# Introduction

## The Origin of these Standards: 1993–2010

The Massachusetts Education Reform Act of 1993 directed the Commissioner and the Department of Elementary and Secondary Education[[1]](#footnote-1) to create academic standards in a variety of subject areas. Massachusetts adopted its first set of Mathematics standards in 1995 and revised them in 2000. In 2007, the Massachusetts Department of Elementary and Secondary Education (ESE) convened a team of educators to revise its 2000 *Mathematics Curriculum Framework.* In 2009 the Council of Chief State School Officers (CCSSO) and the National Governors Association (NGA) began a multi-state standards development project called the *Common Core State Standards* initiative, whereupon the two efforts merged. The pre-kindergarten to grade 12 *Massachusetts Curriculum Framework for Mathematics*, a new framework that included both the Common Core State Standards and unique Massachusetts standards and features, was adopted by the Boards of Elementary and Secondary Education and Early Education and Care in 2010. A similar process unfolded for the English Language Arts/Literacy Framework.

## Review of Mathematics and English Language Arts/Literacy Standards, 2016–2017

In November 2015, the Massachusetts Board of Elementary and Secondary Education voted to move forward with the development of its own next generation student assessment program in mathematics and English Language Arts/Literacy. At the same time, the Board supported a plan to convene review panels comprised of Massachusetts K-12 educators and higher education faculty to review the 2010 Mathematics and English Language Arts/Literacy Curriculum Frameworks. The review panels were also asked to identify any modifications or additions to ensure that the Commonwealth’s standards match those of the most aspirational education systems in the world, thus representing a course of study that best prepares students for the 21st century.

In February 2016, the Department appointed a panel of Massachusetts educators from elementary, secondary, and higher education to review the mathematics and ELA/Literacy standards and suggest improvements based on their experiences using the 2010 Framework. The Department sought comment on the standards through a public survey and from additional content advisors in mathematics and ELA/Literacy.

The 2017 *Massachusetts Curriculum Framework for Mathematics* reflects improvements to the prior Framework that have been informed by the review panel, public comments, and the content advisors. In some cases, the standards have been edited to clarify meaning. Some have been eliminated; others added. The [Glossary](#_Glossary:_Mathematical_Terms,) and [Bibliography](#_Bibliography_and_Resources) have been updated and the Department’s 2010 document titled, [*Making Decisions about Course Sequences and the Model Algebra I Course*](#_Making_Decisions_about)*,* has been revised and is now included in the high school section of the Framework. The intent is to present multiple pathways involving the compression or enhancement of mathematics standards to provide alternative course-taking sequences for students enabling them to be successful and prepared for various college and career pursuits, including mathematic-intensive majors and careers.

The 2017 standards draw from the best of prior Massachusetts standards, and represent the input of hundreds of the Commonwealth’s K-2 and higher education faculty. The 2017 standards present the Commonwealth’s commitment to providing all students with a world-class education.

## The Mathematically Proficient Person of the Twenty-First Century

The standards describe a vision of what it means to be a mathematically proficient person in this century. Students who are college and career ready in mathematics will at a minimum demonstrate the academic knowledge, skills, and practices necessary to enter into and succeed in entry-level, credit bearing courses in College Algebra, Introductory College Statistics, or technical courses. It also extends to a comparable entry-level course or a certificate or workplace training program requiring an equivalent level of mathematics. At the same time, the standards provide for a course of study that will prepare students for a science, technology, engineering, or mathematics career. For example, the level of mathematics preparation necessary to succeed in an engineering program is more ambitious than the preparation needed to succeed in an entry-level, credit-bearing mathematics course as part of a liberal arts program. The standards provide pathways for students who want to pursue a mathematics-intensive career or academic major after high school.

The mathematical skills and understanding that students are expected to demonstrate have wide applicability outside the classroom or workplace. Students who meet the standards are able to identify problems, represent problems, justify conclusions, and apply mathematics to practical situations. They gain understanding of topics and issues by reviewing data and statistical information. They develop reasoning and analytical skills and make conclusions based on evidence that is essential to both private deliberation and responsible citizenship in a democratic society. They understand mathematics as a language for representing the physical world.

