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| Reasoning with Equations |
| Mathematics, High School Algebra I  (Revised February 2019) |
| Equations give us a way of describing a real world situation. When an equation is written and solved it gives us information about the real world. We study equation solving to become skilled at understanding what equations can or cannot tell us about a real world situation. This unit develops student’s conceptual understanding about the algebraic (symbolic) approach to solving linear equations. This unit is not about procedures for solving equations, though students will develop this skill by the end of the unit. The concept of equivalence is developed first with expressions and then with equations. The conceptual understandings students learn in this unit develop their ability to solve any equations in school mathematics. Solving equations is a process of creating equivalent equations using number properties. This unit develops student’s understandings about Properties of Operations and Properties of Equality and the different ways they maintain equivalence in the equation solving process.  This unit addresses the 2017 MA Curriculum Framework for Mathematics Critical Area Number One for the HS Model Algebra I Course regarding linear equations.As stated in the Model Algebra I Course Introduction, **“**By the end of eighth grade, students have learned to solve linear equations in one variable……In Algebra I, students will analyze and explain the process of solving an equation. They master the solution of linear equations (p.108).”  *These Model Curriculum Units are designed to exemplify the expectations outlined in the MA Curriculum Frameworks for English Language Arts/Literacy and Mathematics incorporating the Common Core State Standards, as well as all other MA Curriculum Frameworks. These units include lesson plans, Curriculum Embedded Performance Assessments, and resources. In using these units, it is important to consider the variability of learners in your class and make adaptations as necessary.* |



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| **Stage 1 Desired Results** | | |
| **ESTABLISHED GOALS G**  ***Standards for Mathematical Content***  **A-REI.A.1:** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify or refute a solution method.  **\*A-REI.B.3:** Solve linear equations and inequalities in one variable involving absolute value**.**  **\*A-CED.A.1:** Create equations and inequalities in one variable and use them to solve problems. (Include equations arising from linear and quadratic functions, and simple root and rational functions and exponential functions.)  *Note: This unit addresses only a portion of the standards A. REI.3 and A-CED.1 (underlined sections). The remainder of the standard will be addressed in a later unit.*  ***Standards for Mathematical Practice:***  **SMP1:** Make sense of problems and persevere in solving them.  **SMP2:** Reason abstractly and quantitatively  **SMP3:** Construct viable arguments and critique the reasoning of others.  **SMP6:** Attend to precision.  **SMP7**: Look for and make use of structure  *Connection to ELA Standards:*  **WCA.9-10.2d**  Use precise language and domain-specific vocabulary to manage the complexity of the topic and convey a style appropriate to the discipline and context as well as to the expertise of likely readers. | ***Transfer*** | |
| ***Students will be able to independently use their learning to…***  Express appropriate mathematical reasoning by constructing viable arguments, critiquing the reasoning of others, and attending to precision when making mathematical statements.**T** | |
| ***Meaning*** | |
| **UNDERSTANDINGS U**  ***Students will understand that…***  **U1:** solving linear equations involves a process of creating simpler equivalent equations.  **U2:** there are multiple approaches to solving equations  **U3:** An equation is a statement that two expressions are equivalent  **U5** maintaining equivalence is a necessary component of solving equations  **U6** Equations that look different may be equivalent. | **ESSENTIAL QUESTIONS Q**  **Q1:** How can equations be used?  **Q2** How can different equations have the same solution?  **Q3** How does one reason logically with equations? |
| ***Acquisition*** | |
| ***Students will know…* K**  **K1:** the properties of equality.  **K2:** the properties of operations and how they are used to solve equations.  **K3:** appropriate language[[1]](#footnote-1) in describing the solutions of equations.  **K4:** the difference between an equation and an expression.  **K5:** the process of solving an equation is a series of logical steps verified by the properties of equality.  **K6:** Equations can have no solution, one solution or multiple solutions.    **K7: vocabulary relating to equations, properties, and components of questions**  solution, one solution, no solution, infinitely many solutions, valid solution, equation, expression, equal sign, coefficient, variable, constant, equivalent, inverse operation, reflexive property of equality, symmetric property of equality, transitive property of equality, addition property of equality, subtraction property of equality, multiplication property of equality, division property of equality, substitution property of equality | ***Students will be skilled at…* S**  **S1:** using Properties of Operations and Equality to solve equations  **S2:** solving and justifying the solution process  **S4:** determining the validity of a solution within a given context.  **S5:** using mathematical structures to strategically solve equations |

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| **Stage 2 - Evidence** | |
| **Evaluative Criteria** | **Assessment Evidence** |
| **Meaning Goals:**   * Understand and exhibit equality. * Understand when a value is or is not a solution to a single-variable equation or inequality. * Understand and explain the steps in solving a single-variable equation   **Acquisition Goals:**   * Justify each step of a solution using the properties of equality and inverse operations.   Solutions should focus on the development of a clear and coherent solution explanations | **CURRICULUM EMBEDDED PERFORMANCE ASSESSMENT (PERFORMANCE TASKS):**  ***Intelligent Intelligence***  As a US Counter Intelligence officer, you have been assigned to track down Mathew Wrong, a US defector, and his foreign contacts. Mathew Wrong was a computer programmer working as a subcontractor for WeCompute Inc. He illegally accessed sensitive United States Armed Forces information on combat readiness. U.S. Intelligence sources reported him to be out of the country. It is assumed he will try to sell the information for money. If he is not caught quickly it would put the United States at risk.  To help track Mathew down, you are required to devise a series of math equations that will be used to pass information to others. You will create equations whose solutions may or may not be true. Those with true solutions will be passed along to other US Intelligence officers and those with false solutions will be passed to foreign spies. Your ability to create false solutions that appear to be true is a hallmark of your Intelligence abilities. Your superiors require you to use expressions, equations, and identities in your work and prove that the equations to the allies are solved correctly and the equations to the enemies are solved incorrectly. In order to solve them correctly, you will have to abide by the Properties of Operations Code used by all of the Intelligence officers.  You will create a presentation using precise mathematical language for the commanding officer (for example, report, podcast, script, Jing, PowerPoint/Keynote, etc.) that includes the sets of equations with both faulty and correct solutions. All solutions should be clear and coherent and show appropriate use of the number Properties of both Equality and Operations.  Procedure for equations to be passed along to US Intelligence:   1. Create a variety of equations (minimum of 5) having each of the following characteristics:    1. True for all numbers    2. True for some numbers    3. No solutions 2. Equations should    1. require multiple steps to solve    2. have variables on both sides of the equation    3. require the use of distribution    4. require combining like terms    5. contain fractional coefficients (in two or more equations) 3. Verify (prove) the solutions for each of the equations you create using the number Properties of Operations and the number Properties of Equality and by reasoning logically from each step in the solution process.   Procedure for equations to be passed to foreign spies   1. Create a minimum of 2 equations that have incorrect solutions.    1. Create the equations and a flawed proof by misapplying a number Property of Operations and/or a number Property of Equality.    2. Make it subtle so that it cannot easily be detected and is difficult to check in the original solution |
|  | **OTHER EVIDENCE:**   * Students correct the solutions to equations that were solved incorrectly. Students will write a paragraph communicating how they identified where the mistakes were and how they corrected the mistakes. * Teach a lesson to the class on solving single-variable equations and inequalities (with varying difficulty of equations and inequalities included). Students will be asked to self-assess and classmates will be asked to assess the lesson. * Informal checks for understanding asking students to explain their solution methods. * Write a letter to another student defending their solution to an equation, justifying each step. * Create a poster organizing their explanation of a solution to an equation. * Mid-unit quiz assessing solution of equations with determining if a solution fits a specific context * End-of-unit assessment assessing their ability to look at a solution process and determine errors, determine another method to arrive at the same solution, and determine if a quantity is a solution to an equation. |
| **Stage 3 – Learning Plan** | |
| ***Summary of Key Learning Events and Instruction***  **Lesson 1:** Problem solving approaches  **Lesson 2:** Number Properties  **Lesson 3:** Reasoning Logically  **Lesson 4:** What’s My Number?  **Lesson 5:** Solutions types - Card Sort- Equations may have one, many, or no solutions. <http://map.mathshell.org/materials/lessons.php?taskid=218>  **Lesson 6:** Properties of Equality   1. Solving equations as a process of logic and equivalency, creating their own understanding of the properties of equality; Properties activity; students are given solved equations and have to apply the property used, and vice versa. 2. Connecting the steps and solutions in equation solving to graphic representations   **Lesson 7:** Reasoning about the Equation Solving Process  **Lesson 8:** Rules Gone Wild. Find the flaw in the reasoning given a solution to an equation.   * Give students equations that have been solved by 2 or 3 fictitious students using different methods. For example, can be solved by distributing first or by dividing both sides by 2. * Students work in partners to discuss the different methods and decided if each solution is correct or find errors that were made. Include lesson about common student misconceptions.   **CEPA**  ***Vocabulary to introduce and assess throughout instruction:***  solution, one solution, no solution, infinitely many solutions, valid solution, equation, expression, equal sign, coefficient, variable, constant, equivalent, inverse operation, reflexive property of equality, symmetric property of equality, transitive property of equality, addition property of equality, subtraction property of equality, multiplication property of equality, division property of equality, substitution property of equality | |
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Lesson 1

Reasoning about Equation Solving

**Brief Overview of Lesson:**

Problem solving is at the heart of the usefulness of mathematics. Students learn the usefulness of equations to find solutions to problems. They will solve a problem, share strategies, and then consider the strengths and weaknesses of their strategies. As you plan, consider the variability of learners in your class and make adaptations as necessary.

**Prior Knowledge Required:** Students have explored linear functions/equations graphically, symbolically, and numerically.

**Estimated Time (minutes):** 45 minutes

**Resources for Lesson (list resources and materials):**

Handouts:

*Problem Solving*

*Rule of Four Link*

*Ticket to Leave- Checking Solutions to Equations*

**Content Area/Course:** Mathematics – Algebra 1

**Unit:** Reasoning with Equations

**Time (minutes):** 45 minutes

**Lesson #1**: ***Reasoning about Equation Solving***

**By the end of this lesson students will know and be able to:**

* Make sense of the relationships between tables, graphs and equations.
* Determine the validity of a solution within a given context.

**Essential Question(s) addressed in this lesson:** How can equations be used?

**Lesson Guiding Questions:**

* *What does it mean to solve a problem?*
* *How can equations be used to solve problems?*

**Standard(s)/Unit Goal(s) to be addressed in this lesson:**

A-CED.1: Create equations and inequalities in one variable and use them to solve problems. (Include equations arising from linear and quadratic functions, and simple root and rational functions and exponential functions.)

SMP1: Make sense of problems and persevere in solving them.

SMP3: Construct viable arguments and critique the reasoning of others.

**Instructional Tips/ Strategies/Notes for Teacher:**

Teacher Note: The emphasis in this lesson is to find where students are in their reasoning about problem solving and using equations as a means to solve problems. Because the answer to one part of the problem will be difficult to answer using tables and graphs students will come to conclusions how equations can be used efficiently to solve problems.

In the equation used in this lesson, what does x and A represent in the equation? Note how students respond to the remainder portion for buying a $179.00 cell phone. Does their answer make sense? Do they put the answer back into the context of the problem? How do they determine the answer for buying the $179.00 cell phone if they solved it graphically? Numerically (table)?

**Lesson Sequence**

***Think-Pair-Share Activity (Elicit)*** Ask students to reflect on the following questions:

*What does it mean to find the solution to a problem?*

*What does it mean to find a solution to an equation?*

1. Give students time to think individually (1-3 minutes). Ask them to jot down a couple of thoughts on paper.
2. Have students turn and talk to a partner to share their thinking with each other. Then have each pair share out one reflection with the whole class. The teacher records responses on the white board or chart paper. (8-10 minutes).

* ***Problem Solving Activity*** **(Engage and Explore)** Find a solution by graphing/by table/ symbolically/ by reasoning.

Supply students with a 4-link sheet and the following problem. Ask students to solve the problem (or similar) in as many ways as they can. The teacher way want to model each approach with different problems first. (See attached Problem Solving and Rule of 4 Link handouts. Graphing tools can also be used by teacher discretion). The focus in this problem is not to create equations, but simply to solve them. The intent is for students to look, think, talk, and solve equations and justify their solution.

