# MIModel Integrated Pathway: Model Mathematics I [MI]

## Introduction

The fundamental purpose of the Model Mathematics I course is to formalize and extend the mathematics that students learned in the middle grades.

For the high school Model Mathematics I course, instructional time should focus on six critical areas, each of which is described in more detail below: (1) extend understanding of numerical manipulation to algebraic manipulation; (2) synthesize understanding of function; (3) deepen and extend understanding of linear relationships; (4) apply linear models to data that exhibit a linear trend; (5) establish criteria for congruence based on rigid motions; and (6) apply the Pythagorean Theorem to the coordinate plane.

1. By the end of eighth grade students have had a variety of experiences working with expressions and creating equations. Students become facile with algebraic manipulation in much the same way that they are facile with numerical manipulation. Algebraic facility includes rearranging and collecting terms, factoring, identifying and canceling common factors in rational expressions, and applying properties of exponents. Students continue this work by using quantities to model and analyze situations, to interpret expressions, and to create equations to describe situations.
2. In earlier grades, students define, evaluate, and compare functions, and use them to model relationships among quantities. Students will learn function notation and develop the concepts of domain and range. They move beyond viewing functions as processes that take inputs and yield outputs and start viewing functions as objects in their own right. They explore many examples of functions, including sequences; interpret functions given graphically, numerically, symbolically, and verbally; translate between representations; and understand the limitations of various representations. They work with functions given by graphs and tables, keeping in mind that, depending upon the context, these representations are likely to be approximate and incomplete. Their work includes functions that can be described or approximated by formulas as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.
3. By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. Building on these earlier experiences, students analyze and explain the process of solving an equation, and justify the process used in solving a system of equations. Students develop fluency writing, interpreting, and translating among various forms of linear equations and inequalities, and use them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations. Students explore systems of equations and inequalities, and they find and interpret their solutions. All of this work is grounded on understanding quantities and on relationships among them.
4. Students’ prior experiences with data are the basis for the more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships among quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.
5. In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections, and rotations, and have used these to develop notions about what it means for two objects to be congruent. Students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.
6. Building on their work with the Pythagorean Theorem in eighth grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

## MIModel Integrated Pathway: Model Mathematics I Overview [MI]

#### Number and Quantity

###### Quantities

1. Reason quantitatively and use units to solve problems.

#### Algebra

###### **Standards for Mathematical Practice**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

###### Seeing Structure in Expressions

1. Interpret the structure of linear and exponential expressions with integer exponents.

###### Creating Equations

1. Create equations that describe numbers or relationships.

###### Reasoning with Equations and Inequalities

1. Understand solving equations as a process of reasoning and explain the reasoning.
2. Solve equations and inequalities in one variable.
3. Solve systems of equations.
4. Represent and solve equations and inequalities graphically.

#### Functions

###### Interpreting Functions

1. Understand the concept of a function and use function notation.
2. Interpret linear and exponential functions having integer exponents that arise in applications in terms of the context.
3. Analyze functions using different representations.

###### Building Functions

1. Build a function that models a relationship between two quantities.
2. Build new functions from existing functions.

###### Linear, Quadratic, and Exponential Models

1. Construct and compare linear and exponential models and solve problems.
2. Interpret expressions for functions in terms of the situation they model.

#### Geometry

###### Congruence

1. Experiment with transformations in the plane.
2. Understand congruence in terms of rigid motions.
3. Make geometric constructions.

###### Expressing Geometric Properties with Equations

1. Use coordinates to prove simple geometric theorems algebraically.

#### MIStatistics and Probability

###### Interpreting Categorical and Quantitative Data

1. Summarize, represent, and interpret data on a single count or measurement variable. Use calculators, spreadsheets, and other technology as appropriate.
2. Summarize, represent, and interpret data on two categorical and quantitative variables.
3. Interpret linear models.