They are able to use and apply their mathematical thinking in various contexts and across subject areas, for example, in managing personal finances, designing a robot, or presenting a logical argument and supporting it with relevant quantitative data in a debate. Students should be given opportunities to discuss math’s relevance to everyday life and their interests and potential careers with teachers, parents, business owners, and employees in a variety of fields such as computer science, architecture, construction, healthcare, engineering, retail sales, and education. From such discussions, students can learn that a computer animator uses linear algebra to determine how an object will be rotated, shifted, or altered in size. They can discover that an architect uses math to calculate the square footage of rooms and buildings, to lay out floor dimensions and to calculate the required space for areas such as parking or heating and cooling systems (kumon.org}. They can investigate how public policy analysts use statistics to monitor and predict state, national or international healthcare use, benefits, and costs.

Students who meet the standards develop persistence, conceptual understanding, and procedural fluency; they develop the ability to reason, prove, justify, and communicate. They build a strong foundation for applying these understandings and skills to solve real world problems. These standards represent an ambitious pre-kindergarten to grade 12 mathematics program that ensure that students are prepared for college, careers, and civic life.

## A Coherent Progression of Learning: Pre-Kindergarten through Grade 12

The mathematics content standards are presented by individual grade levels in pre-kindergarten through grade 8 to provide useful specificity. The pre-kindergarten standards apply to children who are four-year-olds and younger five-year-olds. A majority of these students attend education programs in a variety of settings: community-based early care and education centers; family daycare; Head Start programs; and public preschools. In this age group, the foundations of counting, quantity, comparing shapes, adding, and taking apart – and the ideas that objects can be measured – are formed during conversations, play, and with experiences with real objects and situations.

## At the high school level, the standards are presented in two different ways:

1. **Conceptual Categories:** These categories portray a coherent view of high school mathematics standards for the grade-span 9–12. These content standards are organized into six categories: Number and Quantity; Algebra; Functions; Modeling; Geometry; and Statistics and Probability.
2. **Model High School Courses:** Two high school pathways, the Traditional Pathway Model Courses (Algebra I, Geometry, Algebra II) and the Integrated Pathway Model Courses (Mathematics I, II, III), are presented using the content standards in the Conceptual Categories. These model courses were designed to create a smooth transition from the grade-by-grade pre-k–8 content standards to high school courses. All of the college and career ready content standards are included in appropriate locations within each set of three model high school courses. The high school content standards coded with a (+) symbol are optional and identify higher-level mathematics skills and knowledge that students should learn in order to take more advanced mathematics courses. In addition, two Advanced Model High School Courses, Advanced Quantitative Reasoning and Precalculus, are included. Students may choose to take these courses after completing either of the Model High School Pathways. See the section below titled, “Course-Taking Sequences and Pathways for All Students,” for additional advanced mathematics courses and pathways students might pursue in high school.

## Focus, Clarity, and Rigor

In the past, mathematics standards and curricula were often criticized for being “a mile wide and an inch deep” in almost every topic taught each year. The 2010 Framework presented a new design feature for grades Pre-K–12: a focus on three to five critical focus areas per grade or course. The 2017 Framework continues to concentrate on fewer topics in each grade to allow students to deepen and consolidate their understanding in these areas. These critical focus areas are useful in communicating with families and the community and for designing curricula, support services, and programs.

## A Balance of Conceptual Understanding, Procedural Fluency, and Application[[2]](#footnote-2)

The standards strategically develop students’ mathematical understanding and skills. When students are first introduced to a mathematical concept they explore and investigate the concept by using concrete objects, visual models, drawings, or representations to build their understanding. In the early grades they develop number sense while working with numbers in many ways. They learn a variety of strategies to solve problems and use what they have learned about patterns in numbers and the properties of numbers. This serves to develop a strong understanding of number sense, decomposing and composing numbers, the relationship between addition and subtraction, and multiplication and division. In calculations, students are expected to be able to use the most efficient and accurate way to solve a problem based on their understanding and knowledge of place value and properties of numbers. Students reach fluency by building understanding of mathematical concepts – this lays a strong foundation that prepares students for more advanced math work – and by building automaticity in the recall of basic computation facts, such as addition, subtraction, multiplication, and division.