***Justin cuts grass for $5.00 an hour and has saved $35.00 so far. The equation A = 5x + 35 represents the amount (A) of money Justin will have after future grass cuttings (x represents the number of future grass cuttings). How many more grass cuttings will he have to complete to buy a $125.00 cell phone? A $179.00 cell phone? Justify your solutions.***

1. Give students the problem sheet and have students work individually first then share their work in groups of 2-3.
2. Have students justify their solution method.
3. The teacher can help the class connect each method to the others.
4. Summarize students’ responses including the following if students haven’t done so:

* Making sense of the relationships between tables, graphs and equations.
* Determining the validity of a solution within a given context.
* What does it mean to solve a problem?
* How equations can be used to solve problems.

1. In groups of 2-3, have students list the pros and cons of each approach (Table, graph, symbolically, etc.) to solving an equation. This offers an opportunity for students to reflect upon various means of problem solving approaches.

***Formative Assessment-Ticket to Leave - Equation Activity.***

This activity will help teachers discover student’s expertise at working with algebraic expressions (evaluating/order of operations) and solving equations in one variable. (See attached *Checking Solutions to Equations* handout).

The purpose of the *Ticket to Leave* is to see how strong students’ understanding is of using Number Properties and Properties of Operations. Let students know these exercises are for you to gather information so that you can adapt lessons. Collect the student work but do not grade them. You may want to add hints or ask a question about work when they show an error. Note where students are weak or strong in order to guide your next steps.

1. Have students evaluate the statements for the given value of x. Discuss the meaning of the answer in relation to the equation.
2. 4(-3x +1) = - 10 (x – 4) – 14x when x = 3
3. 2t - 4( t – 1) = 2 (2t – 1) + 3t when t = 
4. 1/5(a – 4) = 2/5(3 – 2a) when a = 1

**How skilled are your students at solving equations?**

1. Note students’ strategies: Possible strategies include Guess and Check, Number Sense, Algebraic Reasoning (using Properties of Numbers and Properties of Equality).
2. *4(x – 13) = 4* c. *5p – 9 = 3 ( p – 7)*
3. *( s– 3 )( s – 2) =0* d. *x2 + 3 = 12*

*Discuss strategies using number sense for a, b, and d:*

1. *4 times a number is 4. So, x - 13 must equals 1*
2. *The product of two quantities is 0. So s – 3or s – 2 must be zero*
3. *Since 9 + 3 = 12, the squared number must equal 9.*

*Discuss Guess and check for c. Have students discuss the drawbacks of guess and check. What if the equation was 5p – 9 = 2(p – 7)?*

# Problem Solving

**Use the *Rule of Four Link Sheet* to answer the following problem.**

**Be sure to answer both parts of the question and be prepared to justify you solutions.**

***Justin cuts grass for $5.00 an hour and has saved $35.00 so far. The equation***

***A = 5x + 35 represents the amount (A) of money Justin will have after future grass cuttings***

***(x represents the number of future grass cuttings). How many future grass cuttings will he have to complete to buy a $125.00 cell phone? a $179.00 cell phone? Justify your solutions.***

Rule of 4 Link Sheet

|  |  |
| --- | --- |
| **Problem Solving Approaches to show our reasoning** | |
| **Verbal Description** | **Table** |
| Communicate orally and/or in writing | Communicate Numerically   |  |  | | --- | --- | |  |  | |  |  | |  |  | |  |  | |  |  | |  |  | |  |  | |  |  | |  |  | |  |  | |  |  | |  |  | |  |  | |  |  | |
| **Graph** | **Equation(s)** |
| Communicate Visually   |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | | Communicate Symbolically (algebraically) |

**Ticket to Leave**

**Evaluate the statements for the given value of the variable. Show your work.**

|  |  |  |
| --- | --- | --- |
| **4(-3x +1) = - 10 (x – 4) – 14x** | **2t - 4( t – 1) = 2 (2t – 1) + 3t** | **1/5(a – 4) = 2/5(3 – 2a)** |
| **for x = 3** | **for t =** | **For a = 1** |

1. **Solve the following equations Show all work or explain your reasoning:**

|  |  |  |  |
| --- | --- | --- | --- |
| **4(x – 13) = 4** | **5p – 9 = 3 ( p – 7)** | **( s– 3 )( s – 2) =0** | **x2 + 3 = 12** |
|  |  |  |  |

Lesson 2

Number Properties of Operations Extended to Algebraic Expressions

**Brief Overview of Lesson** *This is an optional lesson* designed to be used to strengthen students’ ability to reason about expressions using Number Properties of Operations. Reasoning about expressions is vital for students to be able to reason about equations. As you plan, consider the variability of learners in your class and make adaptations as necessary.

**Estimated Time** 45 minutes

**Resources for Lesson**

Handouts-

*Number Properties for Addition and Multiplication*

*What about Subtraction and Division Number Properties?*

**Unit: Reasoning with Equations**

**Content Area/Course: Algebra I**

**Lesson #2:** **Number Properties of Operations Extended to Algebraic Expressions**

*Note: This lesson is optional depending upon students’ ability to reason about expressions using Number Properties of Operations. Reasoning about expressions is vital for students to be able to reason about equations.*

**Time (minutes):** 45 minutes

**By the end of this lesson students will know and be able to:**

* Extend Number Properties of Operations to create equivalent expressions
* Justify the process of simplifying expressions
* Reason with properties/operations that do not maintain equivalence

**Essential Question(s) addressed in this lesson:** How is equivalence maintained in working with expressions?

**Standard(s)/Unit Goal(s) to be addressed in this lesson:**

**A-SSE.A.1** Interpret expressions that represent a quantity in terms of its context

a. Interpret parts of an expression such as terms, factors, and coefficients

**A-SSE.A.2** Use the structure of an expression to identify ways to rewrite it. *For example, see x4-y4 as (x2)2-(y2)2, thus recognizing it as a difference of squares that can be factored as (x2-y2)(x2+y2).*

**SMP1:** Make sense of problems and persevere in solving them.

**SMP2:** Reason abstractly and quantitatively

**SMP3:** Construct viable arguments and justify the reasoning of other

**Instructional Tips/ Strategies/Notes for Teacher**

Expressions are foundational for solving equations. Students have learned to solve simple linear equations in one variable in previous grades. In Algebra I, students continue to solve equations in one variable and they analyze, explain, and/or justify the process of solving an equation. They master the solution process for solving linear equations that help them at a later time to apply related solution techniques for solving simple non-linear equations.

**Lesson Sequence**

1. Activator: Students will create expressions that represent area.

Students will simplify the expressions. Discuss the simplification process as applying number properties of operations. Discuss that the area (expressions) remains the same even though even though their symbolic forms differ. Discuss what each term in the expression represents (note the parts of a term such as factors, coefficients etc.

The figure shows two adjoining rectangles. The area of the figure can be represented in two ways:

1. Create an algebraic expression to represent the total area in two ways. Be ready to explain your reasoning.

X 8

1. What property of real numbers does this demonstrate?

Create and simplify an expression for the sum of the areas for the two rectangles below:

3w

3w + 4

w - 1

w

1. **Using Handout I**, *Number Properties for Addition and Multiplication*, students will discuss the Number Properties in Column 1 with a partner and the associated Property Rule in Column 2. Individually, students should complete real number and algebraic examples in Columns 3 & 4. Students *will then share and compare their examples with their partner to discuss* how they are alike and how they are different.

Note to teacher: It’s important that students understand these properties

are true for ALL real numbers. Since variables represent real numbers, these properties are true for all expressions that contain a variable that represents a real number

Students can complete the *Properties in Action* on the bottom of the handout. Students should then discuss their justifications for creating each of their equivalent expressions.

Number Properties for Addition and Multiplication exist for all real numbers. The second part of the lesson activity will help students to reason about Commutative and Associative Properties for Subtraction and Division. Students need to understand that there are no Number Properties involving subtraction and division that are true for ALL real numbers.

1. **Using Handout II**, students will reason about the Commutative and Associative Properties for the operations of subtraction and division. Students will work individually to determine whether statements in the handout are true for all real numbers then share with a partner and compare their responses.

Ask students if there are any numbers that make those statements true for subtraction or for division.

Facilitate a whole class discussion on the reflection questions after students have the opportunity to reason about the questions individually and/or in pairs.

Have students continue to reason about other Properties of Operations by completing the last two problems on Handout II.

Student Practice for Properties of Operations:

<http://regentspreponline.com/algebra/algebra12014>

<http://www.math.com/school/subject2/lessons/S2U2L1GL.html>

**Formative assessment:** Class work/Homework- See handout, *Practice and Applications*

**Preview outcomes for the next lesson:** Properties of Operations including Inverse Properties, translating everyday language into mathematical equations

|  |
| --- |
| 1. **With a partner** discuss the Number Properties in **Column 1** below and the property rule in **Column 2**, 2. **Individually** complete Columns 3 and 4 with your own examples of the rules. (The first one is done for you) 3. **Share and Compare** your examples with your partner. How are they alike and different? |

## I. Number Properties for Addition and Multiplication (are true for *all* real numbers)

|  |  |  |  |
| --- | --- | --- | --- |
| **Number Property** | a, b, and c  represent any real numbers | Real Number  Examples | Algebraic Examples  (for x being **any** real number) |
| **Commutative Property** |  |  |  |
| **of Addition** | a + b = b + a | 6 + 5 = 5 + 6 | -5 + x = x + -5 |
| **of Multiplication** | a  b = b  a |  |  |
|  |  |  |  |
| **Associative Property** |  |  |  |
| **of Addition** | (a + b) + c = a + (b + c) |  |  |
| **of Multiplication** | (a  b)  c = a  (b  c) |  |  |
|  |  |  |  |
| **Distributive Property** | a(b + c) = ab + ac |  |  |
|  |  |  |  |
| **Identity Property** |  |  |  |
| **of Addition** | a + 0 = a, 0 + a = a  (Additive property of zero) |  |  |
| **of Multiplication** | a  1 = a and 1  a = a (Multiplicative property of 1) |  |  |
| **Inverse Properties** |  |  |  |
| **of Addition** | a + ( - a) = 0 |  |  |
| **of Multiplication** |  |  |  |

Properties in Action

Using Properties in Simplifying Algebraic Expressions by ‘Combining Like Terms’

|  |  |  |
| --- | --- | --- |
| Expression | Equivalent Expression (Simplified) | Justification |
| 7x + 4x |  |  |
| 3n2 + n – n2 |  |  |
| 2(x – 1) -3(x - 4) |  |  |

II. What about Subtraction and Division Number Properties???

***Question:* Are *the Commutative and Associative Properties also true for the operations of Subtraction and Division?***

|  |
| --- |
| 1. Individually: Use the space below to justify your reasoning whether the following statements about **Subtraction (Column 1)** and **Division (Column 2)** are true for **all real** numbers.   (Hint: Try real number examples to support your answer.)   1. Share with a partner and compare your responses. |

|  |  |  |
| --- | --- | --- |
| Commutative | a - b = b - a T or F? | a ÷ b = b ÷ a T or F? |
|  |  |
| Associative | (a - b) - c = a - (b - c) T or F? | (a ÷ b) ÷ c = a ÷ (b ÷ c) T or F? |
|  |  |

|  |  |
| --- | --- |
| Reflection: | |
| **Is dividing by 2 commutative? Explain and/or give an example.** | |
| **Is multiplying by ½ Commutative? Explain and/or give an example.** | |
| **Use Substitution to show that the expressions**  and 2x are equal when a) x = 2 and b) x = -8) | **Show how the distributive property justifies the statement**  is true for all values of x. |

## Practice and Applications

Apply properties to simplify the expressions below.

|  |  |
| --- | --- |
| 1. 5(x – 7) + 3x | 1. 9(r + 3r2) + 7(r2 + 4r) |
| 1. -3x + (4x -6) - (3x - 4x) - 6 | 1. x( - x)(14x) |

The expressions below are not equivalent because the Properties of Operations were not applied correctly. Apply the property correctly to make the expressions equivalent.

|  |  |
| --- | --- |
| 1. 3 ( x + 5) ≠ 3x + 5 | 1. -2 ( x + 8 ) ≠ -2x + 16 |
|  |  |

|  |
| --- |
| Write an expression to represent the **perimeter** for the figure described below. Simplify the expression.  (Draw a picture if it helps)  Two sides of a quadrilateral measure *y* centimeters. A third side is 6 centimeters longer. The fourth side is twice the third side. |
| iTunes is having a sale on music. You want to buy either a song album or an individual tune for each of your 10 friends or family members. Song albums cost $ 7.00 each and individual tunes cost $2.00 each.   1. Write an expression (in one variable) for the total amount you must spend. (hint: Let n = the number of friends or family who will get an album) 2. Evaluate the expression if 4 of the people get song albums. |

Lesson 3 – Reasoning Logically

**Brief Overview of Lesson:** Students will develop skills and understandings about reasoning logically in order to construct viable arguments to justify an equation solving process. As you plan, consider the variability of learners in your class and make adaptations as necessary.