## MIModel Integrated Pathway: Model Mathematics I Content Standards [MI]

#### Number and Quantity

##### Quantities MI.N-Q

A. Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems; and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.★
2. Define appropriate quantities for the purpose of descriptive modeling.★
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.★

#### Algebra

##### Seeing Structure in Expressions MI. A-SSE

A. Interpret the structure of linear and exponential expressions with integer exponents.

1. Interpret expressions that represent a quantity in terms of its context.★
	1. Interpret parts of an expression, such as terms, factors, and coefficients.
	2. Interpret complicated expressions by viewing one or more of their parts as a single entity*.*

 ***For example, interpret P(1 + r)n as the product of P and a factor not depending on P.***

##### Creating Equations MI. A-CED

A. Create equations that describe numbers or relationships.

1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and exponential functions with integer exponents.★
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★
3. Represent constraints by linear equations or inequalities, and by systems of linear equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.★

***For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.***

1. Rearrange formulas to highlight a quantity of interest, using the same reasoning (Properties of equality) as in solving equations.★

***For example, rearrange Ohm’s law, V = IR, to solve for resistance, R. Manipulate variables in formulas used in financial contexts such as for simple interest, I=*Pr*t .***

##### Reasoning with Equations and Inequalities MI.A-REI

A. Understand solving equations as a process of reasoning and explain the reasoning.

1. Explain each step in solving a simple linear equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify or refute a solution method.

B. Solve equations and inequalities in one variable.

1. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
	1. Solve linear equations and inequalities in one variable involving absolute value.

C. Solve systems of equations.

1. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
2. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

D. Represent and solve equations and inequalities graphically.

1. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Show that any point on the graph of an equation in two variables is a solution to the equation.
2. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions and/or make tables of values. Include cases where f(x) and/or g(x) are linear and exponential functions.★
3. Graph the solutions of a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set of a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

#### Functions

##### Interpreting Functions MI. F-IF

A. Understand the concept of a function and use function notation.

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

***For example, given a function representing a car loan, determine the balance of the loan at different points in time.***

1. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

***For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1,
f(n + 1) = f(n) + f(n − 1) for n ≥ 1.***

**B. Interpret linear and exponential functions having integer exponents that arise in applications in terms of the context.**

1. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.★
2. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.★

***For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.***

1. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

C. Analyze functions using different representations.

1. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
	1. Graph linear functions and show intercepts.★
2. Graph exponential functions, showing intercepts and end behavior.★
3. Translate among different representations of functions: (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.

***For example, given a graph of one exponential function and an algebraic expression for another, say which has the larger y-intercept.***

##### Building Functions MI.F-BF

A. Build a function that models a relationship between two quantities.

1. Write linear and exponential functions that describe a relationship between two quantities.★
	1. Determine an explicit expression, a recursive process, or steps for calculation from a context.★
	2. Combine standard function types using arithmetic operations.★

***For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.***

1. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★

B. Build new functions from existing functions.

1. Identify the effect on the graph of replacing f(x) by f(x) + k, *k*f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Include linear and exponential models. (Focus on vertical translations for exponential functions). Utilize technology to experiment with cases and illustrate an explanation of the effects on the graph.

##### Linear, Quadratic, and Exponential Models MI.F-LE

A. Construct and compare linear and exponential models and solve problems.

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.★
	1. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.★
	2. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.★
	3. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.★
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).★
3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly.★

B. Interpret expressions for functions in terms of the situation they model.

1. Interpret the parameters in a linear function or exponential function (of the form *f(x) = bx + k*)in terms of a context.★

#### Geometry

##### Congruence MI.G-CO

A. Experiment with transformations in the plane.

1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

B. Understand congruence in terms of rigid motions.

1. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
2. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
3. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

D. Make geometric constructions.

1. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.
2. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

##### Expressing Geometric Properties with Equations MI.G-GPE

B. Use coordinates to prove simple geometric theorems algebraically.

1. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
2. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles (e.g., using the distance formula).★

#### Statistics and Probability

##### Interpreting Categorical and Quantitative Data MI.S-ID

A. Summarize, represent, and interpret data on a single count or measurement variable. Use calculators, spreadsheets, and other technology as appropriate.

