigonometric, and logarithmic functions that arise in applications in terms of the context. | Edited to incorporate a footnote that clarifies the focus and expectations of this cluster for Math III. |
| **F-IF.7b** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. | Graph square root and cube root, ~~and piecewise-defined~~ functions. ~~including step functions and absolute value functions~~. | Edited to incorporate a footnote that clarifies the focus and expectations of this standard for Math III. Step and absolute value functions are an expectation of Model Math II. |
| **F-IF.8.a** Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. | Use the process of factoring ~~and completing the square in a quadratic~~  in a polynomial function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. | Edited to clarify the focus and expectations of this standard for Math III.  |
| **F-IF.8.MA.c**. Translate among different representations of functions and relations: graphs, equations, point sets, and tables. | ~~Translate among different representations of functions and relations: graphs, equations, point sets, and tables.~~ | The content of this standard has been incorporated into the Model Math III revised standard F-IF.9. |
| **F-IF.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* | Translate among different representations of functions ~~and relations: graphs, equations, point sets, and tables.~~ (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way. *For example, given a graph of one ~~quadratic~~ polynomial function and an algebraic expression for another, say which has the larger relative maximum and/or smaller relative minimum.* | Edited to incorporate standard F-IF.MA.8.c which has a related expectation for representing functions in multiple ways. |

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| **F-IF.MA.10** **Standard new to Model Math III** | **F-IF.10** Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, exponential, or trigonometric. | This standard was moved from Model Math I and added to Math III. The expectation for student learning of rational, trigonometric, and logarithmic functions in this standard is a focus of the Math III model course. |
| **F-BF.1** Write a functions that describes a relationship between two quantities. | Write ~~a~~ simple rational and radical functions, logarithmic, and trigonometric functions that describe a relationship between two quantities. | Edited to reflect the focus and expectations of this standard for Algebra Math III. |
| **F-BF.3**  Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *kf*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* | Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *kf*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Include simple rational, radical, logarithmic, and trigonometric functions. Utilize ~~using~~ technology to experiment with cases and illustrate an explanation of the effects on the graph. ~~using~~. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* | Edited to clarify the focus and expectation of this standard for Math III and to reinforce the use of technology to explore effects on graphs of functions when varying k. |
| **F-BF.4.a** Find inverse functions. Solve an equation of the form *f*(*x*) = *c* for a simple function *f* that has an inverse and write an expression for the inverse. *For example, f(x) =2x3 or f(x) = (x + 1)/(x − 1) for x ≠ 1.* | Find inverse functions algebraically and graphically.Solve an equation of the form *f*(*x*) = *c* for a simple ~~linear~~ function *f* that has an inverse and write an expression for the inverse. *For example, f(x) =2x3 or f(x) = (x + 1)/(x − 1) for x ≠ 1.* | Edited to include finding inverse of functions both algebraically by solving equations and graphically. |
| **F-TF.8****NEW TO MATH III.** | Prove and apply trigonometric identities.F-TF.8 Prove the Pythagorean identity *sin2(θ) + cos2(θ) = 1* and use it to find *sin(θ), cos(θ),* or *tan(θ)* given *sin(θ), cos(θ),* or *tan(θ)* and the quadrant. | This standard was moved from Model Math II to Math III where it is an expectation of the Model Math III course. |
| **S-ID Summarize, represent, and interpret data on a single count or measurement variable.**  | Summarize, represent, and interpret data on a single count or measurement variable. Use calculators, spreadsheets, and other technology as appropriate. | Edited to emphasize appropriate technology use in this cluster to support understandings for interpreting the data.  |

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| **S-MD.7** (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game | (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game and replacing the goalie with an extra skater) | Edited to incorporate a footnote to clarify the example in the standard. |

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| High School Model Precalculus |
| **2011 Standard** | **Proposed 2017 Standard****Revisions are in red text** | **Rationale for Revision** |
| **A-APR.6** Rewrite simple rational expressions in different forms; write ***a*(*x*)/*b*(*x*)** in the form q(x) + ***r*(*x*)/*b*(*x*)**, where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system. | ~~Rewrite simple rational expressions in different forms; write~~ ***~~a~~*~~(~~*~~x~~*~~)/~~*~~b~~*~~(~~*~~x~~*~~)~~** ~~in the form q(x) +~~***~~r~~*~~(~~*~~x~~*~~)/~~*~~b~~*~~(~~*~~x~~*~~)~~**~~, where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.~~ | This standard is an expectation in both Model Pathways courses Algebra II and Math III. Deleted from Precalculus.  |
| **G-GMD.4** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. | ~~Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.~~ | This standard is an expectation in both Model Pathways courses Geometry and Math III. Deleted from Precalculus. |