**Prior Knowledge Required:** Properties of Operations, create expressions by translating everyday language into mathematical equations

**Estimated Time** 45 minutes

**Resources for Lesson:**

Handouts:

*Pre-Assessment*

*Why is Reasoning Logically Important in Mathematics?*

**Unit: Reasoning with Equations**

**Content Area/Course: Algebra I**

**Lesson 3:** *Reasoning Logically*

**Time (minutes):** 45 minutes

**By the end of this lesson students will know and be able to:**

* Reason logically using number properties to justify mathematical statements
* Apply properties of operations to maintain the equivalency in a mathematical statement

**Essential Question(s) addressed in this lesson:**

How does one reason logically with equations?

**Guiding Question:** How does one justify their reasoning about equations and their solutions?

**Standard(s)/Unit Goal(s) to be addressed in this lesson:**

**A-REI.A.1:** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify or refute a solution method.

**SMP1:** Make sense of problems and persevere in solving them.

**Instructional Tips/ Strategies/Notes for Teacher**

**Errors in Reasoning:** Errors can be caused by a flaw in the structure of a logical [argument](http://en.wikipedia.org/wiki/Logical_argument) which makes the argument [invalid](http://en.wikipedia.org/wiki/Validity).

Logical form alone can guarantee that given true premises, a true conclusion must follow. However, formal logic makes no such guarantee. If any premise is false, the conclusion can be either true or false. Any formal mistake or logical fallacy similarly invalidates the deductive guarantee.

Properties of Equality and Properties of Operations provide the tools to justify statements are true provided the properties are understood. Highlighting that division by zero results in absurd conclusions helps students to realize the need to know specifics of Properties of Operations particularly the Multiplicative Inverse Property in which a ≠ 0 (see Number Properties handout)

**Division by zero Faulty reasoning**

Since, 0 x 1 = 0 and 0 x 2 = 0

Then 0 x 1 = 0 x 2

But, x 1 =  x 2

So, 1 = 2

**Anticipated Student Preconceptions/Misconceptions:** Thinking of the equal sign as ‘operational’ (a sign connecting the answer to the problem) rather than ‘relational’ (the expression on the left is equal to the expression on the right).

**Lesson Sequence**

1. Open the lesson by administering the Pre-Assessment. (Alternatively the pre-assessment can be given as an exit ticket the day before). You will want to determine what misconceptions students have about reasoning with equations and their ability to apply properties of operations correctly.

Upon completion of the pre-assessment, begin the lesson by connecting what was learned in the previous lesson to what students will learn in this lesson: Prior lesson included:

* + Using Number Properties of Operations to create equivalent expressions and simplifying expressions.
  + Justifying your process for simplifying expressions

In this lesson, students will create a logical argument to determine the truth of mathematical statements (identity equations) by applying the Properties of Operations to justify their reasoning.

1. Use the handout, *Why is Reasoning Logically Important in Mathematics*.

Discuss each step of the justification for ***1 = 2*.** Ask students why one can conclude, in the second line, ***0 x 1 = 0 x 2, and discuss the substitution property that justifies it: Since 0 x 2 = 0 we can substitute 0 x 2 for 0 anytime. For many students substitution has been applied only for evaluating an expression or for guessing/checking the solution to an equation.***

Next, on the board or overhead write the question:

“Is the statement: n + ( n + 2) = 2 ( n + 1 ) true for ALL real numbers?”

(Refer to the *Why is Reasoning Logically Important in Mathematics?* handout)

Have students individually think for a minute about the truth of the statement for **some** numbers and **all** real numbers. After a minute, have students turn and talk about their thinking. Share out students thinking with the whole class.

Teacher Note: Students may think a few numbers will justify the statement. Unless they can justify for each number (an impossible feat) then they cannot answer the question as true for **all** real numbers. Or, unless they can come up with a number that does not make it true (counterexample) then they cannot answer as not true.

Whole class: Give students the *How Does One Reason Logically with Equations?* handout. Use the Think, Pair, Share process to have students answer the following question and justify their answer:

* *Is the statement n + ( n + 2) =2(n + 1) true for some, all, or no real numbers?*
* Students *Think* for about one minute
* Students in pairs Turn and Talk for two minutes to share their thinking

Note: It is important that students have an opportunity to think about how they can justify the solution process before any teacher modeling begins.

Have students share their thinking about the question and their response and reasoning. If necessary, model the justification process with students as shown on the handout. Make sure students understand the relational versus the operational meaning of the equal sign. Have students justify each step using the Number Properties as you take them through the steps. It is important to continually emphasize that this statement is *true for all real numbers.* Use the term, identity, to define the statements that are true for ALL real numbers. Reiterate that there are other sequences of steps that could also have been used to justify.

Note: The first reason in the sequence in the table is the Associative Property of Addition. The second reason “Combine like terms” is informal language**. Formally you can discuss with students how n + n = 2n is justified by the Distribute and Commutative Properties: n + n = n ( 1 + 1) = (1+1) n = 2n, or by Distribution alone: n + n = (1 + 1) n = 2n.**

When revisiting the **1 = 2**, it is not necessary at this point to go too deeply into the Division property of Equality, as this point will be addressed in more depth in a later lesson in this unit.

1. Students then work in pairs to complete the ***You Try*** section of the *How Does One Reason Logically with Equations?* handout. Listen for discussion that justifies the reasoning of the properties they have selected.

Wrap up by having students respond to the Essential Question: ***Why is reasoning logically important in mathematics*?** This can be done using a *Turn and Talk* process or as a writing prompt for an Exit Ticket.

## Pre-Assessment Lesson 3

Do you agree or disagree with the following statements? Explain your reasoning.

|  |  |  |
| --- | --- | --- |
| Statement | Agree/  Disagree | Reasoning/Explanation |
| 1. 4x - (3y+7x) = 11x - 3y |  |  |
| 1. 11x + 6y - 2x + 3y = 9(x + y) |  |  |
| 5n + 10 Area of the triangle = 2.5n + 25 |  |  |

Lesson 3 Why is Reasoning Logically Important in Mathematics?

Since 0 x 1 = 0 and 0 x 2 = 0

Then 0 x 1 = 0 x 2

But, 0 x 1 = 0 x 2

0 0

So, 1 = 2

***Turn and Talk to a classmate and discuss the reasoning used to justify 1 = 2.***

Jot down some ideas to share with the class.

**Is the statement** **true for *some*, a*ll*, or *no* real numbers?**

***Justify your answer*.**

Think about it on your own first and write down your work and ideas, then Turn and Talk with a classmate and share work and ideas. Be prepared to share with the class.

**n + ( n + 2) = 2 ( n + 1)**

**Number Properties of Operations: Our *TOOLS* for Reasoning Logically**

1. **Begin with one of the expressions** (left or right) in the statement
2. **Apply Properties to that expression** until it is the same as the other expression

Variables of the same letter and degree can be added ( a form of distribution)

|  |  |
| --- | --- |
| **Suppose we begin with n + (n + 2)**  Think what you can do with n+(n+2) | **State the Property that we applied** |
| ***n + ( n + 2******)*** = (n + n) + 2 | Why? |
| = 2n + 2 | Combine Like Terms |
| = ***2( n + 1)*** | Why? |

Since we showed a ***logical sequence*** of ***equivalent expressions,*** we have justified that the statement **n + ( n + 2) = 2 ( n + 1 )** is true for **all** real Numbers.

Revisit the activity 1=2 above:

Where does the sequence of reasoning logically break down?

**III. You Try**

Justify the statement is True for all Real Numbers: **2(1 – x) + (x – 1) = 1 – x**

|  |  |
| --- | --- |
| Statements | Property |
| 2(1 –x) + ( x – 1) = |  |
|  |  |
|  |  |
|  |  |
|  |  |
| = 1 - x |  |

**Reflection Activity:**

1. Because the statements above were proved to be true for **all** real numbers they are called **Identities** or Identity Statements. Describe in your own words what an identity statement is. How are identity statements and equations alike? How are they different?
2. ***Why is reasoning logically important in mathematics*?**

Lesson 4

What’s My Number?

**Brief Overview of Lesson:** This lesson focuses on using number problems to create, solve, and check solutions to equations and to reason about the role equivalence plays in the equation solving process.Students develop understandings to move beyond guess and check and reasoning strategies for solving more complicated number problems. As you plan, consider the variability of learners in your class and make adaptations as necessary.

**Prior Knowledge Required:**

Properties of Operations including Inverse Properties

Translating everyday language into mathematical equations

**Estimated Time** 45 minutes

**Resources for Lesson:**

Handouts:

* *Generation N… What’s My Number?*
* *Equivalence in Equation Solving- Revisiting our First Problem*
* *Equivalent Equations*
* *Maintaining Equivalence- Problem 4*

**Unit: Reasoning with Equations**

**Content Area/Course: Algebra I**

**Lesson 4:** *What’s My Number?*

**Time (minutes):** 45 minutes

**By the end of this lesson students will know and be able to:**

Create equations to model number problem situations.

Explain how two different equations can have the same solution.

Create equivalent equations

**Essential Question(s) addressed in this lesson:**

How can two different equations have the same solution?

**Guiding Questions for this lesson:**

How can I model a problem situation algebraically?

How can the model be used to find a solution?

**Standard(s)/Unit Goal(s) to be addressed in this lesson:**

A-REI.B.3: Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters**.**

A-REI.A.1**:** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify or refute a solution method.

A-CED.A.1: Create equations and inequalities in one variable and use them to solve problems. (Include equations arising from linear, and quadratic functions, and simple root rational functions and exponential functions.)

SMP1**:** Make sense of problems and persevere in solving them.

**Instructional Resources/Tools:** *Generation N…… What’s My Number?* Handout

**Instructional Tips/ Strategies/Notes for Teacher**

Other problem ideas for this lesson are problems using consecutive integers, age problems etc. that can be modeled algebraically.

What students need to know and are able to do coming into this lesson :

Students can: solve simple linear equations (variables on one side) using inverse operations and number properties of operation, use the Substitution Property of Equality, translate everyday language into mathematical language to create equations.

**Teacher Note:** This unit extends students’ work in Middle School of applying properties of operations (distributive, commutative, associative, as well as multiplicative and additive identities) to *expressions* that resulted in an equivalent *expression*. In grade 8 students used linear equations/functions to model problems related to rates of change and they solved linear equations using properties of operations and inverse operations of addition and multiplication. This lesson will develop a) students’ ability to create equations for representing a model for solving problems, b) their understanding that a formal approach to solve equations exists and that the approach results in the creation of *equivalent equations* . (Please note the formal process for solving equations will be developed in a later lesson in this unit.

Anticipated Student Preconceptions/Misconceptions

Students may confuse expressions and equations. They may see the equal sign as an ‘operator’ (telling us what operation to perform) rather than as a statement of equivalence between expressions.

**Lesson Sequence**

* Solving number problems that become increasingly challenging
* Creating equations to model number problem situations.
* Comparing equations generated in solving a number problem and reasoning about how two different equations have the same solution can
* Creating equivalent equations.
* Finding solutions to equations by creating an equivalent equation

1. Facilitate a brief discussion with students to address the following questions:

* *How can I model a problem situation algebraically*
* *Why would I want to anyway?*

*Let students discuss the questions as a class before passing out the handout What’s My Number*

* *Introduce the handout and the first problem:* **I am thinking of a number, doubling it, then adding 8. My result is 20.**

Have students **individually think** about the problem and find a solution. (By reasoning and/or by solving an equation.)

*Note: Students have an intuitive sense they use to reason about math. We want to maintain it and also move students towards developing efficient and effective problem solving strategies.*

Next, have students “turn and talk” to a partner and explain their way of thinking about the problem and the way they solved it. Students may draw diagrams, equations, tables or reason backwards

Finally, Ask student pairs to share out one approach to solving the problem. Have pairs only share unique reasoning and equation solving methods (if many pairs have solved the problem the same way). Look for reasoning that involves inverse operations (or working backwards), as well as students who created an equation to model the problem and how they found their solution to the equation. Emphasize the difference between the expression and the equation.

*NOTE:*

In Problem 1 of the *What’s My Number?* activity, although there is an unknown quantity involved, students may easily use backward reasoning. All of the *operations involved are with numbers*: one solution may be to start with 20, take away 8 to get 12 and halve that to get the original number 6.