1. Represent data with plots on the real number line (dot plots, histograms, and box plots).★
2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.★
3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).★

B. Summarize, represent, and interpret data on two categorical and quantitative variables.

1. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.★
2. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.★
	1. Fit a linear function to the data and use the fitted function to solve problems in the context of the data. Use given functions fitted to data or choose a function suggested by the context. Emphasize linear and exponential models.★
	2. Informally assess the fit of a function by plotting and analyzing residuals.★
	3. Fit a linear function for a scatter plot that suggests a linear association.★

C. Interpret linear models.

1. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.★
2. Compute (using technology) and interpret the correlation coefficient of a linear fit.★
3. Distinguish between correlation and causation.★

# MIIModel Integrated Pathway: Model Mathematics II [MII]

## Introduction

The focus of the Model Mathematics II course is on quadratic expressions, equations, and functions; comparing their characteristics and behavior to those of linear and exponential relationships from Model Mathematics I.

For the high school Model Mathematics II course, instructional time should focus on five critical areas: (1) extend the laws of exponents to rational exponents; (2) compare key characteristics of quadratic functions with those of linear and exponential functions; (3) create and solve equations and inequalities involving linear, exponential, and quadratic expressions; (4) extend work with probability; and (5) establish criteria for similarity of triangles based on dilations and proportional reasoning.

1. Students extend the laws of exponents to rational exponents and explore distinctions between rational and irrational numbers by considering their decimal representations. Students learn that when quadratic equations do not have real solutions, the number system must be extended so that solutions exist; analogous to the way in which extending the whole numbers to the negative numbers allows *x* + 1 = 0 to have a solution. Students explore relationships between number systems: whole numbers, integers, rational numbers, real numbers, and complex numbers. The guiding principle is that equations with no solutions in one number system may have solutions in a larger number system.
2. Students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. When quadratic equations do not have real solutions, students learn that that the graph of the related quadratic function does not cross the horizontal axis. They expand their experience with functions to include more specialized functions—absolute value, step, and those that are piecewise-defined.
3. Students begin by focusing on the structure of expressions, rewriting expressions to clarify and reveal aspects of the relationship they represent. They create and solve equations, inequalities, and systems of equations involving exponential and quadratic expressions.
4. Building on probability concepts that began in the middle grades, students use the language of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions.
5. Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean Theorem. Students develop facility with geometric proof. They use what they know about congruence and similarity to prove theorems involving lines, angles, triangles, and other polygons. They explore a variety of formats for writing proofs.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

## MIIModel Integrated Pathway: Model Mathematics II Overview [MII]

#### Number and Quantity

###### The Real Number System

1. Extend the properties of exponents to rational exponents.
2. Use properties of rational and irrational numbers.

###### Quantities

1. Reason quantitatively and use units to solve problems.

###### **Standards for Mathematical Practice**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

###### The Complex Number Systems

1. Perform arithmetic operations with complex numbers.
2. Use complex numbers in polynomial identities and equations.

#### Algebra

###### Seeing Structure in Expressions

1. Interpret the structure of quadratic and exponential expressions.
2. Write quadratic and exponential expressions in equivalent forms to solve problems.

###### Arithmetic with Polynomials and Rational Expressions

1. Perform arithmetic operations on polynomials.

###### Creating Equations

1. Create equations that describe numbers or relationships.

###### Reasoning with Equations and Inequalities

1. Solve equations and inequalities in one variable.
2. Solve systems of equations.

#### Functions

###### Interpreting Functions

1. Interpret quadratic and exponential functions with integer exponents that arise in applications in terms of the context.
2. Analyze functions using different representations.

###### Building Functions

1. Build a function that models a relationship between two quantities.
2. Build new functions from existing functions.

###### Linear, Quadratic, and Exponential Models

1. Construct and compare linear, quadratic and exponential models and solve problems.

#### Geometry

###### Congruence

1. Prove geometric theorems, and when appropriate, the converse of theorems.

###### Similarity, Right Triangles, and Trigonometry

1. Understand similarity in terms of similarity transformations.
2. Prove theorems involving similarity using a variety of ways of writing proofs, showing validity of underlying reasoning.
3. Define trigonometric ratios and solve problems involving right triangles.

