## Organization of the Pre-Kindergarten to Grade 8 Content Standards

The pre-kindergarten through grade 8 content standards in this Framework are organized by **grade level**. Within each grade level, standards are grouped first by **domain**. Each domain is further subdivided into **clusters** of related standards.

* **Standards** define what students should understand and be able to do.
* **Clusters** are groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.
* **Domains** are larger groups of related standards. Standards from different domains may sometimes be closely related.

The table below shows which domains are addressed at each grade level:

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Progression of Pre-K–8 Domains** | | | | | | | | | | |
| **Domain** | **Grade Level** | | | | | | | | | |
| **PK** | **K** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** |
| Counting and Cardinality |  |  |  |  |  |  |  |  |  |  |
| Operations and Algebraic Thinking |  |  |  |  |  |  |  |  |  |  |
| Number and Operations in Base Ten |  |  |  |  |  |  |  |  |  |  |
| Number and Operations – Fractions |  |  |  |  |  |  |  |  |  |  |
| The Number System |  |  |  |  |  |  |  |  |  |  |
| Ratios and Proportional Relationships |  |  |  |  |  |  |  |  |  |  |
| Expressions and Equations |  |  |  |  |  |  |  |  |  |  |
| Functions |  |  |  |  |  |  |  |  |  |  |
| Measurement and Data |  |  |  |  |  |  |  |  |  |  |
| Geometry |  |  |  |  |  |  |  |  |  |  |
| Statistics and Probability |  |  |  |  |  |  |  |  |  |  |

##### Format for Each Grade Level

Each grade level is presented in the same format:

* An introduction and description of the critical areas for learning at that grade.
* An overview of that grade’s domains and clusters.
* The content standards for that grade (presented by domain, cluster heading, and individual standard).

##### Standards Identifiers/Coding

Each standard has a unique identifier that consists of the grade level, (PK, K, 1, 2, 3, 4, 5, 6, 7, or 8), the domain code, and the standard number, as shown in the example below.

##### Grade 1 Content Standards Identifiers/Coding

The first standard above is identified as 1.G.A.1, identifying it as a grade 1 standard in the Geometry Domain, and as the first standard in that domain. Standard 1.G.A.1 is the first standard in this cluster of standards. All of the standards in this Framework use a common coding system.

# PKPre-Kindergarten

## Introduction

The pre-kindergarten standards presented by Massachusetts are guideposts to facilitate young children’s underlying mathematical understanding. The Massachusetts pre-kindergarten standards apply to children who are in the age group of older four- and younger five-year olds. The standards—which correspond with the learning activities in the *Massachusetts Guidelines for Preschool Learning Experiences* (2003)—can be promoted through play and exploration activities, and embedded in almost all daily activities. They should not be limited to “math time.” In this age group, foundations of mathematical understanding are formed out of children’s experiences with real objects and materials.

In preschool or pre-kindergarten, activity time should focus on two critical areas: (1) developing an understanding of whole numbers to 10, including concepts of one-to-one correspondence, counting, cardinality (the number of items in a set), and comparison; and (2) recognizing two-dimensional shapes, describing spatial relationships, and sorting and classifying objects by one or more attributes. Relatively more learning time should be devoted to developing children’s sense of number as quantity than to other mathematics topics.

1. Young children begin counting and quantifying numbers up to 10. They begin with oral counting and recognition of numerals and word names for numbers. Experience with counting naturally leads to quantification. Children count objects and learn that the sizes, shapes, positions, or purposes of objects do not affect the total number of objects in the group. One-to-one correspondence matches each element of one set to an element of another set, providing a foundation for the comparison of groups and the development of comparative language such as *more than, less than,* and *equal to*.
2. Young children explore shapes and the relationships among them. They identify the attributes of different shapes, including length, area, and weight, by using vocabulary such as *long, short, tall, heavy, light, big, small, wide, narrow*. They compare objects using comparative language such as *longer/shorter, same length, heavier/lighter*. They explore and create two- and three-dimensional shapes by using various manipulative and play materials such as Popsicle sticks, blocks, pipe cleaners, and pattern blocks. They sort, categorize, and classify objects and identify basic two-dimensional shapes using the appropriate language.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

## PKPre-Kindergarten Overview

#### Counting and Cardinality

###### **Standards for Mathematical Practice**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
9. Know number names and the counting sequence.
10. Count to tell the number of objects.
11. Compare numbers.

#### Operations and Algebraic Thinking

1. Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

#### Measurement and Data

1. Describe and compare measurable attributes.
2. Classify objects and count the number of objects in each category.
3. Work with money.