* *Repeat the process with number problem 2.* I am thinking of a number, doubling it, adding 8, then adding my original number. My result is 23. *Additionally have students reflect and share their responses to the reflection questions first in pairs and then as a class*

*NOTE:*

* In Problem 2, it is necessary to operate *on the unknown*. Finding the solution to this problem will be more challenging by reasoning alone. Students already have experience with the Substitution Property of Equality when they evaluate an expression. Some students will use Substitution (via guess and check) as a problem solving strategy.
* *Repeat the process with number problem 3.* I am thinking ofa number*, subtracting 10* then *doubling the difference.* The *result is 1 less than my original number.* What’s my number? *Additionally have students reflect and answer the questions;* What was easy / difficultabout the reasoning approach? Were you able to solve it by reasoning? What was easy / difficult about the equation approach? Were you able to solve the equation? How is the equation for problem 3 similar to the equations in problems 1 and 2? How is the equation for problem 3 different to the equations in problems 1 and 2?

*Note:*

In Problem 2 and 3, it is necessary to *operate with an unknown quantity* (to double it in problem 2 and to compare with 1 less than the number in problem 3). The solutions to these problems are more challenging to solve by reasoning. A major conceptual understanding that students need to develop to be successful equation solvers is the difference between operating on equations and operating on expressions. This lesson addresses this important distinction.

**II. EQUIVALENT EQUATIONS**

Generating equivalent equations

Handout-Page-Equivalence in Equation Solving

* Have students verify that performing the same operations on n and on 6 produces equivalent **expressions when n = 6**, namely 2n and 12 are equivalent expressions as are  **2n + 8 and 12 + 8.**

|  |  |  |  |
| --- | --- | --- | --- |
| **A number** | **doubling it,** | **then adding 8.** | **My result is 20.** |
| n = 6 | 2n = 12 | 2n + 8 = 12 + 8  Or 2n + 8 n= 20 |  |

* Now look at the **equations** generated: beginning with n = 6, then 2n = 12 and 2n + 8 = 20. This is the heart of the lesson. Have students think about the 3 questions and discuss with a partner:

What relationship does the equation n = 6 have with the equation 2n = 12? What relationship does the equation n = 6 have with the equation 2n + 8 = 20? What relationship does the equation 2n = 12 have with the equation 2n + 8 = 20?

Give students time to conclude that n = 6 is a solution to all of the equations. **Vocabulary**: We say two **equations are equivalent** when they have exactly the same solutions.

Discuss the formal approach to solving the equation 2n + 8 = 20 as a process of generating ‘equivalent equations’.

Also note for students that even though equations 2n = 12 and 2n + 8 = 20 are equivalent, it does not mean 2n and 2n + 8 are equivalent.

**The final question: How can different equations have the same solution?** This question can be used to summarize the concept of equivalent equations. Have students discuss and share out their responses.

Also this is a good time to review from lesson three the term **Identity** and discuss the difference between **equivalent expressions** and **equivalent equations.** Discuss how equivalent equations are generated by applying Properties of Operations while equivalent equations are generated also by Properties of Equality that will be explored in Lesson 6.

**III. Reinforcement Activity**: Handout-*Equivalent Equations*:

Students create their own equivalent equations. By doing so they are thinking about maintaining equivalence and are naturally applying properties of equality before studying these properties formally.

**IV. Closing Activity**: Handout : *Maintaining Equivalence- Problem 4.*

This final word problem is to set up a sense of need for having a formal equation solving process to help solve problems that can be modeled algebraically and are difficult or tedious to solve by other methods. The quadratic equations here can be solved by applying number sense to the equation to generate an equivalent equation without naming Properties of Equality.

**Preview outcomes for the next lesson:** Pre-assessment Task: ***Equations and Identities*** from the Formative Assessment Lesson Sorting Equations and Identities

<http://map.mathshell.org/materials/lessons.php?taskid=218>

I. Generation N What’s My Number?

|  |  |  |
| --- | --- | --- |
| 1. **I am thinking of *a number*, *doubling* it, then *adding 8*. My *result is 20*. What’s my number?** | | |
| ***Reasoning approach*** | ***Equation approach*** | |
| **What’s my number? Explain your reasoning:**  **Are there any other numbers that work? How do you know?** | **Write an expression that models the situation for the number (use ‘n’ to represent the number).**  **Write an equation that represents the problem situation.**  **Solve for n in the equation.** | |
| **Reflection:**  **What was easy / difficult about the reasoning approach?**  **What was easy / difficult about the equation approach?** | | |
| 1. **I am thinking of *a number*, *doubling* it, *adding 8*, then *adding my original number*. My *result is 23*. What’s my number?** | | |
| ***Reasoning approach*** | | ***Equation approach*** |
| **What’s my number? Explain your reasoning:**  **Are there any other numbers that work? How do you know?** | | **Write an expression that models the situation for the number (use ‘n’ to represent the number).**  **Write an equation that represents the problem situation. Solve for n.** |
| **Reflection:**  **What was easy / difficult about the reasoning approach? Were you able to solve it by reasoning?**  **What was easy / difficult about the equation approach? Were you able to solve the equation?** | | |

|  |  |
| --- | --- |
| 1. **I am thinking ofa number*, subtracting 10,* then *doubling the difference.* The *result is 1 less than my original number.* What’s my number?** | |
| ***Reasoning approach solution*** | **Formal approach solution** |
| **What’s my number? Explain your reasoning:**  **Are there any other numbers that work? How do you know?** | **Write an expression that models the situation for the number (use ‘n’ to represent the number).**  **Write an equation that represents the problem situation. Solve for n.** |
| **Reflection:**  **What was easy / difficult about the reasoning approach? Were you able to solve it by reasoning?**  **What was easy / difficult about the equation approach? Were you able to solve the equation?**  **How is the equation for problem 3 similar to the equations in problems 1 and 2?**  **How is the equation for problem 3 different to the equations in problems 1 and 2?** | |
| **Create your own number problem (as complicated as you like). Swap with a partner and find the solution to each other’s problem.**  **What strategies did you employ to create your problem?**  **What strategies did you employ to solve your partner’s problem?** | |

### II. Equivalence in Equation Solving

|  |  |  |  |
| --- | --- | --- | --- |
| **In Problem 1 from** What’s My Number?, **the solution was the number 6.**  **How is the solution equation, n = 6 related to the problem equation 2n + 8 = 20?** | | | |
| I am thinking of a number | Doubling it | Adding 8 | My result is 20 |
| n | 2n | 2n + 8 | 2n + 8 |
| Checking the solution  n = 6 | 12 | 12 + 8 | 20 |
| What relationship does the equation n = 6 have with the equation 2n = 12?  What relationship does the equation n = 6 have with the equation 2n + 8 = 20?  What relationship does the equation 2n =12 have with the equation 2n + 8 = 20? | | | |
| **Question mark with person leaning against it**  **How can two different equations such as**  **2n = 12 and 2n + 8= 20 have the same solution?**  **Take a minute think about it then share with a partner.**  **Create a pair of equations that are different and have the same solution.**  **What was your strategy to create this pair of equations?** | | | |

|  |
| --- |
| Equivalent Equations |
| ***Determine if the following pairs of equations are equivalent: Explain.***  1. x/3 = 5 and 2x = 30 2. 12x = 18 and 3 = 2x 3. - 7 x = 15 and 14x + 60 = 30 |
| ***Create 3 equations in x that have the solution x = 4.*** |
|  |
|  |
|  |
| ***What was your strategy to create these equations?*** |
| ***Create 2 equations in x that have the solution x = 4 and that have an x on both sides of the equal sign.*** |
|  |
|  |
| ***What was your strategy to create these equations?*** |

|  |  |
| --- | --- |
| 1. **I’m thinking of a number, squaring it, subtracting twice my original number and the result is 8 more than the number squared.** | |
| ***Reasoning approach*** | ***Equation approach*** |
| **What’s my number? Explain your reasoning:**  **Are there any other numbers that work? How do you know?** | **Write an expression that models the situation for the number (use ‘n’ to represent the number).**  **Write an equation that represents the problem situation.**  **Solve for n in the equation.** |
| **Were you able to solve it by reasoning? Explain.**  **Were you able to solve the equation? Explain.** | |
| **Create an equation equivalent to equation: *n2 – 2n = n2 + 8* and explain why your equation is equivalent.** | |

***Teacher Notes Generation N What’s My Number?***

|  |  |  |
| --- | --- | --- |
| 1. **I am thinking of *a number*, *doubling* it, then *adding 8*. My *result is 20*. What’s my number?** | | |
| ***Reasoning approach*** | ***Equation approach*** | |
| **What’s my number? Explain your reasoning:**  **Students may work backwards from 20 by subtracting 8 then divide by 2 or they may try ‘guess and check’ strategy (Middle school strategies)**  **Are there any other numbers that work? How do you know?**  **Students may say the only number doubled to get 12 is the number 6. This question is not easily answered by the guess an check approach.** | **Write an expression that models the situation for the number (use ‘n’ to represent the number).**  **2n + 8 is an expression**  **Write an equation that represents the problem situation.**  **2n + 8 = 20**  **Solve for n in the equation.**  **Students may reason ‘what added to 8 is 20’ or they may use a *procedure* learned in Middle School for solving two step equations of the form px+q=r** | |
| **Reflection: These questions can be a formative assessment to help determine where students are in creating and solving simple linear equations**  **What was easy / difficult about the reasoning approach? The reasoning is easy in this problem but gets more difficult in the next few problems.**  **What was easy / difficult about the equation approach?** | | |
| 1. **I am thinking of *a number*, *doubling* it, *adding 8*, then *adding my original number*. My *result is 23*. What’s my number?** | | |
| ***Reasoning approach*** | | ***Equation approach*** |
| **What’s my number? Explain your reasoning:**  **More difficult to work backwards but still possible: subtracting 8 from 23, and then reasoning about the sum of a number doubled and the original adding to 15. Some students may also try to ‘guess and check’**  **Are there any other numbers that work? How do you know? See first problem note above.** | | **Write an expression that models the situation for the number (use ‘n’ to represent the number).**  **2n + 8 + n expression**  **Write an equation that represents the problem situation. Solve for n.**  **2n + 8 + n = 23 Solving for n is more than a two step procedure and involves applying Properties of Operations in order to ‘combine like terms.’** |
| **Reflection: These questions can be a formative assessment to help determine where students are in creating and solving equations that are beyond a two-step approach**  **What was easy / difficult about the reasoning approach? Were you able to solve it by reasoning?**  **What was easy / difficult about the equation approach? Were you able to solve the equation?** | | |
| 1. **I am thinking ofa number*, subtracting 10,* then *doubling the difference.* The *result is 1 less than my original number.* What’s my number?** | | |
| ***Reasoning approach*** | **Equation approach** | |
| **What’s my number? Explain your reasoning:**  **Difficult to reason about or to guess and check**  **Are there any other numbers that work? How do you know?** | **Write two expressions that model the situation for the number (use ‘n’ to represent the number).**  **2(n – 10)**  **n - 1**  **Write an equation that represents the problem situation. Solve for n.**  **2(n – 10) = n – 1**  **Students have to apply both Properties of Operations and Properties of Equality to solve for n. There is no ‘procedure’ but a general approach will be taught later in this unit so that students will become fluent and able to explain the process of solving** | |
| **Reflection: These questions can be a formative assessment to help determine where students are in creating and solving equations and applying properties of both operations and equality**  **What was easy / difficult about the reasoning approach? Were you able to solve it by reasoning?**  **What was easy / difficult about the equation approach? Were you able to solve the equation?**  **How is the equation for problem 3 similar to the equations in problems 1 and 2?**  **How is the equation for problem 3 different to the equations in problems 1 and 2?**  **Students may not be able to articulate differences. Problem 3 is an equation with the variable on both sides. Students who are taught to solve equations procedurally (combine like terms, isolate the variable, remove parentheses, etc. may not be able to successfully solve equations in various formats.) In this unit students will learn to think about equation solving as a process that maintains equivalency while solving for the variable)** | | |