###### MIICircles

1. Understand and apply theorems about circles.
2. Find arc lengths and areas of sectors of circles.

###### Expressing Geometric Properties with Equations

1. Translate between the geometric description and the equation for a conic section.
2. Use coordinates to prove simple geometric theorems algebraically.

###### Geometric Measurement and Dimension

1. Explain volume formulas and use them to solve problems.

#### Statistics and Probability

###### Conditional Probability and the Rules of Probability

1. Understand independence and conditional probability and use them to interpret data from simulations or experiments.
2. Use the rules of probability to compute probabilities of compound events in a uniform probability model.

## MIIModel Integrated Pathway: Model Mathematics II Content Standards [MII]

#### Number and Quantity

##### The Real Number System MII.N-RN

**A. Extend the properties of exponents to rational exponents.**

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.

***For example, we define 51/3 to be the cube root of 5 because we want (51/3)3 = 5(1/3)3 to hold, so (51/3)3 must equal 5.***

1. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

**B. Use properties of rational and irrational numbers.**

1. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

##### Quantities MII.N-Q

**A. Reason quantitatively and use units to solve problems.**

1. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.★
	1. Describe the effects of approximate error in measurement and rounding on measurements and on computed values from measurements. Identify significant figures in recorded measures and computed values based on the context given and the precision of the tools used to measure

##### The Complex Number System MII.N-CN

**A. Perform arithmetic operations with complex numbers.**

1. Know there is a complex number *i* such that *i*2 = −1, and every complex number has the form *a* + *bi* with *a* and *b* real.
2. Use the relation *i*2 = –1 and the Commutative, Associative, and Distributive properties to add, subtract, and multiply complex numbers.

**C. Use complex numbers in polynomial identities and equations.**

1. Solve quadratic equations with real coefficients that have complex solutions.

#### Algebra

##### Seeing Structure in Expressions MII.A-SSE

**A. Interpret the structure of quadratic and exponential expressions.**

1. Interpret expressions that represent a quantity in terms of its context.★
	1. Interpret parts of an expression, such as terms, factors, and coefficients.
	2. Interpret complicated expressions by viewing one or more of their parts as a single entity*.*

***For example, interpret P(1 + r)t as the product of P and a factor not depending on P.***

1. Use the structure of an expression to identify ways to rewrite it.
 For example, see (x + 2)2 – 9 as a difference of squares that can be factored as ((x + 2) + 3)((x + 2) – 3).

**B. Write quadratic and exponential expressions in equivalent forms to solve problems.**

1. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
	1. Factor a quadratic expression to reveal the zeros of the function it defines.
	2. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
	3. Use the properties of exponents to transform expressions for exponential functions.

***For example, the expression 1.15t can be rewritten as (1.15 1/12)12t ≈ 1.01212t to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.***

##### Arithmetic with Polynomials and Rational Expressions MII.A-APR

**A. Perform arithmetic operations on polynomials.**

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under certain operations.
	1. Perform operations on polynomial expressions (addition, subtraction, multiplication), and compare the system of polynomials to the system of integers when performing operations.
	2. Factor and/or expand polynomial expressions; identify and combine like terms; and apply the Distributive property.

##### Creating Equations MII.A-CED

**A. Create equations that describe numbers or relationships.**

1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from quadratic and exponential functions.
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★
3. Rearrange formulas, including formulas with quadratic terms, to highlight a quantity of interest using the same reasoning as in solving equations (Properties of equality).

***For example, rearrange Ohm’s law*** ***to solve for voltage, V.***

##### Reasoning with Equations and Inequalities MII.A-REI

**B. Solve equations and inequalities in one variable.**

1. Solve quadratic equations in one variable.
	1. Use the method of completing the square to transform any quadratic equation in *x* into an equation of the form (*x* – *p*)2 = *q* that has the same solutions. Derive the quadratic formula from this form.
	2. Solve quadratic equations by inspection (e.g., for *x*2 = 49), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as *a* ± *bi* for real numbers *a* and *b*.

