

# Quick Reference Guide: Standard Algorithms for Addition and Subtraction

Students typically come to school with an informal understanding of addition and subtraction based on experiences like grouping or sharing objects. When students enter school they begin to formalize their understanding of number and operations. They begin with studying patterns and developing strategies for basic operations. Over time these strategies become formalized as algorithms and eventually culminate with the standard algorithm. Students are not likely to invent the standard algorithm on their own, as it is not mathematically intuitive, so direct instruction is necessary. It is vital that students know and can apply the standard algorithm; however, it is critical that they understand its conceptual basis.

**This guide summarizes the progression of standards related to developing a conceptual understanding of the standard algorithms for addition and subtraction in the [Massachusetts Curriculum Framework for Mathematics](#).** Accompanying examples of addition and subtraction strategies and algorithms show how students progress toward the standard algorithms.

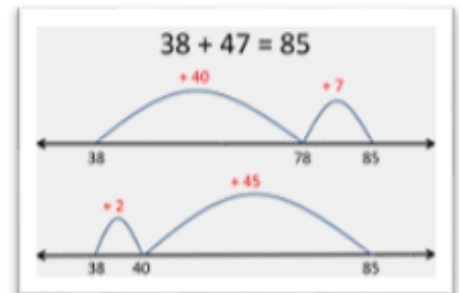
**Algorithm:** A finite set of steps for completing a procedure, e.g., multi-digit operations (addition, subtraction, multiplication, division).

**Standard algorithm:** One of the conventional algorithms used in the United States based on place value and properties of operations for addition, subtraction, multiplication, and division.

**Fluency:** Fluency in the grades 1–6 standards is the ability to carry out calculations and apply numerical algorithms quickly and accurately.

## Grades Pre-K–2: Developing Strategies to Add and Subtract

Pre-kindergarten and kindergarten students begin to establish patterns with operations by understanding addition as putting together and adding to, and subtraction as taking apart and taking from (PK.OA.A and K.OA.A), using objects and drawings to develop their thinking. First and second graders begin to build their understanding of addition and subtraction by developing and using strategies that are based on place value and properties of operations (1.OA.A.1 and 2; 1.OA.B.3 and 4; 1.OA.C.5 and 6; 1.NBT.C.4–6; 2.NBT.B.5–9). Students use concrete objects—counting on their fingers, for example—and drawings as they think through their strategies. Students are able to relate a strategy to a written method and explain their reasoning. By the end of Grade 2, students fluently add and subtract within 100, using strategies based on place value, properties of operations, and/or the relationships between addition and subtraction (2.NBT.B.5).



Open Number Line Addition Strategy

## Grade 3: Introducing Algorithms

In Grade 3, students fluently add and subtract within 1,000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction (3.NBT.A.2). The term *algorithm* first appears in the standards in grade 3.

$$\begin{array}{r} 532 \\ - 398 \\ \hline 200 \text{ (difference of hundreds)} \\ -60 \text{ (difference of tens)} \\ -6 \text{ (difference of ones)} \\ \hline 134 \end{array}$$

Partial Differences Algorithm

$$\begin{array}{r} 537 \\ + 698 \\ \hline 1100 \text{ (sum of hundreds)} \\ 120 \text{ (sum of tens)} \\ 15 \text{ (sum of ones)} \\ \hline 1235 \end{array}$$

Partial Sums Algorithm

## Grade 4: Using the Standard Algorithm

Fourth graders fluently add and subtract multi-digit whole numbers using the standard algorithm (4.NBT.B.4). The standard algorithm represents an efficient and universally applicable method of adding and subtracting numbers; however, it is vital that students understand the properties and place value structures that underlie these procedures.

A vertical subtraction problem: 4712 minus 398. The result is 134. Red annotations show the borrowing process: a '1' is written above the '7', a '1' is written above the '1', and a '2' is written above the '2'. The '7' is crossed out and replaced with '6', and the '1' is crossed out and replaced with '0'.

Standard Algorithm  
for Subtraction

A vertical addition problem: 537 plus 698. The result is 1235. Red annotations show the carrying process: a '1' is written above the '5', and another '1' is written above the '3'.

Standard Algorithm  
for Addition

### Frequently Asked Questions

**Why not just teach the standard algorithm first?** The standard algorithm does not correspond to the way we think about numbers (digits versus place value). If the standard algorithm is taught first, students do not develop an understanding of how place value relates to composing and decomposing numbers. Adults use written calculations for only a small portion of the math they do—most of math done in daily life or the workplace is done mentally or with the help of technology—so supporting students to develop a strong conceptual understanding of addition and subtraction is important long-term.

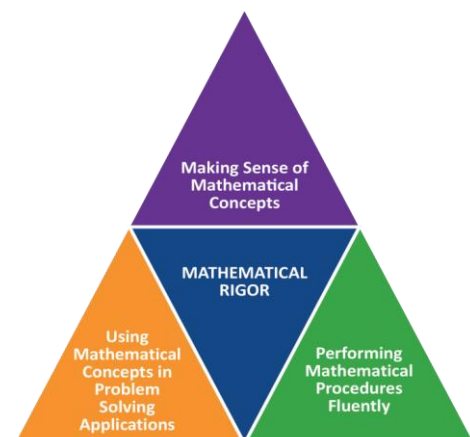
**I learned the standard algorithms for addition and subtraction with words like “borrow” and “carry.” How come these terms are no longer taught?** These words are conceptually misleading. Preferred terms are “compose/decompose” and “trade/regroup/exchange” to correctly explain the processes as they relate to place value.

**Sometimes parents teach students the standard algorithm before we have explored using it in the classroom (e.g., when helping with homework). How should I handle that?** It is helpful to make parents aware of when the standard algorithm will be introduced and when it is expected to be mastered, as well as letting them know which standards support conceptual understanding that leads to the standard algorithm. Acknowledge differences in approaches to mathematics instruction from when they learned mathematics as a child, such as how mastery of an operation now develops over several grades and culminates in fluency with the standard algorithm. Provide strategies for how parents can support learning at home.

### Balanced Mathematical Instruction

To achieve mathematical understanding, students should be actively engaged in **meaningful mathematics**. The standards focus on developing students’ conceptual understanding, procedural fluency, and problem solving applications.

Algorithms, particularly the standard algorithm, provide computational efficiency during mathematical problem solving and are a useful tool when solving application problems (see SMP 5).



### Check It Out!

[Standard Algorithms in the CCSS, NCSM Journal, Fall/Winter 2012-13](#)  
Addition and Subtraction Strategies with Place Value