|  |  |  |  |
| --- | --- | --- | --- |
| II. Equivalence in Equation Solving **I am thinking of *a number*, *doubling* it, then *adding 8*. My *result is 20*. What’s my number?**  **Compare the rows and columns below as they pertain to the question. The goal of revisiting this problem is to help students develop their understanding of how we maintain equivalency in solving equations.** | | | |
| Starting with a  number | Doubling it | Adding 8 | My result is 20 |
| Unknown n | 2n | 2n + 8 | 20 |
| Known solution 6 | 12 | 12 + 8 | 20 |
| Compare the equations below. Answer the questions that follow. Discuss with a partner and be prepared to share with the class. Most students see n = 6 as a solution and not as an equation. You can say it is a solution equation. | | | |
| n = 6 | 2n = 12 | 2n + 8 = 12 + 8  or  2n + 8 = 20 |  |
| **All of these questions lead students towards an understanding of equivalent equations.**  What relationship does the equation n = 6 have with the equation 2n = 12?  Students may simply say n = 6 is the solution for the equation 2n = 12, or that if you solve for n you get n = 6 which is the same equation.  What relationship does the equation n = 6 have with the equation 2n + 8 = 20?  Again, similar to above.  What relationship does the equation 2n = 12 have with the equation 2n + 8 = 20?  Students may say that they both have n = 6 as the solution. | | | |
| **Question mark**  **How can different equations such as**  **2n = 12 and 2n + 8= 20 have the same solution?**  **Take a minute think about the question then share with a partner.**  **We want students to use the phrase ‘equivalent equations’. Discuss that concept with the class. Compare it to ‘equivalent expressions’. Have students summarize the difference between ‘ Equivalent equations’ and Equivalent expressions’. They will study further how each are used in solving equations.**  **Note:**   * **‘Equivalent expressions’ have the same value and are Identities (true for all real numbers) when written as an equation such as; 3x2 – 3x = 3x (x – 1) which is true for all numbers in x** * **‘Equivalent equations’ have the same solution (such as the two equations in the box above)**   + **Important to note: This does not mean we can equate either left sides or right sides of the equations to say that 2n and 2n + 8 are equivalent. It is the equations themselves that are equivalent.** | | | |

|  |
| --- |
| Equivalent Equations |
| ***Determine if the following pairs of equations are equivalent: Explain.***  1. x/3 = 5 and 2x = 30 2. 12x = 18 and 3 = 2x 3. - 7 x = 15 and 14x + 60 = 30 |
| ***Create 3 equations in x that have the solution x = 4.*** |
|  |
|  |
|  |
| ***What was your strategy to create these equations?***  Students responses will show whether they are thinking by guess and check (x + 6 =10), or by reasoning with equivalent forms 2x +2 = 10 and 2(x + 1) = 10 that involve Properties of Operations discussed in Lesson 2 of this Unit. |
| ***Create 2 equations in x that have the solution x = 4 and that have an x on both sides of the equal sign.***  Asking students to next create equations with the variable on both sides will challenge them further. Some students will realize they just have to ‘add’ (or place) an x on both sides of the equation x = 4 as in these examples: ***x + x = x + 4 = 2x = x + 4; x + x + 6 = 10 + x = 2x + 6 = x + 10, etc. By creating these equations students are using the Properties of Equality*** |
|  |
|  |
| ***What was your strategy to create these equations?*** |

### Maintaining Equivalency

|  |  |
| --- | --- |
| 1. **I’m thinking of a number, squaring it, subtracting twice my original number and the result is 8 more than the number squared.** | |
| ***Reasoning approach*** | ***Equation approach*** |
| **What’s my number? Explain your reasoning:**  **Students generally do not guess and check with negative integers making that strategy difficult to use**  **Are there any other numbers that work? How do you know?** | **Write two expressions that model the situation for the number (use ‘n’ to represent the number).**  **x2 – 2x**  **x2 + 8**  **Write an equation that represents the problem situation.**  **x2 – 2x = x2 + 8**  **Solve for x in the equation.**  **Because students created equations with a variable on both sides students may realize they can “undo” the x2 terms on both sides leaving -2x = 8 so x = -4** |
| **Were you able to solve it by reasoning? Explain.**  **Were you able to solve the equation? Explain.** | |
| **Explain how the equation: *x2 – 2x = x2 + 8 is* equivalent to the equation *– 2x = 8*.**  **Students should be able to state that these two equations are equivalent because they have the same solution or because by applying a Property of Equality (Subtraction) equivalency is maintained.** | |

Lesson 5

Sorting Equations and Identities

**Brief Overview of Lesson:**

Students discuss the relationship between equations and identities, by discovering three types of equations: equations with one solution, equations with multiple solutions, and equations with no solution. As you plan, consider the variability of learners in your class and make adaptations as necessary.

**Prior Knowledge Required:**

**Estimated Time (minutes):** 45 minutes

**Resources for Lesson**

*MARS Lesson Sorting Equations and Identities*- lesson found at: <http://map.mathshell.org/materials/lessons.php?taskid=218>

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Title: “[Sorting Equations and Identities](https://www.map.mathshell.org/download.php?fileid=1720)”

Author: [Mathematics Assessment Resource Service University of Nottingham & UC Berkeley](https://www.map.mathshell.org/background.php)

**Unit:** Reasoning with Equations

**Content Area/Course:** Algebra I

**Lesson 5:** *Equations or Identity*

**Time (minutes):** 45 minutes

**By the end of this lesson students will know and be able to:**

* Recognize the differences between equations and Identities
* Test the validity of algebraic statements
* Manipulate algebraic statements correctly and avoid common errors

**Essential Question(s) addressed in this lesson:**

*How does one reason logically to solve equations?*

**Guiding Question addressed in this lesson:**

*How many solutions can an equation have?*

**Instructional Tips/ Strategies/Notes for Teacher**

In addition to the mathematical goals stated in the MARS lesson, students will understand:

* Equations may have no solution, one solution or multiple solutions.
* Some equations are true for all values of the variable.

**Anticipated Student Preconceptions/Misconceptions**

Expressions and equations are synonymous

* Equations can only have one solution
* x and y cannot represent the same number

**Standard(s)/Unit Goal(s) to be addressed in this lesson:**

A-REI.B.3: Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

SFMP 3: Construct viable arguments and critique the reasoning of others

SFMP 7: Look for and make use of structure

**Lesson Sequence**

Refer to MARS *Lesson Sorting Equations and Identities*, and have students explore equations that are always, sometimes or never true.

Suggested process from the lesson introduction: <http://map.mathshell.org/materials/lessons.php?taskid=218>

* ***A day or two before the lesson***, students work individually on an assessment task that is designed to reveal their current understandings and difficulties. You then review their work, and create questions for students to answer in order to improve their solutions.<http://map.mathshell.org/materials/lessons.php?taskid=218> (page S-1)
* A whole-class introduction on finding solutions to equations that may be: always, sometimes, or never true.
* Students work in small groups on a collaborative discussion task. Students sort a card set of equations and create a poster that categorizes the solutions to the equations as always, sometimes, or never true. Students justify within their group the reason for placing an equation in a particular category. Students discuss the reason and come to a consensus about the placement.
* After a whole class discussion of the justifications of methods used in 2-3 equations each group chooses an equation from their poster that meets a given criteria.
* Students revisit their individual *Equations and Identities Posters* and revise their original work based on what they have learned during the lesson.
* Summary activity: *How many solutions can an equation have?* Give examples to support your reasoning, and share them with a partner. Discuss any discrepancies.

**Homework:** Hand back the Pre-Assessment Task (this was used to see where students are). Students will review their responses, make any corrections, and through writing, reflect upon what types of equations were always true, sometimes true, and never true.

Lesson 6

Justifying the Solution Process

**Brief Overview of Lesson:** Properties of Number and Equality are used in the equation solving process. The effects of applying the properties are noted both algebraically (symbolically) and graphically. As you plan, consider the variability of learners in your class and make adaptations as necessary.

**Prior Knowledge Required:** Properties of Operations, equivalent equations, evaluating expressions

**Estimated Time** 90 minutes

**Resources for Lesson**

Handouts:

*Properties of Equality*

*Practice Problems on Comparing Solution Process – (Maria and Jonathan)*

*Graphing Activity Sheets* (created using: [www.graphsketch.com](http://www.graphsketch.com))

*Visualizing the Equation Solving Process*

*Property Matching Activity*

*Equation Solving Graphic Organizer*

**Unit:** Reasoning with Equations

**Content Area/Course:** Algebra I

###### **Lesson 6:** Justifying the Solution Process

**Lesson Overview**: Applying Properties of Operations and Properties of Equality to solve equations, and reasoning about the effects of applying properties by comparing changes in an equation visually.

**Time :** 90 minutes (or 2 class periods)

**By the end of this lesson students will know and be able to:**

* Identify and apply the various properties of equality,
* Use Properties of Equality to generate equivalent equations that lead to solving multi-step equations in one variable;
* Visually compare the transformation on an equation from applying the properties
* Understand the different effects applying Number Properties of Equality and Operations when solving equations algebraically
* Relate steps in the solution process to graphs of each step and the property associated with it.

**Guiding Question for this lesson:**

How does applying Properties of Numbers (Properties of Numbers include both Properties of Operations and Properties of Equality) on equations affect the equation. How can this be represented visually?

**Standard(s)/Unit Goal(s) to be addressed in this lesson:**

A-REI.A.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify or refute a solution method.

A-REI.B.3 Solve Linear equations and inequalities in one variable, including equations with coefficients represented by letters.

SMP.1 Make sense of problems and persevere in solving them

SMP.3 Construct viable arguments and critique the reasoning of others.

**Instructional Tips/ Strategies/Notes for Teacher**

Students will understand the **difference** between operations performed on expressions and operations performed on equations and be able to perform the operations fluently.

* Students will differentiate between Properties of Operations (operations performed on expressions) and Properties of Equality (operations performed on equations), use them in solving multi-step equations and to justify the solution process.
* Students will apply Number Properties of Operations and Properties of Equality to generate equivalent equations.
  + Students will solve equations algebraically using symbolic manipulations and justify the process.
  + Students will reason graphically about the effects of applying Properties of Operations and Properties of Equality in the solution of equations

**Anticipated Student Preconceptions/Misconceptions**

* Students may regard the equal sign operationally rather than equivalently
* Students may regard Properties of Equality as being the same as Properties of Operations
* Students may regard solving equations as “isolating the variable”

**Teacher Notes:** The focus is on Properties of Equality related to operations of addition/subtraction and multiplication/division.

* Students will learn solving equations such as 4(3*m* − 11) + 22 = 42 + 5*m* requires generating equivalent equations using **Properties of Equality and Properties of Operations** . For example, to show how

4(3*m*-11) +22 = 42 + 5*m* is equivalent to 12*m* – 22 = 42 + 5*m* requires **operating** **on an expression (one side of the equation)** using **Properties of Operations** (Distributive property and combining like terms in this case).

But showing 12*m* - 22 = 42 + 5*m* is equivalent to 7*m* = 64

requires **operating on an equation itself** (‘on both sides’ using (addition/ subtraction) using **Properties of Equality**.

* *Students previously (grade 8) were taught to solve*  equations such as: 4(3*m* − 14) + 22 = 42 by Distribution on the expression on the left (It becomes equivalent to 12*m – 22 = 42 )* and then use inverse operations ( reasoning backwards as 12*m must equal 62, ( such as; If 12m=62 then what can I multiply 12 by to get 62), so 6212 = m.* In Algebra I students will know and use Properties of Equality as properties that operate on (across) the equation (equal sign).
* Equations are statements that equate two expressions. Operating on an equation requires students to view an equation as a single object and not as separate expressions. However, solving equations requires being able to work with expressions appropriately.

An important idea for students to understand is that the solution to an equation is also a solution of each new equation created in the solution process.(See Lesson 4)

**Lesson Sequence**

**Guiding Question:** *How is equality maintained in the solution process of solving algebraic equations?*

#### **Properties of Equality Handout**

1. Give students a *Properties of Equality* handout sheet. Ask students to rephrase in their own words and to give their own examples of the Properties. These Properties may have been studied before but should not be trivialized. You can use white boards to have students write down their examples and you may want to share some student responses.
2. Students will complete Column 3 individually by creating examples of the properties stated in the left two columns. Students can share their examples in partners or groups of 3-4. Have groups share out example.

This activity does not “solve” an equation but develops students’ understanding that an equation is a single object that states an equivalency between two expressions. They learn that the Properties of Equality are applied to the equation (“across the equal sign”) and that those properties guarantee that the equivalency is maintained.

Note: Before students complete columns 4 and 5, students need to understand an equation is ‘solved’ when they have created the **simplest *equivalent equation*** (such as y = 5) (See Lesson 4). They need to consider the question: How do I get there? (Using Properties of Number) It is also important for students to know an answer can’t ‘hang out there’. It’s not just “5”. A solution can look like an equation, i.e. a “solution equation,” or it can be put back into the context of the problem when the problem has a real world connection. We also don’t want to restrict students’ natural intuitive reasoning when it comes to finding the solution to an equation. What sense can students make of solving an equation if the goal is to “isolate the x”? For that reason it is best to emphasize the process of generating equivalent equations eventually results in x (with coefficient of one) being the only symbol on one side of the equation.

1. Students will complete column 4 individually at first and then share with a partner. Have students share out their equivalent equations. Students complete Column 5 (Solve for x) by applying an appropriate Property of Equality and write the accompanying equivalent equation until a solution equation (the simplest equivalent equation) is found. Students can share their work and solutions in partners or groups of 3-4. Then have groups share out an example.