**C. Solve systems of equations.**

1. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

***For example, find the points of intersection between the line y = –3x and the circle x2 + y2 = 3.***

#### Functions

##### Interpreting Functions MII.F-IF

**B. Interpret quadratic and exponential functionswith integer exponents that arise in applications in terms of the context.**

1. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.★
2. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.★

***For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.***

1. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

**C. Analyze functions using different representations.**

1. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
	1. Graph quadratic functions and show intercepts, maxima, and minima.★
	2. Graph piecewise-defined functions, including step functions and absolute value functions.★
2. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
3. Use the process of factoring and completing the square in a quadratic function to show zeros, minimum/maximum values, and symmetry of the graph and interpret these in terms of a context.
4. Use the properties of exponents to interpret expressions for exponential functions. Apply to financial situations such as Identifying appreciation/depreciation rate for the value of a
house or car some time after its initial purchase:.

***For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, and y = (1.2) t ∕10, and classify them as representing exponential growth or decay.***

1. Translate among different representations of functions (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.

***For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.***

##### Building Functions MII.F-BF

**A. Build a function that models a relationship between two quantities.**

1. Write linear, quadratic, and exponential functions that describe relationships between two quantities.★
	1. Determine an explicit expression, a recursive process, or steps for calculation from a context.★
	2. Combine standard function types using arithmetic operations.★

***For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.***

**B. Build new functions from existing functions.**

1. Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *k f*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Include exponential, quadratic, and absolute value functions. Utilize technology to experiment with cases and illustrate an explanation of the effects on the graph.
2. Find inverse functions algebraically and graphically.
	1. Solve an equation of the form *f*(*x*) = *c* for a linear function *f* that has an inverse and write an expression for the inverse.

##### Linear, Quadratic, and Exponential Models MII.F-LE

**A. Construct and compare linear, quadratic, and exponential models and solve problems.**

1. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.★

#### MIIGeometry

##### Congruence MII.G-CO

**C. Prove geometric theorems and, when appropriate, the converse of theorems.**

1. Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent, and conversely prove lines are parallel; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment’s endpoints.
2. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
3. Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.
	1. Prove theorems about polygons. Theorems include the measures of interior and exterior angles. Apply properties of polygons to the solutions of mathematical and contextual problems.

##### Similarity, Right Triangles, and Trigonometry MII.G-SRT

**A. Understand similarity in terms of similarity transformations.**

1. Verify experimentally the properties of dilations given by a center and a scale factor:
	1. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
	2. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
3. Use the properties of similarity transformations to establish the Angle-Angle (AA) criterion for two triangles to be similar.

**B. Prove theorems involving similarity using a variety of ways of writing proofs, showing validity of underlying reasoning.**

1. Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.
2. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

**C. Define trigonometric ratios and solve problems involving right triangles.**

1. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
2. Explain and use the relationship between the sine and cosine of complementary angles.
3. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★

##### Circles MII.G-C

**A. Understand and apply theorems about circles.**

1. Prove that all circles are similar.
2. Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral and other polygons inscribed in a circle.
4. (+) Construct a tangent line from a point outside a given circle to the circle.

**B. Find arc lengths and areas of sectors of circles.**

1. Derive, using similarity, the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

##### Expressing Geometric Properties with Equations MII.G-GPE

**A. Translate between the geometric description and the equation for a conic section.**

1. Derive the equation of a circleof given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
2. Derive the equation of a parabola given a focus and directrix.

**B. Use coordinates to prove simple geometric theorems algebraically.**

1. Use coordinates to prove simple geometric theorems algebraically including the distance formula and its relationship to the Pythagorean Theorem*.*

***For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1,******) lies on the circle centered at the origin and containing the point (0, 2).***

1. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

##### Geometric Measurement and Dimension MII.G-GMD

**A. Explain volume formulas and use them to solve problems.**

1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri’s principle, and informal limit arguments.
2. (+) Give an informal argument using Cavalieri’s principle for the formulas for the volume of a sphere and other solid figures.
3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★

#### Statistics and Probability

##### Conditional Probability and the Rules of Probability MII.S-CP

**A. Understand independence and conditional probability and use them to interpret data from simulations or experiments.**