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| High School Model Advanced Quantitative Reasoning |
| **2011 Standard** | **Proposed 2017 Standard****Revisions are in red text** | **Rationale for Revision** |
| **F-TF.5** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. | ~~Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.~~ | This standard is an expectation in both Model Pathways courses Algebra II and Math III. Deleted from Advanced Quantitative Reasoning. |
| **G-GMD.4** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. | ~~Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.~~ | This standard is an expectation in both Model Pathways courses Geometry and Math III. Deleted from Advanced Quantitative Reasoning. |
| **G-MG.3** Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). | ~~Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).~~ | This standard is an expectation in both Model Pathways courses Geometry and Math III. Deleted from Advanced Quantitative Reasoning. |
| **G-MG.4** Use dimensional analysis for unit conversions to confirm that expressions and equations make sense. | ~~Use dimensional analysis for unit conversions to confirm that expressions and equations make sense.~~ | This standard is an expectation in both Model Pathways courses Geometry and Math III. Deleted from Advanced Quantitative Reasoning. |
| **S-ID.9** Distinguish between correlation and causation. Distinguish between correlation and causation. | ~~Distinguish between correlation and causation.~~ | This standard is an expectation in both Model Pathways courses Algebra I and Math I. Deleted from Advanced Quantitative Reasoning. |

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| **S-IC.4** Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. | ~~Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.~~ | This standard is an expectation in both Model Pathways courses Algebra II and Math III. Deleted from Advanced Quantitative Reasoning. |
| **S-IC.5** Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. | ~~Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.~~ | This standard is an expectation in both Model Pathways courses Algebra II and Math III. Deleted from Advanced Quantitative Reasoning. |
| **S-IC.6** Evaluate reports based on data. | ~~Evaluate reports based on data.~~ | This standard is an expectation in both Model Pathways courses Algebra II and Math III. Deleted from Advanced Quantitative Reasoning. |
| **S-MD.7** (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). | (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game and replacing the goalie with an extra skater). | Edited to incorporate a footnote to clarify the example in the standard. |

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| High School Conceptual Category Standards**The standards in the high school Conceptual Categories are the foundation for the high school model courses.** **These standards appear in multiple model courses****(See pp. 76-101 in the Math Curriculum Framework 2017)** |
| **2011 Standard** | **Proposed 2017 Standard****Revisions are in red text** | **Rationale for Revision** |
| **N.Q.MA.3.a** | **N.Q.** ~~MA~~ **.3.a** | Edited to remove “MA” (Massachusetts) designation. |
| **A-SSE Interpret the structure of expressions.** | Interpret the structure of expressions (linear, quadratic, exponential, polynomial, rational). | Edited this cluster heading to align with the revised expectations across the high school model courses. |
| **A-APR.1** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. | Understand that polynomials form a system analogous to the integers, namely, they are closed under certain operations. a. Perform operations on polynomial expressions (addition, subtraction, multiplication, division), and compare the system of polynomials to the system of integers when performing operations. b. Factor and/or expand polynomial expressions, identify and combine like terms, and apply the Distributive Property.  | Edited this standard to align with the revised expectations in the high school model courses. (Algebra II and Math III.) |
| **A-CED.1**  Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. | Create equations and that describe numbers or relationships. Include equations arising from linear and quadratic functions, simple root and rational functions and exponential functions. | Edited this standard to align with the revised expectations across the high school model courses. |