#### Geometry

1. Identify and describe shapes (squares, circles, triangles, rectangles).
2. Analyze, compare, create, and compose shapes.

## PKPre-Kindergarten Content Standards

**Counting and Cardinality** **PK.CC**

A. Know number names and the counting sequence.

1. Listen to and say the names of numbers in meaningful contexts.
2. Recognize and name written numerals 0–10.

B. Count to tell the number of objects.

1. Understand the relationships between numerals and quantities up to 10.

C. Compare numbers.

1. Count many kinds of concrete objects and actions up to ten, using one-to-one correspondence, and accurately count as many as seven things in a scattered configuration. Recognize the “one more,” “one less” patterns.
2. Use comparative language, such as *more/less than, equal to,* to compare and describe collections of objects.

#### Operations and Algebraic Thinking PK.OA

A. Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

1. Use concrete objects to model real-world addition (putting together) and subtraction (taking away) problems up through five.

#### Measurement and Data PK.MD

A. Describe and compare measurable attributes.

1. Recognize the attributes of length, area, weight, and capacity of everyday objects using appropriate vocabulary (e.g., *long, short, tall, heavy, light, big, small, wide, narrow*).
2. Compare the attributes of length and weight for two objects, including longer/shorter, same length; heavier/lighter, same weight; holds more/less, holds the same amount.

B. Classify objects and count the number of objects in each category.

1. Sort, categorize, and classify objects by more than one attribute.

C. Work with money.

1. Recognize that certain objects are coins and that dollars and coins represent money.

#### Geometry PK.G

A. Identify and describe shapes (squares, circles, triangles, rectangles).

1. Identify relative positions of objects in space, and use appropriate language (e.g., *beside, inside, next to, close to, above, below, apart*).
2. Identify various two-dimensional shapes using appropriate language.

B. Analyze, compare, create, and compose shapes.

1. Create and represent three-dimensional shapes (ball/sphere, square box/cube, tube/cylinder) using various manipulative materials (such as Popsicle sticks, blocks, pipe cleaners, pattern blocks).

# KKindergarten

## Introduction

In kindergarten, instructional time should focus on two critical areas: (1) representing, relating, and operating on whole numbers, initially with sets of objects; and (2) describing shapes and space. More learning time in kindergarten should be devoted to number than to other topics.

1. Students use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects, or eventually with equations such as 5 + 2 = 7 and 7 – 2 = 5. (Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.) Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.
2. Students describe their physical world using geometric ideas (e.g., shape, orientation, spatial relations) and vocabulary. They identify, name, and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles, and hexagons, presented in a variety of ways (e.g., with different sizes and orientations), as well as three-dimensional shapes such as cubes, cones, cylinders, and spheres. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

## KKindergarten Overview

###### **Standards for Mathematical Practice**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

#### Counting and Cardinality

1. Know number names and the counting sequence.
2. Count to tell the number of objects.
3. Compare numbers.

#### Operations and Algebraic Thinking

1. Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

#### Number and Operations in Base Ten

1. Work with numbers 11–19 to gain foundations for place value.

#### Measurement and Data

1. Describe and compare measurable attributes.
2. Classify objects and count the number of objects in each category.

#### Geometry

1. Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).
2. Analyze, compare, create, and compose shapes.

## KKindergarten Content Standards

**Counting and Cardinality** **K.CC**

A. Know number names and the count sequence.

1. Count to 100 by ones and by tens.
2. Count forward beginning from a given number within the known sequence   
   (instead of having to begin at one).
3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).

B. Count to tell the number of objects.

1. Understand the relationship between numbers and quantities; connect counting to cardinality.
   1. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
   2. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
   3. Understand that each successive number name refers to a quantity that is one larger. Recognize the one more pattern of counting using objects.
2. Count to answer “how many?” questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

C. Compare numbers.

1. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group for groups with up to 10 objects, e.g., by using matching and counting strategies.
2. Compare two numbers between 1 and 10 presented as written numerals.

#### Operations and Algebraic Thinking K.OA

A. Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

1. Represent addition and subtraction with objects, fingers, mental images, drawings,[[1]](#footnote-1) sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.
2. Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.
3. Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).
4. For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.
5. Fluently add and subtract within 5, including zero.

#### Number and Operations in Base Ten K.NBT

A. Work with numbers 11–19 to gain foundations for place value.

1. Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

#### KMeasurement and Data K.MD

A. Describe and compare measurable attributes.

1. Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.
2. Directly compare two objects with a measurable attribute in common, to see which object has “more of”/“less of” the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.*

B. Classify objects and count the number of objects in each category.

1. Classify objects into given categories; count the numbers of objects in each category (up to and including 10) and sort the categories by count.