As students apply their mathematical knowledge and skills to solve real-world problems, they also gain an understanding of the importance of mathematics throughout their lives.

Mathematics Complexity Triangle:

Making Sense of Mathematical Concepts at the top

Using Mathematical Concepts in Problem Solving Application, MATHEMATICAL RIGOR, and Performing Mathematical Procedures Fluently at the bottomTo achieve mathematical understanding, students should be actively engaged in meaningful mathematics. The content and practice standards focus on developing students in the following areas:

* **Conceptual understanding** – make sense of the math, reason about and understand math concepts and ideas
* **Procedural fluency** – know mathematical facts, compute and do the math
* **Capacity** – solve a wide range of problems in various contexts by reasoning, thinking, and **applying** the mathematics they have learned.

## Middle and High School Course-Taking Sequences and Pathways for All Students

The Massachusetts High School Program of Studies ([MassCore](http://www.doe.mass.edu/ccr/masscore/)) is a recommended program of studies that includes four years of mathematics coursework in grades 9–12. MassCore describes other learning opportunities, such as Advanced Placement (AP) classes, dual enrollment, senior projects, online courses for high school or college credit, and service or work-based learning.

The Framework provides an opportunity for districts to revisit and plan course sequences in middle and high school mathematics along with educators, middle and high school guidance counselors, parents, college mathematics faculty, and mathematics leaders. This Framework includes a new section titled, [*Making Decisions about Course Sequences and the Model Algebra I Course*](#_Making_Decisions_about)*.* This section presents several options for pathways for students ready to move at an accelerated rate.

**On Grade Sequence:** Students who follow the Framework grade-level sequence pre-K–8 will be prepared for the Traditional or Integrated Model Course high school pathway, beginning with Algebra I or Mathematics I in grade 9. Students following this pathway will be prepared to take a fourth year advanced course in grade 12, such as the Model Precalculus Course, Model Quantitative Reasoning Course, or other advanced courses offered in their district.

**Model Algebra I in Grade 8 and High School Acceleration:** One option for accelerating learning is to take the Model Algebra I course in grade 8. This pathway option compresses the standards for grade 7 with part of the grade 8 standards in grade 7. That allows grade 8 students to complete the grade 8 standards related to algebra and the Algebra I model high school standards in one year (grade 8). Considerations for assigning a student this pathway include two factors: grade 8 standards are already rigorous and students are expected to learn the grade 8 standards in order to be prepared for the Algebra I model course.

This section also presents pathways for students who are ready to accelerate their learning, beginning in grade 9. Some of these pathways lead to calculus in grade 12 while others offer a sequence to other advanced courses such as Quantitative Reasoning, Statistics, Linear Algebra, AP courses, Discrete Mathematics, or participating in a dual enrollment program.

All pathways should aspire to meet the goal of ensuring that no student who graduates from a Massachusetts High School will be placed into a remedial mathematics course in a Massachusetts public college or university. Achieving this goal may require mathematics secondary educators and college faculty to work collaboratively to select or co-develop appropriate 12th grade coursework and assessments. Presenting a variety of course-taking pathways encourages students to persist in their mathematical studies. It also helps them realize that there are multiple opportunities to make course-taking decisions as they continue to advance mathematically and pursue their interests and career and college goals.

## Mathematics in the Context of a Well-Rounded Curriculum

Strong mathematics achievement is a requisite for studying the sciences (including social sciences), technology (including computer science), and engineering. The centrality of mathematics to the pursuit of STEM careers is well documented.

In addition, an effective mathematics program builds upon and develops students’ mathematical knowledge and literacy skills. Reading, writing, speaking, and listening skills are necessary elements of learning and engaging in mathematics, as well as in other content areas. The English Language Arts/Literacy standardsrequire that instruction in reading, writing, speaking, listening, and language is a shared responsibility within the school. The pre-K–5 ELA/Literacy standards include expectations for reading, writing, speaking, listening, and language applicable to a range of subjects, including mathematics, social studies, science, the arts, and comprehensive health. Grades 6–12 ELA/Literacy standards are divided into two sections, one for ELA and the other for history/social studies, science, mathematics, and technical subjects. This division reflects the unique, time-honored role of ELA teachers in developing students’ literacy skills, while at the same time recognizing that teachers in other disciplines also contribute in this development.