Share with the class student’s various strategies relate to the following

* Students may begin to solve an equation with either side of the equation
* Some students may want to work with variables first, while others may want to work with numbers first.
* Some may ask “should I distribute first? Divide first?

1. Practice Problems. (Maria and Jonathan problems).
   1. Students will reason about the equivalent equations in Part A. Have students complete the first problem in Part A individually by reflecting on the operation that was performed to generate each equivalent equation. Have students share with a partner. Then students will complete the second problem in Part A and reflect on the question posed in part A.
   2. In Part B students will first reason about the Property. Have students fill in the equivalent equation for each property individually and then share with a partner. Have students complete the second Part B problem and discuss as a class the approaches to solving the equations presented by the problems

## Visualizing the Equation Solving Process

## Notes:

Students will compare graphs of the equivalent equations generated in the equation solving process. Using the Maria and Jonathan problems, students will see a visualization of the equivalency at each step in the solution process. When properties of operations are used such as distribution and combining like terms students will see no change in the graph. When properties of Equality are used to ‘operate’ on the equal sign students will see that change visually. Throughout students will see the x value of the point of intersection defines the value of the equivalency of the expressions on both sides the equation.

Previously in this lesson, students created a logical argument for solving an equation. (See Maria and Jonathan activity) This activity will visualize for students the effect of applying properties to equations and/or expressions during the equation solving process.

The process of solving an equation generally involves both ‘operating’ on the equation itself using Properties of Equality and operating on one side (an expression) using Properties of operations. This distinction is important in avoiding misconceptions related to working on ‘both sides’ of an equation.

Students will now see these symbolic transformations graphically. Each side of the original equation is graphed as a separate function. Subsequent equivalent equations are also graphed in this way.

##### Handouts (color copies preferred)

-Visualizing the Equation Solving Process Maria Problems

-Visualizing the Equation Solving Process Jonathan Problems

***Color is helpful to keep track of the original equation’s expressions on each side and their respective graphs. If handouts are in black and white, have students draw over one of the lines representing the left or right side of the equation in ink or with colored pencils. (Graphs were generated at: http://www.graphsketch.com/)***

1. Using the Maria Visual Handout, have students compare changes in the graphs at each step to the Property applied. (Compare the left side and right side of the equations as properties are applied at each step). Have students jot down their own thoughts about the graphs at each step in the last column. Have students turn and talk to a partner and then have a whole class discussion about the comparisons. (Note the point of intersection if students do not and the fact that the x value remains the same throughout and is the solution to the equation)
2. Continue the activity with the Jonathan Visual Handout. Additionally students will complete the reflection questions to compare changes in graphs when a property of Equality was applied (to both sides) compared to when a Property of Operations was applied (to one side). Students will also reflect upon the points of intersection and how the solution is represented in the graph.

NOTE: Unlike the first example which only involved Properties of Equality, the second example involves both Properties of Operations and Equality. Students should see no change in the graph when a Property of Operation is involved (Step 2-applying the Distribution Property resulted in no change to the graph). This is an opportunity to expand on the difference between expressions and equations and working on one side (expressions) versus both sides together.

Summary: The graphic organizer can be given to students but it is advised to have students first reflect or compare and contrast the properties and create their own graphic organizer to aid in reflecting on and developing a deeper understanding of the properties and their use in solving equations.

**Formative assessment:** **Property Matching Activity**

In this activity small groups of students will arrange Equation Cards and Property Cards to justify the solution process in a logical sequence.

Alternatively, have half the class solve the equations on their own (without the activity) cards while the other half completes the activity. Compare sequences of solutions.

You may also just use the equations themselves on an exit ticket.

**Preview outcomes for the next lesson:**

In the next lesson students will reflect on equation solving strategies and approaches.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Property** | **a, b, and c represent real number quantities** | **Example** | **Apply it to generate an equivalent equation** | **Solve for x** |
| **Addition Property of Equality** | **If a = b**, then a + c = b + c. | y = 5  y + 4 = 5 + 4 | 4x – 7= 27 |  |
|  | Create your own example 🡪 |  |
| **Subtraction Property of Equality** | **If a = b**, then a – c = b – c. | -5 = t  -5 - 1/2 = t – 1/2 | 3x + 2 = 4x -5 |  |
|  | Create your own example 🡪 |  |
| **Multiplication Property of Equality** | **If a = b**, then a  c = b  c. | r = 2  r  - 3 = 2  - 3 | + 3 = 2 (x + 3) |  |
|  | Create your own example 🡪 |  |
| **Division Property of Equality** | **If a = b**, then a/c = b/c, c ≠ 0 | d = 1 | 7(x + 3) = 14(2x + 9) |  |
|  | Create your own example 🡪 |  |
| **Substitution Property of Equality** | **If a = b**, then you may replace b with a (or a with b) in any expression | If x = 5 **and** x + y = 9 then 5+y = 9 | 2y = 7 and 2y + 3x = 14 |  |
|  | Create your own example 🡪 |  |

#### Properties of Equality Tools for maintaining Equivalency in Solving Equations

***A. Maria used the following equivalent equations to solve for x. Fill in the properties she used to solve for x in the equation.***

|  |  |  |
| --- | --- | --- |
|  | **4 - 3x = x + 9** | **Property** |
| *1.* | *4 = 4x + 9* |  |
| *2.* | *-5 = 4x* |  |
| 3. | *= x* or x = |  |

***Use this space to check her solution:***

***B. Jonathan used the properties in column 2 to solve for x. What equivalent equations did he generate using those properties***

|  |  |
| --- | --- |
| **2(x+5) = -4(x-7)** | **Property** |
| 1. | Distributive Property  (applied twice!) |
| 2. | Addition Property of Equality |
| 3. | Subtraction Property of Equality |
| 4. | Division Property of Equality |

***Use this space to check YOUR solution:***

***What Property of Equality are you using when checking your solution?***

***Maria’s partner used the following equivalent equations to solve for x. Fill in the properties she***

***used to solve for x in the equation.***

|  |  |  |
| --- | --- | --- |
|  | **4 - 3x = x + 9** | **Property** |
| *1.* | *4 - 4x = 9* |  |
| *2.* | *-4x = 5* |  |
| 3. | *= x* or x = |  |

***How is Maria’s equation solving different from her partner’s?***

***Jonathan’s partner used the following sequence of equations to solve for x. What properties did he generate using those properties.***

|  |  |
| --- | --- |
| **2(x+5) = -4(x-7)** | **Property** |
| 1. (x + 5) = -2 (x - 7) |  |
| 2. x + 5 = -2x + 14 |  |
| 3. 3x + 5 = 14 |  |
| 4. 3x = 9 |  |
| . x = 3 |  |

***Is x = 3 a solution to each equations in the steps above?***

***Explain***

1. Card Matching Variable on Both Sides with Distribution

Make copies of the sheet below. Cut out the table cells and put in envelopes one per group. Students in groups of 2-3 will order the set of equivalent equations and matching properties.

|  |  |
| --- | --- |
| **4(*x* + 3) = 3*x* – 2** | **Original equation.** |
| **4*x* + 12 = 3*x* – 2** | **Distributive property of Operations.** |
| **4*x* + 12 – 3*x* = 3*x* – 2 – 3*x*** | **Subtraction Property**  **of Equality** |
| ***x* + 12 = – 2** | **Combine like terms**  **(Involves Commutative and Associative Properties of Operations)** |
| ***x* + 12 – 12 = – 2 – 12** | **Subtraction Property**  **of Equality** |
| ***x* = – 14** | **Simplify each side (Properties of Numbers)** |

1. Card Matching Variable and Distribution on Both Sides

Make copies of the sheet below. Cut out the table cells and put in envelopes one per group. Students in groups of 2-3 will order the set of equivalent equations and matching properties.

|  |  |
| --- | --- |
| **4(3x-5) = -2(-x + 8) -6x** | **Original equation.** |
| **12x – 20 = 2x -16 -6x** | **Distributive property of Operations.** |
| **12x – 20 = -4x -16** | **Combine like terms**  **(Involves Commutative and Associative Properties of Operations)** |
| **16x – 20 = -16** | **Addition Property**  **of Equality** |
| **16x = 4** | **Addition Property**  **of Equality** |
| **x =** | **Division Property**  **of Equality** |

1. Card Matching Equations With Fractions

Make copies of the sheet below. Cut out the table cells and put in envelopes one per group. Students in groups of 2-3 will order the set of equivalent equations and matching properties.

|  |  |
| --- | --- |
|  | **Original equation.** |
|  | **Multiplication Property**  **of Equality** |
| **4x + 3 = 12x - 2** | **Distributive Property**  **of Operations.** |
| **3 = 8x - 2** | **Subtraction Property**  **of Equality** |
| **5 = 8x** | **Addition Property**  **of Equality** |
|  | **Division Property**  **of Equality** |

I. Visualizing the Equation Solving Process:

Revisiting the equation solving process and the Properties of Numbers.

*The graphs of each side of the equation are shown. At each step consider how the graphs changed and/or stayed the same. The blue line represents the graph of the expression on the left side of the equation. (The Maria Problems)*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Equation** | **Property** | **Graph** | **Notes about the Graphs** |
|  | **4 - 3x = x + 9** | **Original equation** | graph on a coordinate plane showing two lines- one red and one blue. The red line has a positive slope and an equation of g of x equals x plus nine. The blue line has a negative slope and an equation of f of x equals 4 minus three x. | **f(x) = 4-3x**  **g(x) = x + 9** |
| *1.* | ***4 - 4x = 9*** | **Addition/Subtraction Property of Equality** | A graph on the coordinate plane with two lines- one red line and one blue line. The red line is horizontal passing through y = 9. The blue line has a negative slope and passes through the points (8, -1) and (1, 0). |  |
| *2.* | ***-4x = 5*** | **Addition/Subtraction Property of Equality** | A graph on the coordinate plane with two lines- one red line and one blue line. The red line is horizontal and intercepts the y axis at (0, 3). The blue line has a negative slope and passes through the origin. |  |
| 3. | *=* **x**  or **x** = | **Division/Multiplication Property of Equality** | A graph on the coordinate plane with one red line and one blue line. The red line has a positive slope of 1 and passes through the origin.  The blue line is horizontal and crosses the y- axis at (0, -5/4). |  |

Reflection: How can it be that Addition or Subtraction could be used? How can it be that Multiplication or Division could be used? Jot down a few thoughts then explain your reasoning to another person.

### **II. Visualizing the Equation Solving Process:**

Revisiting the equation solving process and the Properties of Numbers.

*The graphs of each side of the equation are shown. At each step consider how the graphs changed and/or stayed the same. The blue line represents the graph of the expression on the left side of the equation (The Jonathan Problems)*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Equation** | **Property** | **Graph** | **Notes about the Graphs** |
|  | **2(x + 5) = -4(x – 7)** | **Original equation** | A graph on the coordinate plane with one red line and one blue line. The red line has a slope of negative 4 and crosses the y-axis at (0, 28). The blue line has a slope of positve two and crosses the y-axis at (0,10) The lines intersect at (3, 8). | **f(x) =2(x+5)**  **g(x)= -4(x-7)** |
| *1.* | **X + 5 = -2(x -7)** | **Division/Multiplication Property of Equality** | A graph on the coordinate plane with one red line and one blue line. The red line has a slope of negative 2 and crosses the y-axis at (014). The blue line has a slope of 1 and crosses the y-axis at (0,5).  The lines intersect at (3, 8) |  |
| *2.* | **X + 5 = -2x + 14** | **Distribution Property** | A graph on the coordinate plane with one red line and one blue line. The red line has a slope of negative 2 and crosses the y-axis at (0,14). The blue line has a slope of 1 and crosses the y-axis at (0,5).  The lines intersect at (3, 8). |  |
| *3.* | *3X + 5 = 14* | **Addition/ Subtraction Property of Equality** | A graph on the coordinate plane with one red line and one blue line. The red line is horizontal and crosses the y-axis at (0,14). The blue line has a slope of 3 and crosses the y-axis at (0,5).  The lines intersect at (3, 14) |  |
| 4. | *3x = 9* | **Addition/Subtraction property of Equality** | A graph on the coordinate plane with one red line and one blue line. The red lineis horizontal and crosses the y-axis at (0, 9). The blue line has a slope of 3 and passes through the origin. The lines intersect at (3, 9). |  |
| 5. | *x = 3* | **Division/Multiplication Property of Equality** | A graph on the coordinate plane with one red line and one blue line. The red line is horizontal  crosses the y-axis at (014). The blue line has a slope of 1 and passes through the origin. The lines intersect at (3, 3) |  |

|  |
| --- |
| Reflection:  What was the resulting change in the graph when a ***Property of Equality*** was applied to the equation: |
| 1. **Multiplication/Division?** |
| 1. **Addition/Subtraction?** |

|  |
| --- |
| What was the resulting change in the graph when the ***Property of Operations*** was applied to the equation? |

|  |
| --- |
| Look at each graph, find the point of intersection, and write down the coordinates of the points of intersection for each graph. |
| How did the point of intersection change after each property was applied? |
| How is the solution to the equation represented in the graph? |

Solving Equations and

Justifying the process

**Properties of Equality**

**Operate on Equations**

**Properties of Operations**

**Operate on Expressions**

Performed on ***Both sides simultaneously*** to maintains equivalency

Performed on ***Left or Right******side*** of equation to rewrite an expression

**PROPERTIES OF REAL NUMBERS**

***Most frequently used Properties of Operations*** ***Most frequently used Properties of Equality***

|  |  |
| --- | --- |
| **Commutative** | **Addition** |
| **Associative** | **Subtraction** |
| **Distributive** | **Multiplication** |
| **Identity and Inverse** | **Division (by a non-zero real number)** |
| **Additional Properties of Real Numbers: Properties of Zero, Properties of Negatives** | |

Lesson 7 Reasoning about the Equation Solving process

**Brief Overview of Lesson:** This lesson develops a formal symbolic approach to solving linear equations in one variable. Students will reason about different algebraic approaches for solving equations and reason about particular strategies used when solving equations. As you plan, consider the variability of learners in your class and make adaptations as necessary.