#### G**eometry K.G**

A. Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

1. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*.
2. Correctly name shapes regardless of their orientation or overall size.
3. Identify shapes as two-dimensional (lying in a plane, “flat”) or three-dimensional (“solid”).

B. Analyze, compare, create, and compose shapes.

1. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/“corners”) and other attributes (e.g., having sides of equal length).
2. Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.
3. Compose simple shapes to form larger shapes.

**For example, “Can you join these two triangles with full sides touching to make a rectangle?”**

# 1Grade 1

## Introduction

In grade 1, instructional time should focus on four critical areas: (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; (3) developing understanding of linear measurement and measuring lengths as iterating length units; and (4) reasoning about attributes of, and composing and decomposing geometric shapes.

1. Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use a variety of models, including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction, and to develop strategies to solve arithmetic problems with these operations. Students understand connections between counting and addition and subtraction (e.g., adding two is the same as counting on two). They use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., “making tens”) to solve addition and subtraction problems within 20. By comparing a variety of solution strategies, children build their understanding of the relationship between addition and subtraction.
2. Students develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10. They compare whole numbers (at least to 100) to develop an understanding of and solve problems involving their relative sizes. They think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes.
3. Students develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement.[[2]](#footnote-2)
4. Students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

## 1Grade 1 Overview

#### Operations and Algebraic Thinking

1. Represent and solve problems involving addition and subtraction.

###### **Standards for Mathematical Practice**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
9. Understand and apply properties of operations and the relationship between addition and subtraction.
10. Add and subtract within 20.
11. Work with addition and subtraction equations.

#### Number and Operations in Base Ten

1. Extend the counting sequence.
2. Understand place value.
3. Use place value understanding and properties of operations to add and subtract.

#### Measurement and Data

1. Measure lengths indirectly and by iterating length units.
2. Tell and write time.
3. Represent and interpret data.
4. Work with money.

#### Geometry

1. Reason with shapes and their attributes.

## 1Grade 1 Content Standards

#### Operations and Algebraic Thinking 1.OA

**A. Represent and solve problems involving addition and subtraction.**

1. Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations (number sentences) with a symbol for the unknown number to represent the problem.[[3]](#footnote-3)
2. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

**B. Understand and apply properties of operations and the relationship between addition and subtraction.**

1. Apply properties of operations to add. [[4]](#footnote-4)

**For example, when adding numbers order does not matter. If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known (Commutative property of addition). To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12 (Associative property of addition). When adding zero to a number, the result is the same number (Identity property of zero for addition).**

1. Understand subtraction as an unknown-addend problem. *For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.*

**C. Add and subtract within 20.**

1. Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).
2. Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use mental strategies such as counting on; making 10 (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a 10 (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

**D. Work with addition and subtraction equations.**

1. Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false.

**For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.**

1. Determine the unknown whole number in an addition or subtraction equation relating three whole numbers.

**For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = – 3, 6 + 6 = .**

#### Number and Operations in Base Ten 1.NBT

**A. Extend the counting sequence.**

1. Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.

**B. Understand place value.**

1. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:
   1. 10 can be thought of as a bundle of ten ones—called a “ten.”
   2. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
   3. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).
2. Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.

**C. Use place value understanding and properties of operations to add and subtract.**

1. Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings, and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.
2. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used. Identify arithmetic patterns of 10 more and 10 less than using strategies based on place value.
3. Subtract multiples of 10 in the range 10–90 from multiples of 10 in the range 10–90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

#### Measurement and Data 1.MD

**A. Measure lengths indirectly and by iterating length units.**

1. Order three objects by length; compare the lengths of two objects indirectly by using a third object.
2. Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.*

**B. Tell and write time.**

1. Tell and write time in hours and half-hours using analog and digital clocks.

**C. Represent and interpret data.**

1. Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.

**D. Work with money.**

1. Identify the values of all U.S. coins and know their comparative values (e.g., a dime is of greater value than a nickel). Find equivalent values (e.g., a nickel is equivalent to five pennies). Use appropriate notation (e.g., 69¢). Use the values of coins in the solutions of problems (up to 100¢).

#### Geometry 1.G

**A. Reason with shapes and their attributes.**

1. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes that possess defining attributes.
2. Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.[[5]](#footnote-5)
3. Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of.* Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

# 2Grade 2

## Introduction

In grade 2, instructional time should focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes.