1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).★
2. Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.★
3. Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.★
4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities*.*★

***For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.***

1. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.★

***For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.***

**B. Use the rules of probability to compute probabilities of compound events in a uniform probability model.**

1. Find the conditional probability of *A* given *B* as the fraction of *B*’s outcomes that also belong to *A*, and interpret the answer in terms of the model.★
2. Apply the Addition Rule, *P*(*A* or *B*) = *P*(*A*) + *P*(*B*) – *P*(*A* and *B*), and interpret the answer in terms of the model.★
3. (+) Apply the general Multiplication Rule in a uniform probability model, *P*(*A* and *B*) = *P*(*A*)*P*(*B*|*A*) = *P*(*B*)*P*(*A*|*B*), and interpret the answer in terms of the model.★
4. (+) Use permutations and combinations to compute probabilities of compound events and solve problems.★

# MIIIModel Integrated Pathway: Model Mathematics III [MIII]

## Introduction

It is in the Model Mathematics III course that students integrate and apply the mathematics they have learned from their earlier courses. For the high school Model Mathematics III course, instructional time should focus on four critical areas: (1) apply methods from probability and statistics to draw inferences and conclusions from data; (2) expand understanding of functions to include polynomial, rational, and radical functions;[[1]](#footnote-1) (3) expand right triangle trigonometry to include general triangles; and (4) consolidate functions and geometry to create models and solve contextual problems.

1. Students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data— including sample surveys, experiments, and simulations—and the roles that randomness and careful design play in the conclusions that can be drawn.
2. The structural similarities between the system of polynomials and the system of integers are developed. Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students identify zeros of polynomials and make connections between zeros of polynomials and solutions of polynomial equations. Rational numbers extend the arithmetic of integers by allowing division by all numbers except zero. Similarly, rational expressions extend the arithmetic of polynomials by allowing division by all polynomials except the zero polynomial. A central theme of the Model Mathematics III course is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers. This critical area also includes exploration of the Fundamental Theorem of Algebra.
3. Students derive the Laws of Sines and Cosines in order to find missing measures of general (not necessarily right) triangles. They are able to distinguish whether three given measures (angles or sides) define 0, 1, 2, or infinitely many triangles. This discussion of general triangles opens up the idea of trigonometry applied beyond the right triangle, at least to obtuse angles. Students build on this idea to develop the notion of radian measure for angles and extend the domain of the trigonometric functions to all real numbers. They apply this knowledge to model simple periodic phenomena.
4. Students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of underlying function. They identify appropriate types of functions to model a situation; they adjust parameters to improve the model; and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. The description of modeling as “the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions” is at the heart of this Model Mathematics III course. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

## MIIIModel Integrated Pathway: Model Mathematics III Overview [MIII]

#### Number and Quantity

###### **Standards for Mathematical Practice**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

###### The Complex Number Systems

1. Use complex numbers in polynomial identities and equations.

###### Vector and Matrix Quantities

1. Represent and model with vector quantities.
2. Perform operations on matrices and use matrices in applications.

#### Algebra

###### Seeing Structure in Expressions

1. Interpret the structure polynomial and rational expressions.
2. Write expressions in equivalent forms to solve problems.

###### Arithmetic with Polynomials and Rational Expressions

1. Perform arithmetic operations on polynomials.
2. Understand the relationship between zeros and factors of polynomials.
3. Use polynomial identities to solve problems.
4. Rewrite rational expressions.

###### Creating Equations

1. Create equations that describe numbers or relationships.

###### Reasoning with Equations and Inequalities

1. Understand solving equations as a process of reasoning and explain the reasoning.
2. Represent and solve equations and inequalities graphically.

#### Functions

###### Interpreting Functions

1. Interpret functions that arise in applications in terms of the context (rational, polynomial, square root, cube root, trigonometric, logarithmic).
2. Analyze functions using different representations.