Consistent with emphasizing the importance of an interdisciplinary approach to literacy, the Mathematics Guiding Principles recognize that reading, writing, speaking, and listening skills are necessary elements of learning and engaging in mathematics. Mathematics students learn specialized vocabulary, terms, notations, symbols, representations, and models relevant to the grade level. Being able to read, interpret, and analyze mathematical information from a variety of sources and communicating mathematically in written and oral forms are critical skills to college and career readiness, citizenship, and informed decision-making.

In essence, mathematics is a language for describing and understanding the physical world. Notably, the recent revision of the *Massachusetts Curriculum Framework for Science and Technology/Engineering* (2016) also highlights literacy in its Guiding Principles and Practices.

To achieve a well-rounded curriculum at all grade levels, the standards in this Framework are meant to be used with the *Massachusetts Curriculum Framework for English Language Arts/Literacy, the Arts, History and Social Science, Science and Technology/Engineering, Comprehensive Health and Physical Education, Foreign Languages*. In grades 9–12, the standards are also meant to be used with the *Framework for Career and Vocational and Technical Education* to achieve a truly rich and well-rounded curriculum.

## What the Mathematics Curriculum Framework Does and Does Not Do

The standards define what all students are expected to know and be able to do, not how teachers should teach. While the standards focus on what is most essential, they do not describe all that can or should be taught. A great deal is left to the discretion of teachers and curriculum developers.

No set of grade-level standards can reflect the great variety of abilities, needs, learning rates, and achievement levels in any given classroom. The standards define neither the support materials that some students may need, nor the advanced materials that others should have access to. It is also beyond the scope of the standards to define the full range of support appropriate for English learners and for students with disabilities. Still, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills that will be necessary in their post-high-school lives.

The standards should be read as allowing for the widest possible range of students to participate fully from the outset with appropriate accommodations to ensure maximum participation of students with special education needs. For example, for students with disabilities, *reading* math texts and problems should allow for the use of Braille, screen-reader technology, or other assistive devices. *Writing* should include the use of a scribe, computer, or speech-to-text technology that includes mathematical terms, notations, and symbols. In a similar manner, *speaking* and *listening* should be interpreted broadly to include sign language (see Appendix 1).

While the mathematics described herein is critical to college, career, and civic readiness, they do not define the “whole” of readiness. Students require a wide-ranging, rigorous academic preparation and attention to such matters as social, emotional, and physical development and approaches to learning.

## Document Organization

**Eight** **Guiding Principles for Mathematical Programs in Massachusetts** follows this introductory section. The Guiding Principles are philosophical statements that underlie the standards and resources in this Framework.

Following the Guiding Principles are the eight **Standards for Mathematical Practice.** These standardsdescribe the varieties of expertise that all mathematics educators at all levels should seek to develop in their students.

Following the Standards for Mathematical Practice are the **Standards for Mathematical Content,** presented in three sections:

1. **Pre-kindergarten through grade 8 content standards by *grade level***
2. **High school content standards by *conceptual category***
3. **High school content standards by *model high school courses****—*includes six model courses outlined in two pathways (Traditional and Integrated) and two model advanced courses, Precalculus and Advanced Quantitative Reasoning.

As described above, this Framework also includes a section in the high school content standards, entitled “Making Course Decisions about Course Sequences and the Model High School Algebra I Course.*”* This new section provides options for middle and high school course-taking sequences, including pathways that accelerate learning in order to allow students to reach advanced courses, such as Calculus, by the end of grade 12.

**The supplementary resources** that follow the learning standards address both engaging learners in content through the Standards for Mathematical Practice, and guidance in applying the standards for English language learners and students with disabilities. A [Glossary](#_Glossary:_Mathematical_Terms,) of mathematical terms, tables, illustrations, and a list of references is also included.

1. At the time, the agency was called the Department of Education. [↑](#footnote-ref-1)
2. Sealey, Cathy. *Balance is Basic, A 21st Century View of a Balanced Mathematical Program* [↑](#footnote-ref-2)