1. Students extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1,000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).
2. Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1,000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.
3. Students recognize the need for standard units of measure (centimeter and inch) and they use rulers and other measurement tools with the understanding that linear measure involves an iteration of units. They recognize that the smaller the unit, the more iterations they need to cover a given length.
4. Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

## 2Grade 2 Overview

#### Operations and Algebraic Thinking

###### **Standards for Mathematical Practice**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
9. Represent and solve problems involving addition and subtraction.
10. Add and subtract within 20.
11. Work with equal groups of objects to gain foundations for multiplication.

#### Number and Operations in Base Ten

1. Understand place value.
2. Use place value understanding and properties of operations to add and subtract.

#### Measurement and Data

1. Measure lengths indirectly and by iterating length units.
2. Relate addition and subtraction to length.
3. Work with time and money.
4. Represent and interpret data.

#### Geometry

1. Reason with shapes and their attributes.

## 2Grade 2 Content Standards

#### Operations and Algebraic Thinking 2.OA

**A. Represent and solve problems involving addition and subtraction.**

1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[[6]](#footnote-6)

**B. Add and subtract within 20.**

1. Fluently add and subtract within 20 using mental strategies.[[7]](#footnote-7) By end of grade 2, know from memory all sums of two single-digit numbers and related differences.

**For example, the sum 6 + 5 = 11 has related differences of 11 – 5 = 6 and 11 – 6 = 5.**

**C. Work with equal groups of objects to gain foundations for multiplication.**

1. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.
2. Use addition to find the total number of objects arranged in rectangular arrays with up to five rows and up to five columns; write an equation to express the total as a sum of equal addends.

#### Number and Operations in Base Ten 2.NBT

**A. Understand place value.**

1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:
   1. 100 can be thought of as a bundle of ten tens—called a “hundred.”
   2. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
2. Count within 1,000; skip-count by 5s, 10s, and 100s. Identify patterns in skip counting starting at any number.
3. Read and write numbers to 1,000 using base-ten numerals, number names, and expanded form.
4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

**B. Use place value understanding and properties of operations to add and subtract.**

1. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
2. Add up to four two-digit numbers using strategies based on place value and properties of operations.
3. Add and subtract within 1,000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.
4. Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.
5. Explain why addition and subtraction strategies work, using place value and the properties of operations.[[8]](#footnote-8)

#### 2Measurement and Data 2.MD

**A. Measure and estimate lengths in standard units.**

1. Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.
2. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.
3. Estimate lengths using units of inches, feet, centimeters, and meters.
4. Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

**B. Relate addition and subtraction to length.**

1. Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.
2. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, …, and represent whole-number sums and differences within 100 on a number line diagram.

**C. Work with time and money.**

1. Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.
   1. Know the relationships of time, including seconds in a minute, minutes in an hour, hours in a day, days in a week; days in a month and a year and approximate number of weeks in a month and weeks in a year.
2. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies (up to $10), using $ and ¢ symbols appropriately and whole dollar amounts.

**For example, if you have 2 dimes and 3 pennies, how many cents do you have? If you have $3 and 4 quarters, how many dollars or cents do you have? (Students are not expected to use decimal notation.)**

**D. Represent and interpret data.**

1. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Organize and record the data on a line plot (dot plot) where the horizontal scale is marked off in whole-number units.
2. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems,[[9]](#footnote-9) using information presented in a bar graph.

#### Geometry 2.G

**A. Reason with shapes and their attributes.**

1. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.[[10]](#footnote-10) Identify triangles, squares, rectangles, rhombuses, trapezoids, pentagons, hexagons, and cubes.
2. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.
3. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves*, *thirds*, *half of*, *a third of*, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

# 3Grade 3

## Introduction

In grade 3, instructional time should focus on four critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes.

1. Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.
2. Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, **1**∕**2** of the paint in a small bucket could be less paint than **1**∕**3** of the paint in a larger bucket, but **1**∕**3** of a ribbon is longer than **1**∕**5** of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.
3. Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps; a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.
4. Students describe, analyze, and compare properties of two-dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

## 3Grade 3 Overview

#### Operations and Algebraic Thinking

1. Represent and solve problems involving multiplication and division.

###### **Standards for Mathematical Practice**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
9. Understand properties of multiplication and the relationship between multiplication and division.
10. Multiply and divide within 100.
11. Solve problems involving the four operations, and identify and explain patterns in arithmetic.

#### Number and Operations in Base Ten

1. Use place value understanding and properties of operations to perform multi-digit arithmetic.

#### Number and Operations—Fractions

1. Develop understanding of fractions as numbers.

#### Measurement and Data

1. Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.
2. Represent and interpret data.
3. Geometric measurement: understand concepts of area and relate area to multiplication and to addition.
4. Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

#### Geometry

1. Reason with shapes and their attributes.

## 3Grade 3 Content Standards

#### Operations and Algebraic Thinking 3.OA

A. Represent and solve problems involving multiplication and division.

1. Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in five groups of seven objects each.