###### Building Functions

1. Build a function that models a relationship between two quantities.
2. Build new functions from existing functions.

###### Linear, Quadratic, and Exponential Models

1. Construct and compare linear, quadratic and exponential models and solve problems.

###### Trigonometric Functions

1. Extend the domain of trigonometric functions using the unit circle.
2. Model periodic phenomena with trigonometric functions.
3. Prove and apply trigonometric identities.

#### MIIIGeometry

###### Similarity, Right Triangles, and Trigonometry

1. Apply trigonometry to general triangles.

###### Geometric Measurement and Dimension

1. Visualize relationships between two-dimensional and three-dimensional objects.

###### Modeling with Geometry

1. Apply geometric concepts in modeling situations.

#### Statistics and Probability

*Interpreting Categorical and Quantitative Data*

1. Summarize, represent, and interpret data on a single count or measurement variable. Use calculators, spreadsheets, and other technology as appropriate.

*Making Inferences and Justifying Conclusions*

1. Understand and evaluate random processes underlying statistical experiments.
2. Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

*Using Probability to Make Decisions*

1. Use probability to evaluate outcomes of decisions.

## MIIIModel Integrated Pathway: Model Mathematics III Content Standards [MIII]

#### Number and Quantity

##### The Complex Number System MIII.N-CN

**C. Use complex numbers in polynomial identities and equations.**

1. (+) Extend polynomial identities to the complex numbers.

***For example, rewrite x2 + 4 as (x + 2i)(x – 2i).***

1. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

##### Vector and Matrix Quantities MIII.N-VM

**A. Represent and model with vector quantities.**

1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., ***v***, |***v***|, ||***v***||, *v*).
2. (+) Solve problems involving velocity and other quantities that can be represented by vectors.

**C. Perform operations on matrices and use matrices in applications.**

1. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
2. (+) Add, subtract, and multiply matrices of appropriate dimensions.
3. (+) Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

#### Algebra

##### Seeing Structure in Expressions MIII.A-SSE

**A. Interpret the structure of polynomial and rational expressions.**

1. Interpret expressions that represent a quantity in terms of its context.★
	1. Interpret parts of an expression, such as terms, factors, and coefficients.
	2. Interpret complicated expressions by viewing one or more of their parts as a single entity*.*

 ***For example, interpret P(1 + r)n as the product of P and a factor not depending on P.***

1. Use the structure of an expression to identify ways to rewrite it.

***For example, see x4 – y4 as (x2)2 – (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 – y2)(x2 + y2) and as (x-y)(x+y)(x-yi)(x+yi).***

**B. Write expressions in equivalent forms to solve problems.**

1. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.★

***For example, calculate mortgage payments.***

##### Arithmetic with Polynomials and Rational Expressions MIII.A-APR

**A. Perform arithmetic operations on polynomials.**

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under certain operations.
	1. Perform operations on polynomial expressions (addition, subtraction, multiplication, and division), and compare the system of polynomials to the system of integers when performing operations.

**B. Understand the relationship between zeros and factors of polynomials.**

1. Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).
2. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

**C. Use polynomial identities to solve problems.**

1. Prove polynomial identities and use them to describe numerical relationships.

***For example, the polynomial identity (x2 + y2)2 = (x2 – y2)2 + (2xy)2 can be used to generate Pythagorean triples.***

1. (+) Know and apply the Binomial Theorem for the expansion of (*x* + *y*)*n* in powers of *x* and *y* for a positive integer *n*, where *x* and *y* are any numbers, with coefficients determined for example by Pascal’s Triangle.

**D. Rewrite rational expressions.**

1. Rewrite simple rational expressions in different forms; write ***a*(*x*)∕*b*(*x*)** in the form q(x) + ***r*(*x*)∕*b*(*x*)**, where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.
2. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

##### Creating Equations MIII.A-CED

**A. Create equations that describe numbers or relationships.**

1. Create equations and inequalities in one variable and use them to solve problems. (Include equations arising from simple root and rational functions.)★
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★
3. Represent constraints by equations or inequalities and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.★

***For example, represent equations describing satellites orbiting earth and constraints on earth’s size and atmosphere.***

