Quick Reference Guide:
Standard Algorithms for Multiplication and Division

The conceptual foundations for multiplication and division are developed in grades pre-K–2 through the study of addition, subtraction, and place value. The formal study of multiplication and division starts in grade 3 and includes analysis of operational patterns, grouping of numbers, arrays, and area-based strategies. These strategies become formalized as algorithms over time and culminate with the standard algorithms. Students are not likely to invent the standard algorithms on their own as they are not mathematically intuitive, so direct instruction is necessary. It is vital that students know and can apply the standard algorithms; however, it is critical that they understand their conceptual basis. **This guide summarizes the progression of standards related to developing a conceptual understanding of the standard algorithms for multiplication and division in the *Massachusetts Curriculum Framework for Mathematics*.** Accompanying examples of multiplication and division strategies and algorithms show how students progress toward the standard algorithms in grades 2–6.

Algorithm: A finite set of steps for completing a procedure: e.g., a multi-digit operation (addition, subtraction, multiplication, division).

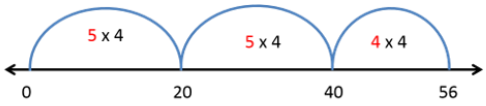
Standard algorithm: One of the conventional algorithms used in the United States based on place value and properties of operations for addition, subtraction, multiplication, and division.

Fluency: Fluency in the grades 1–6 standards is the ability to carry out calculations and apply numerical algorithms quickly and accurately.

Grades 2–3: Laying the Foundation

Second-grade students explore the foundational concepts of multiplication and division through work with equal groups of objects (2.OA.C) and patterns of repeated addition. Students begin their formal study of multiplication and division in grade 3, continuing their work with patterns by understanding the relationship between multiplication and division (3.OA.B.6). Students also learn and apply the properties of multiplication (3.OA.B.5) and develop strategies using drawings and equations (3.OA.A.3). By the end of grade 3, students fluently multiply and divide within 100 and know from memory all products of two single-digit numbers and related division facts (3.OA.C.7). Automaticity with these facts, combined with fluent application of the properties of operations, is crucial for successful use of the standard algorithms in later grades.

Open Number Line Strategy for Division



56 ÷ 4 = 14

5 + 5 + 4 = 14

Grade 4: Introducing Algorithms

In grade 4, students begin more formal work with algorithms—called "strategies" in the standards—for multiplication and division. Drawings, particularly area models, are still critical in making sense of multiplication and division (4.NBT.B.5 and 6). Algorithms based on place value, such as partial products and partial quotients, begin to figure more prominently in grade 4.

Area Model for Multiplication

	30	4
20	600	80
8	240	32

34 x 28 = 952

600 + 240 + 80 + 32 = 952

Partial Products Algorithm for Multiplication

34
x 28
600 (product of 30 x 20)
240 (product of 30 x 8)
80 (product of 4 x 20)
+ 32 (product of 4 x 8)
952

Grade 5: Using the Standard Multiplication Algorithm

By the end of grade 5, students fluently multiply multi-digit numbers using the standard algorithm (5.NBT.B.5). While students may be taught the standard algorithm for division—both "long" and "short"—they will continue to illustrate and solve problems using equations, rectangular arrays, and/or area models (5.NBT.B.6) as well as the partial quotients algorithm.

Grade 6: Using the Standard Division Algorithm

Moving from left to right and involving multiple steps, the standard algorithm for division is the most complex of all the operations. Grade 6 students fluently apply the standard division algorithm for multi-digit numbers (6.NS.B.2) and perform all operations with multi-digit decimals (6.NS.B.3).

Partial Quotient Algorithm for Division

32|1560		
-320	10 X 32	
1240		10
-320	10 X 32	10
920		10
-320	10 X 32	10
600		+ 8
-320	10 X 32	48 R24
280		
-256	8 X 32	
24		

Standard Algorithm for Multiplication

```
  3
  34
x 28
 272
+680
 952
```

Standard Algorithm for Division

```
      48 R24
32 | 1560
   - 128
     280
   - 256
      24
```

Frequently Asked Questions

Why not just teach the standard algorithm first? The standard algorithm does not correspond to the way we think about numbers (digits versus place value). If the standard algorithm is taught first, students do not develop an understanding of how place value relates to composing and decomposing numbers. Adults use written calculations for only a small portion of the math they do—most math done in daily life or the workplace is done mentally or with the help of technology—so supporting students to develop a strong conceptual understanting of multiplication and dvision is important long-term.

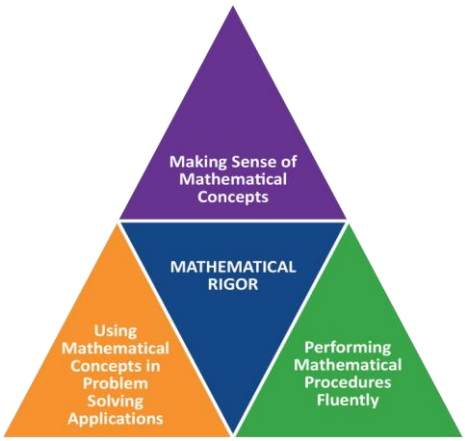
I teach students not originally from the United States who are able to compute accurately without using the standard algorithm. Do I need to teach them the standard algorithm too? Yes, the MA Frameworks require fluency with the standard algorithm for multiplication of whole numbers in grade 5, division of whole numbers in grade 6, and all operations with decimals in grade 6.

What about parents who teach students the standard algorithm (such as when helping with homework) before we have explored using it in the classroom? It is helpful to make parents aware of when the standard algorithm will be introduced and when it is expected to be mastered, as well as letting them know which standards support the development of conceptual understanding that leads to the standard algorithm. The timing may be different from when they learned mathematics as a child, particularly as facility with an operation now develops over several grades and culminates in fluency with the standard algorithm.

Balanced Mathematical Instruction

To achieve mathematical understanding, students should be actively engaged in **meaningful mathematics**. The standards focus on developing students' conceptual understanding, procedural fluency, and problem solving applications.

Algorithms, particularly the standard algorithms, provide computational efficiency during mathematical problem solving and are a useful tool when solving application problems (see SMP 5).



Check It Out!
Standard Algorithms in the CCSS, NCSM Journal, Fall/Winter 2012-13: http://bit.ly/1YjYKnD
Progression Number and Operations in Base Ten: http://bit.ly/2mueF8V