**For example, describe a context in which a total number of objects can be expressed as 5 × 7.**

1. Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each.

**For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.**

1. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities,e.g., by using drawings and equations with a symbol for the unknown number to represent the problem*.*[[11]](#footnote-11)
2. Determine the unknown whole number in a multiplication or division equation relating three whole numbers.

**For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = ÷ 3, 6 × 6 = ?.**

B. Understand properties of multiplication and the relationship between multiplication and division.

1. Apply properties of operations to multiply.[[12]](#footnote-12)

**For example: When multiplying numbers order does not matter. If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known (Commutative property of multiplication); The product 3 × 5 × 2 can be found by 3 × 5 = 15 then 15 × 2 = 30, or by 5 × 2 = 10 then 3 × 10 = 30 (Associative property of multiplication); When multiplying two numbers either number can be decomposed and multiplied; one can find 8 x 7 by knowing that 7 = 5 + 2 and that 8 × 5 = 40 and 8 × 2 = 16, resulting in 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56 (Distributive property); When a number is multiplied by 1 the result is the same number (Identity property of 1 for multiplication).**

1. Understand division as an unknown-factor problem.

**For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.**

C. Multiply and divide within 100.

1. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of grade 3, know from memory all products of two single-digit numbers and related division facts.

**For example, the product 4 x 7 = 28 has related division facts 28 ÷ 7 = 4 and 28 ÷ 4 = 7.**

D. Solve problems involving the four operations, and identify and explain patterns in arithmetic.

1. Solve two-step word problems using the four operations for problems posed with whole numbers and having whole number answers. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies, including rounding.[[13]](#footnote-13)
2. Identify arithmetic patterns (including patterns in the addition table or multiplication table) and explain them using properties of operations.

**For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.**

#### 3Number and Operations in Base Ten 3.NBT

A. Use place value understanding and properties of operations to perform multi-digit arithmetic. [[14]](#footnote-14)

1. Use place value understanding to round whole numbers to the nearest 10 or 100.
2. Fluently add and subtract within 1,000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
3. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

#### Number and Operations—Fractions 3.NF

A. Develop understanding of fractions as numbers for fractions with denominators 2, 3, 4, 6, and 8.

1. Understand a fraction 1∕*b* as the quantity formed by 1 part when a whole (a single unit) is partitioned into *b* equal parts; understand a fraction *a*∕*b*as the quantity formed by *a* parts of size 1∕*b*.
2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.
   1. Represent a unit fraction, 1∕*b,* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1∕*b* and that the fraction 1∕*b* is located 1∕*b* of a whole unit from 0 on the number line.
   2. Represent a fraction *a*∕*b* on a number line diagram by marking off *a* lengths 1∕*b* from 0. Recognize that the resulting interval has size *a*∕*b* and that its endpoint locates the number *a*∕*b* on the number line.
3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
   1. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
   2. Recognize and generate simple equivalent fractions, e.g., 1∕2 = 2∕4, 4∕6 = 2∕3. Explain why the fractions are equivalent, e.g., by using a visual fraction model.
   3. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.

**For example, express 3 in the form 3 = 3∕1; recognize that 6∕1 = 6; locate 4∕4 and 1 at the same point of a number line diagram.**

* 1. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

#### Measurement and Data 3.MD

A. Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

1. Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.
2. Measure and estimate liquid volumes and masses of objects using standard metric units of grams (g), kilograms (kg), and liters (l).[[15]](#footnote-15) Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same metric units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.[[16]](#footnote-16)

B. Represent and interpret data.

1. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs.

**For example, draw a bar graph in which each square in the bar graph might represent five pets.**

1. Generate measurement data by measuring lengths of objects using rulers marked with halves and fourths of an inch. Record and show the data by making a line plot (dot plot), where the horizontal scale is marked off in appropriate units—whole numbers, halves, or fourths. *(*[*See Glossary for example*](#_Glossary:_Mathematical_Terms,)*.)*

C. Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

1. Recognize area as an attribute of plane figures and understand concepts of area measurement.
   1. A square with side length of one unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.
   2. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.
2. Measure areas by counting unit squares (square cm, square m, square in., square ft., and non-standard units).
3. Relate area to the operations of multiplication and addition.
4. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
5. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
6. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths *a* and *b*+ *c* is the sum of *a* × *b* and *a* × *c*. Use area models to represent the distributive property in mathematical reasoning.
7. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems.