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| **A-REI.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. | **A-REI.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify or refute a solution method. | Edited this standard to align with the revised expectations in high school model courses. ( Algebra I and Math I) |
| **A-REI.MA.3.a**  | A-REI.~~MA~~.3.a  | Edited to remove “MA” (Massachusetts) designation. |
| **A-REI.MA.4.c**  Demonstrate an understanding of the equivalence of factoring, completing the square, or using the quadratic formula to solve quadratic equations. | ~~Demonstrate an understanding of the equivalence of factoring, completing the square, or using the quadratic formula to solve quadratic equations.~~ | The content of this standard is contained in the standard A-REI.4.b.  |
| **A-REI.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).  | Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Show that any point on the graph of an equation in two variables is a solution to the equation. | Edited to include an expectation of applying the understanding between graphs, equations, and solutions of equations.  |
| **A.REI.12** Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. | Graph the solutions ~~to~~ of a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set ~~to~~ of a system of linear inequalities in two variables as the intersection of the corresponding half-planes. | Edited to use language consistent with solutions of equations and inequalities. |
| **F-IF.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. | Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. For example, given a function representing a car loan, determine the balance of the loan at different points in time. | Edited this standard to align with the revised expectations in high school model courses. (Algebra I and Math I.) |
| **F-IF Interpret functionsthat arise in applications in terms of the context.**  |  Interpret functions that arise in applications in terms of the context. (linear, quadratic, exponential, rational, polynomial, square, cube root, trigonometric, logarithmic) | Edited this cluster heading to align with the revised expectations across the high school model courses. |
| **F-IF.8.a** Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. | Use the process of factoring and/or completing the square in quadratic and polynomial functions, where appropriate, to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. | Edited this standard to align with the revised expectations across the high school model courses. |
| **F-IF.8.b** Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, and y = (1.2)t/10, and classify them as representing exponential growth or decay.* | Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, and y = (1.2)t/10, and classify them as representing exponential growth or decay. Apply to financial situations such as identifying appreciation and depreciation rate for the value of a house or car some time after its initial purchase. | Edited this standard to align with the revised expectations across the high school model courses. |
| **F-IF.8.MA.c**. Translate among different representations of functions and relations: graphs, equations, point sets, and tables. | ~~Translate among different representations of functions and relations: graphs, equations, point sets, and tables.~~ | The content of this standard has been incorporated into the revised standard F-IF.9. |
| **F-IF.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* | **F-IF.9** Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way. *For example, given a graph of one ~~quadratic~~ polynomial function (including quadratic functions) and an algebraic expression for another, say which has the larger/smaller relative maximum and/or minimum.* | Edited this standard to align with the revised expectations across the high school model courses |
| **F-BF.1** Write a functions that describes a relationship between two quantities. | Write a function (linear, quadratic, exponential, simple rational, radical, logarithmic, and trigonometric) that describes a relationship between two quantities.  | Edited this standard to align with the revised expectations across the high school model courses |
| **F-BF.3**  Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *kf*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* | Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *kf*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs.( Include, linear, quadratic, exponential, absolute value, simple rational and radical, logarithmic and trigonometric functions.) Utilize ~~using~~ technology to experiment with cases and illustrate an explanation of the effects on the graph~~.~~ *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* | Edited this standard to align with the revised expectations across the high school model courses and to reinforce the use of technology.  |
| **F-BF.4.a** Find Inverse Functions Solve an equation of the form *f*(*x*) = *c* for a simple function *f* that has an inverse and write an expression for the inverse. *For example, f(x) =2x3 or f(x) = (x + 1)/(x − 1) for x ≠ 1.* | F-BF.4. Find .Inverse functions algebraically and graphically. a. Solve an equation of the form *f*(*x*) = *c* for a simple function *f* that has an inverse and write an expression for the inverse. *For example, f(x) =2x3 or f(x) = (x + 1)/(x − 1) for x ≠ 1.(Include linear and simple polynomial, rational, and* exponential functions.) | Edited this standard to include the expectation of finding inverses of functions both algebraically and graphically. across the high school model courses |
| **F-LE.5** Interpret the parameters in a linear or exponential function in terms of a context. | Interpret the parameters in a linear or exponential function (of the form f*(x) = bx + k)* in terms of a context. | Edited this standard to align with the revised expectations across the high school model courses. |
| **G-CO Prove geometric theorems**  | G-CO Prove geometric theorems and, when appropriate, the converse of theorems. | Edited this cluster heading to align with the revised expectations in the high school Model Courses (Geometry and Math II). |

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| G-CO.9 Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment’s endpoints. | G-CO.9 Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent, and conversely prove lines are parallel; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment’s endpoints. | Edited this standard to align with the revised expectations in the high school Model Courses (Geometry and Math II). |
| **G-CO.10** Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent, the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.  | G-CO.10Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.  | Edited this standard to align with the revised expectations in the high school Model Courses (Geometry and Math II). |
| **G-CO.MA.11.a** Prove theorems about polygons. *Theorems include: measures of interior and exterior angles.* | G-CO. ~~MA~~ .11.a Prove theorems about polygons. *Theorems include: measures of interior and exterior angles.* Apply properties of polygons to the solutions of mathematical and contextual problems. | Edited this standard to align with the revised expectations in the high school Model Courses (Geometry and Math II).Edited to remove “MA” (Massachusetts) designation. |
| **G-C.3** Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. | Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral and other polygons inscribed in a circle. | Edited this standard to align with the revised content in the high school Model Courses (Geometry and Math II). |
| **G-GPE.MA.3.a** | G.GP. ~~MA~~ .3.a | Edited to remove “MA” (Massachusetts) designation. |

