Note: Not all slides have talking points provided.

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| **Slide** | **Talking Points** |
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| 5 | We’re about to look at several examples of ways to start with a simple math problem and revise the problem slightly to increase the level of mathematical rigor.As you go through these examples, think about what characterizes the problems that are more rigorous, and how that fits with your initial brainstormed definition of rigor. |
| 6 | (Click space bar to reveal each bullet):**6 + 5 = \_\_\_:** This is the kind of math problem that most people are accustomed to seeing. It’s a straightforward computation problem, and the goal is simply to accurately find the answer. This is a cognitively low-demand task; the only mathematical task required is computation and there is a proscribed procedure to follow to do the computation.**What pairs of numbers….:** If you change the problem slightly to instead ask for the pairs of numbers that add up to 11, all of a sudden the problem is a little richer. You are still achieving the goal of computing correctly that you had in the previous version of the problem, but now there are more opportunities to do so embedded in the same problem. While this is still a cognitively low-demand task, it is more rigorous than the previous one because now you need to think about different ways to make 11 out of two quantities. There is still a proscribed procedure to follow, but the procedure now involves computation, making comparisons and keeping track of the list of pairs.**What pairs can you find if you don’t…:** Many people will think (at least at first) only about whole numbers. If you ask the question slightly differently now, you increase the rigor by having to consider different classes of numbers: whole numbers, positive and negative numbers, fractions, decimals and others.This is a cognitively higher-demand task because now you have to think more conceptually about how different KINDS of numbers can be combined to make 11. There is not merely a single correct answer to find, but a variety of correct answers to find, and those answers can be used to find others. (I.e. If I know that 9½ + 1½ makes 11, I might reason that 8½ + 2½ also makes 11. Or if I know that 8.5 + 2.5 makes 11, I might reason that 7.5 + 3.5 makes 11.) The process for arriving at a solution is not as prescribed as in the prior two examples.**How do you know…:** By adding this final question, you increase the level of rigor yet again. This is a very cognitively demanding task. Not only is there no prescribed way to arrive at a solution, but the process involves analyzing, testing, comparing and justifying. |
| 7 | Rigor does not necessarily mean going faster or going further ahead in the material. It means delving deeper into the mathematics.To get a little more sense of what “going deeper” means, we’re going to look at two examples of math problems adapted in four different ways – like the 6+5 example – to show increasing levels of rigor. It’s not important that all of you feel like you know how to solve the problems – what matters more here is to look for the difference in the kind of mathematical reasoning needed to solve the problem. |
| 8 | Here’s the first example. Answering this question involves a straightforward memorization task. It’s a cognitively low-demand task.(Note that cognitively low-demand mathematical tasks don’t necessarily mean that they are somehow “inferior” or less desirable than higher cognitively demanding tasks. They are an important part of being able to do mathematics. However, what IS less desirable is a steady diet for students of only low cognitively demand tasks.) |
| 9 | We can change the question in this way to increase the level of rigor somewhat. Again, there’s still a prescribed procedure (or choice of several procedures) that can be used to solve the problem. However, now there is a greater mathematical requirement than memorization needed to solve the problem. |
| 10 | If we revise the problem in this way, the rigor increases yet again. In this version of the problem, you have to draw on your understanding of “average” and what an average tells you about a set of data in order to solve the problem. You might use some familiar procedures about finding averages to figure this out, but the point of the problem is not to simply complete a computation. |
| 11 | This problem now raises the level of rigor yet again. Again, there is no prescribed way to go about solving this problem; you need to draw on what you know about averages to think about this problem. While you might call it a “word problem,” it differs from many textbook word problems. Often textbook word problems just involve completing a known procedure for a problem couched in a “real-world” situation – but there is still a specific procedure to practice in order to solve the problem. This problem is more rigorous than that sort of textbook example because it requires the learner to reason about averages, to make choices about what mathematical tools to use, and to justify their thinking using mathematics. |
| 12 | Here’s another example – this time in geometry.This problem is an example of a memorization task. |
| 13 | We can raise the level of rigor slightly by asking this kind of very familiar problem. Now there is memorization required to recall the formula, and computation skills to apply to find the area. However, it is still a cognitively low-demand task. |
| 14 | Changing the problem in this way increases the rigor yet again and makes the task a cognitively higher demand task. Students have to not just complete a procedure but use that procedure in the service of answering a more complex question. They have to test their ideas, organize their work and thinking, compare their results and draw a conclusion. |
| 15 | This problem increases the rigor yet again. There is no prescribed way to solve the problem, you need to draw on what you know about area and perimeter to reason about the problem, and you have to describe your thinking. |
| 16 | In summary, we want to move away from a vision of “rigor” that means doing more and doing it faster to a vision of “rigor” that means going into greater depth with mathematical ideas, and paying attention to learning and understanding concepts, not JUST completing procedures. Procedures are an important companion to conceptual understanding, but a steady diet of procedures alone is not enough for students to learn mathematics. |
| 17 | The new MA mathematics curriculum frameworks have 2 important parts:1. The Standards for Mathematical Content and
2. The Standards for Mathematical Practice.

The Standards for Mathematical Practice are the ones we’ll be paying attention to today. These are a part of every grade level, PreK-12 and capture a vision of the “habits of mind” of a mathematically proficient student – ways of approaching mathematical tasks, and reasoning and making sense of them. |
| 18 | There are 8 Standards for Mathematical Practice, listed here. You received a handout with the text of these 8 standards before this meeting to give yourself time to read them. We’ll be paying special attention to #3 today. |
| 19 | The 8 math practices describe the essence of what is expected from these frameworks. While they drew on other descriptions of mathematical processes from other professional organizations in mathematics and education, they are not identical to those that have been supported. They go a step further in their rigor, depth and specificity in describing ways that students engage with mathematics.Paying attention only to the content standards and overlooking the math practices would result in falling far short of meeting the new math standards. |
| 20 | This is more than “advanced problem solving.” The Standards for Mathematical Practice define a next step in the level of expectation that all students have an opportunity to engage with rigorous mathematics. |
| 21 | These practices or mathematical “habits of mind” describe what students –**at all grades**– must develop and use. |
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| 23 | We’re going to take 5-7 minutes to give you an opportunity to start work on the problem. It’s important for everyone to at least give it a try for 2 reasons: 1) you will see students grappling with this problem in the video, and it really helps to understand what the problem is about as you listen to them; 2) You are not expected to finish it; just get into it enough to have a sense of what the problem is about and to pay attention to how you are thinking about the pattern in the problem. |
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| 29 | Today’s protocol was intended to help the team build shared knowledge about the Standards for Mathematical Practice that are part of the new MA curriculum frameworks in mathematics.The next two protocols will now lay out a framework for making this mathematics accessible to students with disabilities. The next one will introduce the framework in the context of just one student, and the final protocol will extend that framework to a classroom of students with varied learning needs. |