**D. Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.**

1. Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

#### Geometry 3.G

A. Reason with shapes and their attributes.

* 1. Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Compare and classify shapes by their sides and angles (right angle/non-right angle). Recognize rhombuses, rectangles, squares, and trapezoids as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
  2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole.

**For example, partition a shape into four parts with equal areas and describe the area of each part as ¼ of the area of the shape.**

# 4Grade 4

## Introduction

In grade 4, instructional time should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; (3) and understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

1. Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.
2. Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., 15∕9 = 5∕3), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.
3. Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

## 4Grade 4 Overview

#### Operations and Algebraic Thinking

###### **Standards for Mathematical Practice**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
9. Use the four operations with whole numbers to solve problems.
10. Gain familiarity with factors and multiples.
11. Generate and analyze patterns.

#### Number and Operations in Base Ten

1. Generalize place value understanding for multi-digit whole numbers less than or equal to 1,000,000.
2. Use place value understanding and properties of operations to perform multi-digit arithmetic on whole numbers less than or equal to 1,000,000.

#### Number and Operations—Fractions

1. Extend understanding of fraction equivalence and ordering for fractions ordering for fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.
2. Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers for fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.
3. Understand decimal notation for fractions, and compare decimal fractions.

#### Measurement and Data

1. Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
2. Represent and interpret data.
3. Geometric measurement: Understand concepts of angle and measure angles.

#### Geometry

1. Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

## 4Grade 4 Content Standards

#### Operations and Algebraic Thinking 4.OA

**A. Use the four operations with whole numbers to solve problems.**

1. Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.
2. Multiply or divide to solve word problems involving multiplicative comparison*,* e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison*.*[[17]](#footnote-17)
3. Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
   1. Know multiplication facts and related division facts through 12 x 12.

**B. Gain familiarity with factors and multiples.**

1. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

**C. Generate and analyze patterns.**

1. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.

**For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.**

#### Number and Operations in Base Ten 4.NBT

**A. Generalize place value understanding for multi-digit whole numbers less than or equal to 1,000,000.**

1. Recognize that in a multi-digit whole number, a digit in any place represents 10 times as much as it represents in the place to its right.

**For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.**

1. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
2. Use place value understanding to round multi-digit whole numbers to any place.

**B. Use place value understanding and properties of operations to perform multi-digit arithmetic on whole numbers less than or equal to 1,000,000.**

1. Fluently add and subtract multi-digit whole numbers using the standard algorithm.
2. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
3. Find whole-number quotients and remainderswith up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

#### 4Number and Operations—Fractions 4.NF

**A. Extend understanding of fraction equivalence and ordering for fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.**

1. Explain why a fraction a∕b is equivalent to a fraction **(*n*×*a*)**∕**(*n*×*b*)**by using visual fraction models, with attention to how the numbers and sizes of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions, including fractions greater than 1.
2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as **1**∕**2**. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

**B. Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers for fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.**

1. Understand a fraction ***a***∕***b*** with *a* > 1 as a sum of fractions **1**∕***b***.
   1. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. (The whole can be a set of objects.)
   2. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using drawings or visual fraction models. *Examples:* ***3****∕****8****=****1****∕****8****+****1****∕****8****+****1****∕****8*** *;* ***3****∕****8****=****1****∕****8****+****2****∕****8*** *; 2* ***1****∕****8****= 1 + 1 +****1****∕****8****=****8****∕****8****+****8****∕****8*** *+****1****∕****8****.*
   3. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
   4. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using drawings or visual fraction models and equations to represent the problem.
2. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
3. Understand a fraction *a∕b* as a multiple of 1*∕b*.

For example, use a visual fraction model to represent 5∕4 as the product 5 × (1∕4), recording the conclusion by the equation 5∕4 = 5 × (1∕4).

1. Understand a multiple of *a∕b* as a multiple of 1∕*b*, and use this understanding to multiply a fraction by a whole number.

**For example, use a visual fraction model to express 3 × (2∕5) as 6 × (1∕5), recognizing this product as 6∕5. (In general, n × (a∕b) = (n × a)∕b.)**

1. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.

**For example, if each person at a party will eat 3∕8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?**

**C. Understand decimal notation for fractions, and compare decimal fractions.**

1. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.[[18]](#footnote-18)

**For example, express 3∕10 as 30∕100, and add 3∕10 + 4∕100 = 34∕100.**

1. Use decimal notation to represent fractions with denominators 10 or 100.

**For example, rewrite 0.62 as 62∕100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.**

1. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

#### Measurement and Data 4.MD

**A. Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.**

1. Know relative sizes of measurement units within one system of units, including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table.

**For example, know that 1 ft. is 12 times as long as 1 in. Express the length of a 4 ft. snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), …**

1. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.
2. Apply the area and perimeter formulas for rectangles in real-world and mathematical problems.