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| G-GPE.4 Use coordinates to prove simple geometric theorems algebraically*. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, ) lies on the circle centered at the origin and containing the point (0, 2).* | Use coordinates to prove simple geometric theorems algebraically including the distance formula and its relationship to the Pythagorean Theorem*. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, ) lies on the circle centered at the origin and containing the point (0, 2).* | Edited this standard to align with the revised expectations in the high school Model Courses (Geometry and Math II). |
| **G-MG.MA.4** | G-MG. ~~MA~~ .4 | Edited to remove “MA” (Massachusetts) designation. |
| **S-ID Summarize, represent, and interpret data on a single count or measurement variable.**  | Summarize, represent, and interpret data on a single count or measurement variable. Use calculators, spreadsheets, and other technology as appropriate. | Edited this cluster heading to align with the revised expectations in the high school Model Courses. (Algebra I and Math I.) |
| **S-ID.6.** a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Uses given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* Footnote: Linear focus; learn as general principal in Math I.  Footnote: Focus on linear applications; learn as a general principle to be expanded in Math II and III. | Fit a linear function to the data and use the fitted function~~s~~ to solve problems in the context of the data. Uses ~~given~~ functions fitted to data or choose *a function suggested by the context. Emphasize linear~~, quadratic,~~ and exponential models.*Footnote: ~~Footnote: Linear focus; discuss as general principal in Algebra I.~~~~Footnote: Focus on linear applications; learn as a general principle to be expanded in Math II and III.~~ | Edited this standard to align with the revised expectations in the high school Model Courses. (Algebra I and Math I.) |
| S-CP Understand independence and conditional probability and use them to interpret data.  | Understand independence and conditional probability and use them to interpret data from simulations or experiments. | Edited this cluster heading to align with the revised expectations in the high school Model Courses. (Geometry and Math II.) |
| **S-MD.7** (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game | (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game and replacing the goalie with an extra skater) | Edited this standard to align with the revised expectations in the high school Model Courses. (Algebra II and Math III.) |

# Summary of Proposed Changes to Other Sections of the Framework

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| **Section** | **Change and Rationale** |
| Introduction | Updated to include Massachusetts standards development and review processes 1993–2017.Strengthened and expanded the connection between student learning expectations and college and career readiness and civic preparation. |
| Guiding Principles | All Guiding Principles were revised; updated to address Social and Emotional Learning in the mathematics classroom. |
| Pre-K to 8 Content Standards Introduction | Updated example to reflect current framework . |
| Pre-Kindergarten | Edited coding of standards to reflect grade and domain. |
| Grade-level Introductions grades 5, 6, 7 | Updated grade-level introductions to reflect revisions. |
| High School; Conceptual Categories  | Updated introduction and included charts to provide a visual display showing which conceptual category standards are addressed in each model course; charts show the progression of content in the model course pathways. Edited Conceptual Category Standards as appropriate to match any new content added to model course standards. |
| High School Model Course introduction | Updated to reflect revisions made to model course content standards. |
| Model Course Introductions | Each Model Course introduction was updated to reflect the revisions in the model courses. |
| High School: Making Decision document | Added a new section to describe course-taking sequences and a variety of pathways that accelerate learning in middle school and high school, including Algebra I in grade 8.  |
| English Learners and Students with Disabilities | Updated language to reflect recent understandings for supporting English Learners and Students with Disabilities. |
| Appendices: Standards for Mathematical Practice | Customized the descriptions of the Mathematical Practice Standards; presented by narrower grade-spans (Pre-K-5; 6-8; high school) with specific examples of connections between the content standards at these grade-spans and the mathematical practice standards. |
| Appendices: Glossary | Included additional mathematical definitions and terms referenced in the standards. |
| Coding  | Coded model course high school content standards with appropriate course designations: AI., GEO., AII., MI., MII., MIII., PC., QR.  |