**Prior Knowledge Required:** Solving an equation algebraically preserves the equivalence of the expressions in the equation. (See also General principles about expressions and equations in this lesson)

**Estimated Time:** 45 minutes

**Resources for Lesson:**

Handouts

*Equation Solving First Steps*

*Applications*

*Strategies*

*Further Reasoning with Equations*

**Unit: Reasoning With Equations**

**Content Area/Course: Algebra I**

**Lesson # 7**: Reasoning about the Equation Solving Process

**Time:** 45 minutes

**By the end of this lesson students will know and be able to:**

Solve equations efficiently to avoid making errors

**Essential Question(s) addressed in this lesson:**

Howdoes one reason logically with equations?

How can different equations have the same solution?

**Guiding Questions for this lesson:**

Is there a formal algebraic process for solving linear equations? If so, what does it look like?

Can an equation be solved in more than one way? If so, is there a ’best’ way to solve it?

**Standard(s)/Unit Goal(s) to be addressed in this lesson:**

A-REI.B.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters

SMP.1 Make sense of problems and persevere in solving them

SMP.3 Construct viable arguments and critique the reasoning of others.

SMP.7 Look for and make use of structure

**Instructional Tips/ Strategies/Notes for Teacher**

As students are working on problems individually, the teacher should walk around the room to observe a variety of approaches students are using to solve equations. Allow time during the lesson to discuss the multiple approaches. Have students share or show their work using a document reader so that students may see the various strategies, as well as begin to make strategic choices when solving equations.

For example: Ways to generate equivalent equations (isolate x):

3x+2 = 5x – 9

3x + 2 – 5x + 9 = 0 bring all on one side

2 = 5x – 3x – 9 solve for positive x

1/5 ( x-1) = 4 + (x+2) multiply by five first before distributing

While modeling, be sure to discuss (ask students) which property is being applied. It is important to have students explicitly identify the properties used in their solution process. All Properties of Real Numbers are an important part of Algebra. Knowing how and when to apply each property is more important than knowing the names of properties. Repeated use of the correct names by teachers and students will help students remember the names of properties.

Preferred methods may not necessarily be those that are efficient or elegant. For some students the method they prefer is that

which helps them avoid making mistakes.

**Anticipated Student Preconceptions/Misconceptions**

Pre-conceptions:

* There is only one way to solve an equation
* to solve an equation means you ‘get the x’ alone on one side

### Required Prior Knowledge

### General Principles: Expressions and Equations

1. Algebraic ***expressions*** involve numbers, variables, and operation

Example: *7(2p – 1)* and *10 – 3p* are both **expressions**.

1. Number Properties of Operations are used to create **equivalent expressions**.

Example: *7(2p – 1)* is equivalent to *14p -7* (by Distribution)

1. An **Equation** is a statement that two expressions are equivalent.

Example: *7(2p -1)* ***=*** *10 – 3p*

1. Number Properties of Equality and Operations are used to create **equivalent equations**.

Example:

*14p – 7* ***=*** *10 – 3p*

*14p – 7* ***+ 3p******=*** *10 – 3p* ***+ 3p*** *(*Addition Property of Equality)

14p + 3p – 7 ***=*** 10 (Properties of Operations)

17p – 7 ***=*** 10 (Properties of Operations)

1. **17p – 7 = 10** and **14p – 7 = 10 – 3p** and **7(2p - 1) = 10 – 3p** are all equivalent equations

### General principles for solving Linear Equations:

1. Solving a linear equation requires finding a value of the variable for which the two expressions are equivalent.
2. Equivalent Equations have the same solution.
   * **Example: The equation 7(2p - 1) = 10 – 3p**  has the same solution as equation 17p – 7 = 10
3. To solve a linear equation algebraically requires creating (simpler) equivalent equations until eventually there is at most a single variable term and a single constant term.

7(2p – 1) = 10 – 3p

14p – 7 = 10 – 3p Property of Operation (Distributive)

17p – 7 = 10 Property of Equality (Addition)

17p = 17 Property of Equality (Addition)

p = 1 Property of Equality (Division)

* The simplest equivalent equation (such as p = 1 above) has a single variable term and a single constant term.
* The constant term in the simplest equivalent equation of a linear equation is the value of the variable that solves the equation or the solution to the equation.
* For the original equation, 7(2p – 1) = 10 – 3p, **p = 1 is the simplest equivalent equation**.
* The **solution** to the equation 7(2p-1) = 10 – 3p is p = 1. **1** is the value of the variable, **p**, for which the two expressions are equivalent and **1** is also a solution to each equivalent equation created in the **equation solving** process.

**Lesson Sequence** :

See Above: *Required Prior Knowledge* before starting this lesson. Do not start this lesson until students have the necessary prior knowledge. This lesson builds on the prior knowledge noted above. Students must be able to work with expressions and equations and know the difference. They must know that solving an equation is a process of creating equivalent equations. Creating equivalent equations is done by applying Properties of Equality.

***Once begun, half done***: For students to be fluent equation solvers they have to be able to formulate an approach to the equation that will allow them to begin the process of creating simpler equivalent equations.

THERE ARE MANY WAYS FOR THIS TO HAPPEN.

1. Handout-*Equation Solving First Steps*
2. In this activity, students will first reflect on a “first step” to begin to solve an equation. They will share their ideas with a partner. Have a whole class share out so students can hear about the various ways students are thinking.
3. Students will complete the table then share with a partner. Select a variety of student work to show on a document reader (or whiteboard) to share with the class.
4. Students will ‘take the first step’ in the next 6 problems. The teacher will strategically select student work to share with the class. Select two problems for students to complete individually using a few different approaches. Compare answers from these different approaches
5. Conclude with a discussion about a general guideline for solving single variable linear equations. For example:

**Create simpler equivalent equations until the variable term is alone on one side of the equation and has a coefficient of 1**

Avoid being too formulaic about first steps. Emphasize the word **simpler** to mean any approach that helps simplify the expressions or the terms in the equation itself will lead to a solution eventually.

1. Two application problems are provided for practice. The first problem requires students to understand some mathematical terms such as consecutive even and perimeter. The answer is 234 meters (the sides are 76-78-80). The second problem answer is 60 years old. (The life events are sequential: 1/5x + ¼ x + ½ x + 3 = x)
2. Handout- *Equation Solving Strategies*

The guidelines above for solving equations algebraically (or symbolically)

do not specify the order that operations are applied nor which operations to apply. This activity will give an opportunity for students to reason about solution strategies that are more efficient and can reduce the chance of student errors. The handout begins with strategizing with the constant terms and variable terms followed by considering mathematical structures

1. Have students complete question 1 then compare the strategies in the table to their own strategy.
2. Students will reflect on the questions asked and discuss with a partner
3. Continue with the second problem. Emphasize there is no ‘right’ or ‘wrong’ way to solve a problem. The process needs to be logical and maintain equivalence at each step.

Note the strategies in the examples:

* Begin with variable terms
  + Some students prefer variables on the left
  + Some students prefer a positive coefficient
* Begin with non-variable terms
  + Subtracting so numbers remain positive
  + Move numbers to the left
* Set the equation equal to zero
  + Move all terms to one side of the equal sign
* Begin with both variables and non-variable terms
  + Moving all variable terms to one side and all non-variable terms to the other side

Some students may demonstrate an “inefficiency” in equation solving (for example, in solving the equation 3x – 4 = 9, students may choose to subtract 9 from both sides as a first step.) While this is not an incorrect approach to equation solving, it improves the likelihood of a mistake in calculations. When students have more fluency in justifying the steps of solving equations, encourage them to choose more efficient solving methods.

1. **Considering the structure of the terms in an equation**

The next three problems consider the structure of the equation in terms of common factors. Students will practice problems on their own and discuss the strategies in their solutions.

**Formative assessment): The last three problems and the writing prompt can be used to assess where students are in understanding solving linear equations**

1. Handout- *Further Reasoning* *with Equations*

Students will think about the structure and terms in an equation to answer the question. (This will help students in CEPA task to create equations)

### Equation Solving First Steps Reasoning about the Equation Solving Process

* 1. Consider how to create a simpler equivalent equation from the given equation below. List some ways you are considering here:
  2. Turn and talk with a partner. Be prepared to share out with the class.
  3. Solve the given equation by the process of **creating simpler equivalent equations.**  Write each new equivalent equation in the left column and your justification for it in the right column. Justifications can be naming the property you applied or describing the action or operation you applied.

|  |  |
| --- | --- |
| Equation to solve | Justification |
| 7(x + 1) – 4(5 -2x) = - 5(x – 9) | Given Equation |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

* 1. **Take the First Step**: Write the first step you would take to solve each equation below. Do not solve completely. Be prepared to discuss your first step with the class.

|  |  |  |
| --- | --- | --- |
| 1. 5 w = 11 | 1. 19z -10 + 4z = 3z – 4 | 1. 5(x + 3) = (x – 1) |
| 1. 10a = 6 +2(5a – 1) | 1. 8x - 12 = 4( x – 3) + 4x | 1. -2a – 4 = 6(1 – a) -1 |

Applications

Create an equation to model the situation. Then solve the equation to find the solution:

* 1. The lengths in meters of the sides of a triangle are three consecutive even integers whose sum is 154 more than the greatest integer. What is the perimeter of the triangle?

1. A man has lived a fifth of his life as a boy, a fourth as a young man, a half as a married man, and has been a widow for 3 years. How old is he?

# **II. Equation Solving Strategies**

Constants and variable terms

1. Solve the equation **3g – 70 = 10g -7.** Show the steps in your solution process.
2. Compare your solution process to the 3 different solution processes in the table below:

|  |  |  |
| --- | --- | --- |
| **Student 1** | **Student 2** | **Student 3** |
| **3g – 70 = 10g – 7** | **3g – 70 = 10g – 7** | **3g – 70 = 10g – 7** |
| **3g = 10g + 63** | **-7g – 70 = -7** | **3g – 70 – 10g +7 = 0** |
| **-7g = 63** | **-7g = 63** | **-7g - 63 = 0** |
| **g = -9** | **g = -9** | **-7g = 63** |
|  |  | **g = -9** |
| 1. How are the solutions similar? How are they different? How do they compare to your solution? 2. How do they compare in terms of efficiency? 3. Turn and share with a partner your thoughts about the similarities and differences of each. Be prepared to share your insights with the class. | | |

|  |
| --- |
| 1. Solve the problem below and show your steps   **6x – 5 + 2 (4x - 1) = -5 (2x – 3) + 6x – 7**   1. Compare your solution with another student. Discuss any similarities and/or differences in your strategies. |

|  |
| --- |
| 1. Consider this solution by another student:   6x – 5 + 2 (4x - 1) = -5 (2x – 3) + 6x – 7  6x – 5 + 8x - 2 = - 10x + 15 + 6x - 7  6x + 8x + 10x – 6x = 15 – 7 + 5 + 2  18x = 15     1. How was your solution similar and/or different? 2. What was this student’s strategy? Turn and share your thoughts with a table partner. Be prepared to share with the class. |

Considering Structure in the solutions of Equations:

1. Solve the equation ***7 (3x – 8) = 14?*** Show the steps that lead to your solution.
2. Compare the Student solutions in the table below to your solution. Describe the first step by each student below in terms of the property applied.

|  |  |
| --- | --- |
| Student 1 | Student 2 |
| *7 (3x – 8) = 14* | *7 (3x – 8) = 14* |
| 21x -56 = 14 | 3x – 8 = 2 |
| 21x = 70 | 3x = 10 |
| x = 10/3 | x = 10/3 |

1. How did the use of the structure of the equation show in Student 2’s solution?
2. How do the two solutions compare in terms of efficiency?