**For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor. (Note: When finding areas of rectangular regions answers will be in square units. For example, the area of a 1 cm x 1 cm rectangular region will be 1 square centimeter (1 cm2, students are not expected to use this notation.) When finding the perimeter of a rectangular region answers will be in linear units. For example, the perimeter of the region is: 1cm + 1cm + 1cm +1cm = 4 cm or 2(1cm) + 2(1cm) = 4 cm).**

**B. Represent and interpret data.**

1. Make a line plot (dot plot) representation to display a data set of measurements in fractions of a unit (**1**∕**2**, **1**∕**4**, **1**∕**8**). Solve problems involving addition and subtraction of fractions by using information presented in line plots (dot plots).

**For example, from a line plot (dot plot) find and interpret the difference in length between the longest and shortest specimens in an insect collection.**

**C. Geometric measurement: Understand concepts of angle and measure angles.**

1. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:
   1. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through **1**∕**360** of a circle is called a “one-degree angle,” and can be used to measure angles.
   2. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.
2. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.
3. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

#### Geometry 4.G

**A. Draw and identify lines and angles, and classify shapes by properties of their lines and angles.**

1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.
2. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.
3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

# 5Grade 5

## Introduction

In grade 5, instructional time should focus on four critical areas:  (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of measurement systems and determining volumes to solve problems; and (4) solving problems using the coordinate plane

1. Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)
2. Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.
3. Students convert among different-sized measurement units within a given measurement system allowing for efficient and accurate problem solving with multi-step real-world problems as they progress in their understanding of scientific concepts and calculations. Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real-world and mathematical problems.
4. Students learn to interpret the components of a rectangular coordinate system as lines and understand the precision of location that these lines require. Students learn to apply their knowledge of number and length to the order and distance relationships of a coordinate grid and to coordinate this across two dimensions. Students solve mathematical and real world problems using coordinates.

The Standards for Mathematical Practice complement the content standards so that students increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.

## 5Grade 5 Overview

#### Operations and Algebraic Thinking

1. Write and interpret numerical expressions.

###### **Standards for Mathematical Practice**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
9. Analyze patterns and relationships.

#### Number and Operations in Base Ten

1. Understand the place value system.
2. Perform operations with multi-digit whole numbers and with decimals to hundredths.

#### Number and Operations—Fractions

1. Use equivalent fractions as a strategy to add and subtract fractions.
2. Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

#### Measurement and Data

1. Convert like measurement units within a given measurement system.
2. Represent and interpret data.
3. Geometric measurement: Understand concepts of volume and relate volume to multiplication and to addition.

#### Geometry

1. Graph points on the coordinate plane to solve real-world and mathematical problems.
2. Classify two-dimensional figures into categories based on their properties.

## 5Grade 5 Content Standards

#### Operations and Algebraic Thinking 5.OA

**A. Write and interpret numerical expressions.**

1. Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols, e.g.,(6 x 30) + (6 x 1∕2).
2. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.

**For example, express the calculation “Add 8 and 7, then multiply by 2” as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.**

**B. Analyze patterns and relationships.**

1. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.

**For example, given the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.**

#### Number and Operations in Base Ten 5.NBT

**A. Understand the place value system.**

1. Recognize that in a multi-digit number, including decimals, a digit in any place represents 10 times as much as it represents in the place to its right and **1**∕**10** of what it represents in the place to its left.
2. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.
3. Read, write, and compare decimals to thousandths.
   1. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (**1**∕**10**) + 9 × (**1**∕**100**) + 2 × (**1**∕**1000**).
   2. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
4. Use place value understanding to round decimals to any place.

**B. Perform operations with multi-digit whole numbers and with decimals to hundredths.**

1. Fluently multiply multi-digit whole numbers. (Include two-digit x four-digit numbers and, three-digit x three-digit numbers) using the standard algorithm.
2. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
3. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction and between multiplication and division; relate the strategy to a written method and explain the reasoning used.

#### Number and Operations—Fractions 5.NF

**A. Use equivalent fractions as a strategy to add and subtract fractions.**

1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.

**For example, 2∕3 + 5∕4 = 8∕12 + 15∕12 = 23∕12. (In general, a∕b + c∕d = (ad + bc)∕bd.)**

1. Solve word problems involving addition and subtraction of fractions referring to the same whole (the whole can be a set of objects), including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem.Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.