##### Reasoning with Equations and Inequalities MIII.A-REI

**A. Understand solving equations as a process of reasoning and explain the reasoning.**

1. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

**D. Represent and solve equations and inequalities graphically.**

1. Explain why the *x*-coordinates of the points where the graphs of the equations *y* = *f*(*x*) and *y* = *g*(*x*) intersect are the solutions of the equation *f*(*x*) = *g*(*x*); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f*(*x*) and/or *g*(*x*) are polynomial, rational, and logarithmic functions.★

#### Functions

##### Interpreting Functions MIII.F-IF

**B. Interpret functions that arise in applications in terms of the context (rational, polynomial, square root, cube root, trigonometric, logarithmic).**

1. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★
2. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.★

***For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.***

1. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

**C. Analyze functions using different representations.**

1. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
2. Graph square root and cube root functions.★
3. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.★
4. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.★
5. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
	1. Use the process of factoring in a polynomial function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
6. Translate among different representations of functions: (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way.

***For example, given a graph of one polynomial function and an algebraic expression for another, say which has the larger relative maximum and/or smaller relative minimum.***

1. Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, exponential, or trigonometric.

##### Building Functions MIII.F-BF

**A. Build a function that models a relationship between two quantities.**

1. Write simple rational and radical functions, logarithmic, and trigonometric functions that describes a relationship between two quantities.★
2. Combine standard function types using arithmetic operations.★

***For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model (include simple rational and radical functions, logarithmic, and trigonometric functions).***

**B. Build new functions from existing functions.**

1. Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *kf*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Include simple rational, radical, logarithmic, and trigonometric functions. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
2. Find inverse functions algebraically and graphically.
	1. Solve an equation of the form *f*(*x*) = *c* for a simple function *f* that has an inverse and write an expression for the inverse.

***For example, f(x) =2x3 or f(x) = (x + 1)∕(x − 1) for x ≠ 1.***

##### Linear, Quadratic, and Exponential Models MIII.F-LE

**A. Construct and compare linear, quadratic, and exponential models and solve problems.**

1. For exponential models, express as a logarithm the solution to *abct* = *d* where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology.★

##### Trigonometric Functions MIII.F-TF

**A. Extend the domain of trigonometric functions using the unit circle.**

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

**B. Model periodic phenomena with trigonometric functions.**

1. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★

**C. Prove and apply trigonometric identities.**

1. Prove the Pythagorean identity sin2(θ) + cos2(θ) = 1 and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant.

#### Geometry

##### Similarity, Right Triangles, and Trigonometry MIII.G-SRT

**D. Apply trigonometry to general triangles.**

1. (+) Derive the formula *A* = ½ *ab* sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
2. (+) Prove the Laws of Sines and Cosines and use them to solve problems.
3. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

##### Geometric Measurement and Dimension MIII.G-GMD

**B. Visualize relationships between two-dimensional and three-dimensional objects.**

1. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

##### Modeling with Geometry MIII.G-MG

**A. Apply geometric concepts in modeling situations.**

1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★
2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★
3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★
4. Use dimensional analysis for unit conversions to confirm that expressions and equations make sense.★

#### Statistics and Probability

##### Interpreting Categorical and Quantitative Data MIII.S-ID

**A. Summarize, represent, and interpret data on a single count or measurement variable. Use calculators, spreadsheets, and other technology as appropriate.**

1. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.★

##### Making Inferences and Justifying Conclusions MIII.S-IC

**A. Understand and evaluate random processes underlying statistical experiments.**

1. Understand statistics as a process for making inferences to be made about population parameters based on a random sample from that population.★
2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.★

***For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of five tails in a row cause you to question the model?***

**B. Make inferences and justify conclusions from sample surveys, experiments, and observational studies.**

1. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.★
2. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.★
3. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.★
4. Evaluate reports based on data.★

##### Using Probability to Make Decisions MIII.S-MD

**B. Use probability to evaluate outcomes of decisions.**

1. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).★
2. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game and replacing the goalie with an extra skater).★
1. In this course, rational functions are limited to those whose numerators are of degree at most 1 and denominators are of degree at most 2; radical functions are limited to square roots or cube roots of at most quadratic polynomials. [↑](#footnote-ref-1)