Compare the solutions by two students in each of the following problems.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Student 1 | Student 2 |  | Student 1 | Student 2 |
| 1. 4(x – 1) + 3 (x -1) = 7 | |  | 1. (x + 5) +(x + 5) = 7 | |
| 4x – 4 + 3x – 3 = 7 | 7(x – 1) =7 |  | x + 2 +x + 3 = 7 | x + 5 = 7 |
| 7x – 7 = 7 | x – 1 = 1 |  | x + 5 = 7 | x = 2 |
| 7x = 14 | x = 2 |  | x = 2 |  |
| x = 2 |  |  |  |  |
|  |  |  |  |  |
| How was structure used by Student 2 to solve each equation? What was efficient about the solutions? | | | | |

|  |
| --- |
| Find the solution to the following equations. Plan your strategy and be prepared to discuss it. |
| 1. 12z -15z -8 + 6 = 4z + 6 -1 |
| 1. -9m – (4+ 3m) = - (2m – 1) -5 |
|  |
| Writing prompt. Suppose you have to help a new student to solve linear equations. Explain in your own words the process for finding the solution to a linear equation. |

### Further reasoning with Equations

|  |  |  |  |
| --- | --- | --- | --- |
| For the equations below, replace each **?** with a number or an expression so that each of the following conditions are satisfied. | | | |
| 1. The solution is 2 | 1. The equation is an Identity | 1. No solution | 1. The solution is 0 |
| * 1. 4 – (t – 2) = \_\_**? \_\_**  1. b) 2. d) | | | |
| * 1. – 7x + \_\_**?\_\_** = 3x -7  1. b) 2. d) | | | |
| * 1. Turn and talk to a partner and discuss the strategies you used to satisfy each of the conditions above.   What condition(s) was the easiest to satisfy? What condition(s) was most difficult to satisfy? | | | |

Lesson 8

Rules Gone Wild

**Brief Overview of Lesson:** In this lesson, students will learn touncover faulty reasoning in the equation solving process. As you plan, consider the variability of learners in your class and make adaptations as necessary.

**Prior Knowledge Required:** Understand Properties of Operations, Properties of Equality, and the equation solving process.

**Estimated Time** 45 minutes

**Resources for Lesson**

Handouts:

Rules Gone Wild

Errors Activity Sheet

**Unit:** Reasoning with Equations

**Content Area/Course:** Algebra 1

**Time (minutes):** 45 minutes

**Lesson 8: *Rules Gone Wild***

**Overview:** Students will use logical reasoning to find flaws in a solution solving process. Students will find errors in solutions to equations.

**By the end of this lesson students will know and be able to:**

Identify the faulty reasoning in an equation solving process and justify the correct process.

**Essential Question addressed in this lesson:**

How does one reason logically with equations?

**Standard(s)/Unit Goal(s) to be addressed in this lesson (type each standard/goal exactly as written in the framework):**

**A-REI.A.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify or refute a solution method.

**SMP**.**3** Construct viable arguments and critique the reasoning of others.

Anticipated Student Preconceptions/Misconceptions

Common Errors: Operating with rational numbers; following the order of operations, misapplying properties of Equality and Properties of operations

**Teacher Notes:**

There is a distinction between a simple *mistake* and a *mathematical fallacy* in a proof: a mistake in a proof leads to an invalid proof just in the same way, but in the best-known examples of mathematical fallacies, there is some concealment in the presentation of the proof. For example the reason validity fails here is a [division by zero](http://en.wikipedia.org/wiki/Division_by_zero) that is hidden by algebraic notation. Although the proofs are flawed, the errors, are comparatively subtle, or designed to show that certain steps are conditional, and should not be applied in the cases that are the ***exceptions to the rules.***

The traditional way of presenting a mathematical fallacy is to give an invalid step of deduction mixed in with valid steps. Beyond pedagogy, the resolution of the fallacy can lead to deeper understanding of the properties.

In [high school algebra](http://en.wikipedia.org/wiki/Elementary_algebra), other typical examples may involve where a [root](http://en.wikipedia.org/wiki/Root_of_a_function) is incorrectly extracted or, more generally, where different values of a [multiple valued function](http://en.wikipedia.org/wiki/Multiple_valued_function) are equated.

### Fallacies based on division by zero

It is possible to disguise a special case of division by zero in an algebraic argument, leading to proofs of obvious contradictions such as 1 = 2 as in the following:

With the following assumptions:

0 ∙ ( x + 1) = 0

0 ∙ ( x + 2) = 0

Then the following must be true: 0 ∙ (x + 1) = 0 ∙ ( x + 2)

Dividing by zero gives**:**

. Solving, yields: x+1 = x+2

So, 1 = 2. The fallacy is the implicit assumption that dividing by 0 is a legitimate operation

Instructional Tips/Strategies/Suggestions:

The 2=1 activity is a great lesson for students to come to understanding of the division equality property and the caveat of division by 0. This lesson will help students understand the Division Property of Equality and the exclusion of zero.

1. Handout – Rules Gone wild
2. Students will complete column 2 with an appropriate reason using the properties of operations and/or the properties of equality.
3. Students will discuss their reasons with a small group and come to a consensus for column 2.

# RULES GONE WILD EXPOSING FAULTY REASONING

Proof that 2 = 1:

Study the proof below. Compare the statements and reasons in column 2. Then answer the questions below.

|  |  |
| --- | --- |
| Statement | Property |
| 1. a = b | 1. You are given that two numbers called a and b are equal |
| 1. a2 = a b | 1. Multiplication Prop. of Equality |
| 1. a2 - b2 = a b - b2 | 1. Subtraction Prop. of Equality |
| 1. (a - b)(a + b) = b (a - b) | 1. Distribution (Reversed) |
| 1. a + b = b | 1. Division |
| 1. b + b = b | 1. Substitution |
| 1. 2b = b | 1. “Combine like terms” (By Distribution:   b + b = b ( 1 + 1) = b(2) = 2b |
| 1. 2 = 1 | 1. Division Prop. of Equality |

Can you find the flaw in the reasoning? Consider both the statements and the reasons.

Discuss your thoughts with your group .

### Error Analysis

### Some of the problems below have errors. Some do not. Determine which have errors and which do not. You will then work with a small group to compare your results. For any discrepancies among your work you will need to come to a consensus. As a group you will pass in your results with corrections made and justified.

|  |  |  |  |
| --- | --- | --- | --- |
| 1 |  | 2 |  |
| 3 |  | 4 |  |
| 5 |  | 6 | No Solution |

|  |
| --- |
| ***Find the lengths of the sides of the triangle if the perimeter is 3 times the length of side AC***  A    11 - x 3x – 2  B x + 2 C |

**List of Unit Resources**

###### Lesson 1 Reasoning about Equation Solving

Problem Solving

Rule of Four Link

Ticket to Leave- Checking Solutions to Equations

###### Lesson 2 Number Properties of Operations Extended to Algebraic Expressions

Number Properties for Addition and Multiplication

What about Subtraction and Division Number Properties?

Practice and Applications

###### Lesson 3 Reasoning Logically

Pre-Assessment

Why is Reasoning Logically Important in Mathematics

You Try

###### Lesson 4 What’s My Number?

Generation N… What’s My Number?

Equivalence in Equation Solving

Equivalent Equations

Maintaining Equivalence

###### Lesson 5 Equation or Identity

MARS Lesson Sorting Equations and Identities - lesson found at: <http://map.mathshell.org/materials/lessons.php?taskid=218>

###### Lesson 6 Justifying the Solution Process

Properties of Equality

Practice Problems on Comparing Solution Process

Graphing Activity Sheets (created at graphsketch.com)

Property Matching Activity

Visualizing the Equation Solving process

Graphing Tool: Graphsketch.com

###### Lesson 7 Equation Solving Strategies

Equation solving Process

Applications

Strategies

Further reasoning

###### Lesson 8 Rules Gone Wild

Exposing Faulty reasoning

Error Analysis

**Curriculum Embedded Performance Assessment (CEPA)**

As a US Counter Intelligence officer, you have been assigned to track down Mathew Wrong, a US defector, and his foreign contacts. Mathew Wrong was a computer programmer working as a subcontractor for WeCompute Inc. He illegally accessed sensitive United States Armed Forces information on combat readiness. U.S. Intelligence sources reported him to be out of the country. It is assumed he will try to sell the information for money. If he is not caught quickly it would put the United States at risk.

To help track Mathew down, you are required to devise a series of math equations that will be used to pass information to others. You will create equations whose solutions may or may not be true. Those with true solutions will be passed along to other US Intelligence officers and those with false solutions will be passed to foreign spies. Your ability to create false solutions that appear to be true is a hallmark of your Intelligence abilities. Your superiors require you to use expressions, equations, and identities in your work and prove that the equations to the allies are solved correctly and the equations to the enemies are solved incorrectly. In order to solve them correctly, you will have to abide by the Properties of Operations Code used by all of the Intelligence officers.

You will create a presentation using precise mathematical language for the commanding officer (for example, report, podcast, script, Jing, PowerPoint/Keynote, etc.) that includes the sets of equations with both faulty and correct solutions. All solutions should be clear and coherent and show appropriate use of the number Properties of both Equality and Operations.

Procedure for equations to be passed along to US Intelligence:

1. Create a variety of equations (minimum of 5) having each of the following characteristics:
   1. True for all numbers
   2. True for some numbers
   3. No solutions
2. Equations should
   1. require multiple steps to solve
   2. have variables on both sides of the equation
   3. require the use of distribution
   4. require combining like terms
   5. contain fractional coefficients (in two or more equations)
3. Verify (prove) the solutions for each of the equations you create using the number Properties of Operations and the number Properties of Equality and by reasoning logically from each step in the solution process.

Procedure for equations to be passed to foreign spies

1. Create a minimum of 2 equations that have incorrect solutions.
   1. Create the equations and a flawed proof by misapplying a number Property of Operations and/or a number Property of Equality.
   2. Make it subtle so that it cannot easily be detected and is difficult to check in the original solution

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| CATEGORY | 4 | 3 | 2 | 1 |
| Equations and Solutions Concept | Equations are varied and show complete understanding of the nature of solutions to equations | Equations are varied and show substantial understanding of the nature of solutions to equations | Equations are varied and show some understanding of the nature of solutions to equations | Equations show very limited understanding of the nature of solutions to equations |
| Equation Characteristics | All equations meet the requirements of having multiple steps to solve, variables on both sides, require the use of distribution and combining like terms | All equations meet most of the requirements of having multiple steps to solve, variables on both sides, require the use of distribution and combining like terms | Some equations meet the requirements of having multiple steps to solve, variables on both sides, require the use of distribution and combining like terms | Equations meet some of the requirements of having multiple steps to solve, variables on both sides, require the use of distribution and combining like terms |
| Mathematical Reasoning | Uses complex and refined mathematical reasoning in applying number Properties of both Operations and Equality | Uses effective mathematical reasoning in applying number Properties of both Operations and Equality | Some evidence of mathematical reasoning in applying number Properties of both Operations and Equality | Little evidence of mathematical reasoning in applying number Properties of both Operations and Equality |
| Mathematical Language | Uses precise mathematical language in all verbal and/or written communication | Uses precise mathematical language in most of the verbal and/or written communication | Uses precise mathematical language in some of the verbal and/or written communication | Uses little precise mathematical language in verbal and written communication |
| Completion | All work is complete for both equations with correct solutions and equations with incorrect solutions. | Most of the work is complete for both equations with correct solutions and equations with incorrect solutions. | Half of the work is completed | Several of the equations are not completed. |
| Mathematical Errors | 90-100% of the steps and solutions have no mathematical errors. | Almost all (85-89%) of the steps and solutions have no mathematical errors. | Most (75-84%) of the steps and solutions have no mathematical errors. | More than 75% of the steps and solutions have mathematical errors. |

**CEPA Rubric**

1. Language encompasses the use of appropriate vocabulary, sentence structure, and coherent discourse. [↑](#footnote-ref-1)