**For example, recognize an incorrect result 2∕5 + 1∕2 = 3∕7, by observing that 3∕7 < 1∕2.**

**B. Apply and extend previous understandings of multiplication and division to multiply and divide fractions.**

1. Interpret a fraction as division of the numerator by the denominator (*a*∕*b* = *a* ÷ *b*). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

**For example, interpret ¾ as the result of dividing 3 by 4, noting that ¾ multiplied by 4 equals 3, and that when three wholes are shared equally among four people each person has a share of size ¾. If nine people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?**

1. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
   1. Interpret the product (*a∕b*) × *q* as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a* × *q* ÷ *b*.

**For example, use a visual fraction model and/or area model to show (2∕3) × 4 = 8∕3, and create a story context for this equation. Do the same with (2∕3) × (4∕5) = 8∕15 . (In general, (a∕b) × (c∕d) = ac∕bd.)**

* 1. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

1. Interpret multiplication as scaling (resizing), by:
2. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

**For example, without multiplying tell which number is greater: 225 or ¾ x 225; 11∕50 or 3∕2 x 11∕50?**

1. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence   
   *a*/*b* = (*n* × *a*)/(*n* × *b*)to the effect of multiplying *a*∕*b* by 1.
2. Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
3. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[[19]](#footnote-19)
   1. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients.

**For example, create a story context for (1∕3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1∕3) ÷ 4 = 1∕12 because (1∕12) × 4 = 1∕3.**

* 1. Interpret division of a whole number by a unit fraction, and compute such quotients.

**For example, create a story context for 4 ÷ (1∕5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1∕5) = 20 because   
20 × (1∕5) = 4.**

* 1. Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem.

**For example, how much chocolate will each person get if three people share ½ lb. of chocolate equally? How many 1∕3-cup servings are in two cups of raisins?**

#### Measurement and Data 5.MD

**A. Convert like measurement units within a given measurement system.**

1. Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real-world problems.

**B. Represent and interpret data.**

1. Make a line plot (dot plot) to display a data set of measurements in fractions of a unit. Use operations on fractions for this grade to solve problems involving information presented in line plot (dot plot).

**For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.**

**C. Geometric measurement: Understand concepts of volume and relate volume to multiplication and to addition.**

1. Recognize volume as an attribute of solid figures and understand concepts of volume measurement.
   1. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.
   2. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.
2. Measure volumes by counting unit cubes, using cubic cm, cubic in., cubic ft., and non-standard units.
3. Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume.
4. Find the volume of a right rectangular prism with whole-number edge lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
5. Apply the formula *V* = *l* × *w* × *h* and *V* = *B* × *h* (where *B* stands for the area of the base) for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems.
6. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems.

#### Geometry 5.G

**A. Graph points on the coordinate plane to solve real-world and mathematical problems.**

1. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the zero on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate).
2. Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

**B. Classify two-dimensional figures into categories based on their properties.**

1. Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category.

**For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.**

1. Classify two-dimensional figures in a hierarchy based on properties.

**For example, all rectangles are parallelograms because they are all quadrilaterals with two pairs of opposite sides parallel.**

1. Drawings need not show details, but should show the mathematics in the problem. [↑](#footnote-ref-1)
2. Students should apply the principle of transitivity of measurement to make indirect comparisons, but they need not use this technical term. [↑](#footnote-ref-2)
3. See Glossary, Table 1. [↑](#footnote-ref-3)
4. Students need not use formal terms for these properties. [↑](#footnote-ref-4)
5. Students do not need to learn formal names such as “right rectangular prism.” [↑](#footnote-ref-5)
6. See Glossary, Table 1. [↑](#footnote-ref-6)
7. Strategies such as counting on; making tens; decomposing a number; using the relationship between addition and subtraction; and creating equivalent but easier or known sums. [↑](#footnote-ref-7)
8. Explanations may be supported by drawings or objects. [↑](#footnote-ref-8)
9. See Glossary, Table 1. [↑](#footnote-ref-9)
10. Sizes are compared directly or visually, not compared by measuring. [↑](#footnote-ref-10)
11. See Glossary, Table 2. [↑](#footnote-ref-11)
12. Students need not use formal terms for these properties. Students are not expected to use distributive notation. [↑](#footnote-ref-12)
13. Students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations). [↑](#footnote-ref-13)
14. A range of algorithms may be used. [↑](#footnote-ref-14)
15. Excludes compound units such as cm3 and finding the geometric volume of a container. [↑](#footnote-ref-15)
16. Excludes multiplicative comparison problems (problems involving notions of “times as much”; see Glossary, Table 2). [↑](#footnote-ref-16)
17. See Glossary, Table 2. [↑](#footnote-ref-17)
18. Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade. [↑](#footnote-ref-18)
19. Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade. [↑](#footnote-ref-19)