Mathematics Learning Community
Number Sense
Session 11

Title: *Representing and Interpreting Fractions*

Common Core State Standards Addressed in the LASW Problem:

4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $1/2$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Standards for Mathematical Practice Addressed in the MLC Session:

2: Reason abstractly and quantitatively.   **4:** Model with Mathematics.

MLC members use a number line model during Math Metacognition to reason about fractions as locations on a number line. Students use models to help compare quantities represented as fractions in the LASW problem. Once created, these models need to be interpreted as students use their reasoning skills to determine which quantity is greatest.

Standards-Based Teaching and Learning Characteristics in Mathematics Addressed in the MLC Session:

- 5.1 Depth of content knowledge is evident throughout the presentation of the lesson.
- 5.2 Through the use of probing questions and student responses, decisions are made about what direction to take, what to emphasize, and what to extend in order to build students' mathematical understanding.
- 5.3 Students' prior knowledge is incorporated as new mathematical concepts are introduced.
- 5.4 Student misconceptions are anticipated /identified and addressed.
- 5.5 Classroom strategies incorporate multiple forms of representation.

Session Agenda:

Part I: Mathematical Background
Part II: Math Metacognition
Part III: Looking at Student Work   • *Time Spent on Test* Problem   (Grade 5)
Part IV: Our Learning
Part V: Feedback and Wrap-up

Materials Needed for this Session:

- ✓ Nametags
- ✓ Index cards
- ✓ Chart paper and markers
- ✓ Refreshments
- ✓ Copies of handouts
- ✓ Highlighters
- ✓ Student Work Samples

Possible Ways to Personalize this Session

- Beginning with this session, student work samples are no longer provided for the LASW problem. Instead, MLC members will provide the samples for the given problem. See Page 5 in Part III for more details.
- Two Guiding Questions are provided in the Student Work Analysis Grid – the other two can be selected by you or by the group.
- If time is short at the end of the session, have group members take their Exit Card "to go" and drop it off later in your mailbox.

Part I: Mathematical Background
Approximate Time: 20 Minutes
Grouping: Whole Group

A. **Welcome** members of your group to the Math Learning Community. Remind group of **established norms.**

B.   **Today's Content**:
   a. The mathematics during this session focuses on representing and interpreting fractions.
   b. What do we need to know in order to be able to represent and interpret fractions?
   c. Chart ideas to refer to during the Protocol for LASW.
   d. There should be significant development of fraction sense prior to operating with fractions, including: working with different models for fractions, exploring part/whole relationships and other types of fraction representations.
   e. **Types of fraction representations:**
      i. Ratios
      ii. Percents
      iii. Fraction of a Region
      iv. Probability
      v. Decimals
      vi. Measurement on a number line
      vii. Operations with Fractions (division)
      viii. Clocks (time measure, degree measure)
      ix. Proportion (common denominators, common numerators)
      x. Literal translation (story, event)
   f. How do the ways in which children learn to represent and interpret fractions relate to the ways in which they understand whole numbers?

C. **Relating Content to the Three C's Theme:**
   a. Students need experience in defining the unit based upon a problem context. Teachers should discuss what decisions are involved in the process of defining the unit. How do these decisions affect what results as a solution to the problem? Both of the problems in this session involve defining a unit and then stating an answer as a result of the defined unit.
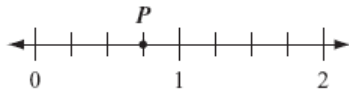
Part II: Math Metacognition
Approximate Time: 25 minutes
Grouping: Whole Group

A. **Problem:** The source of the problem used for this session is a multiple-choice MA DESE Released Test Item from the 2006 Grade 5 MCAS.

> Point *P* is located on the number line shown below.
>
>
>
> Which of the following fractions best represents the location of point *P*?
>
> A. $\dfrac{1}{4}$ B. $\dfrac{3}{8}$ C. $\dfrac{3}{4}$ D. $\dfrac{4}{5}$

B. **Solution to Problem:** C

➤ **Problem Intent:** (*Note: The problem intent for all Math Metacognition problems is the same*). See Session 2 for more information. In addition, this problem serves as a good start to discussing representing fractional quantities, in particular on a number line. We previously explored the number line as a tool to notate a strategy. Here, the number line is used to represent a quantity. The length of the segment from zero to Point P represents the fractional amount ¾ of a unit. However, many of the same issues still arise:
   a. Once again, we need to consider counting: does one count zero on a number line? Does one count tic marks or the spaces in between the marks?
   b. Is the number found on, above, or below the number line? Is the answer in the number of jumps or a point on the number line?

C. **Discuss** each of the four possible answer choices:
   a. What would be the reasoning behind choices A, B and D? What mathematical ideas are inherent in each of these 3 options?

Choice **A:** $\dfrac{1}{4}$	Choice **B:** $\dfrac{3}{8}$
Reasoning behind this choice:	Reasoning behind this choice:
▪ P is ¼ unit away from 1. ▪ Intervals are marked at ¼ units.	▪ P is 3/8 of the way to 2. ▪ Consider 2 to be the whole, rather than 1.

Choice **D:** $\dfrac{4}{5}$
Reasoning behind this choice:
▪ 5 tic marks from 0 to 1   ▪ P is on the 4th tic mark   ▪ Counting tic marks rather than considering the length as the quantity to be measured

Part III: Looking at Student Work (LASW)
Approximate Time: 50 minutes
Grouping: Refer to protocol

A. Complete the **MLC protocol** with the group.

B. **Problem**: The source of the problem used for this session is the stem of a multiple-choice MA DESE Released Test Item from the 2006 Grade 5 MCAS. This item was a multiple choice question, but the choices were simply the names of each of the 4 students. Instead, the direction: "Show how you figured out your answer" was added.

The table below shows the amount of time each of four students spent on a mathematics test yesterday.

Time Spent on Mathematics Test

Name	Time (in hours)	Name	Time (in hours)
Joe	$\dfrac{1}{2}$	Lena	$\dfrac{1}{4}$
Keith	$\dfrac{2}{3}$	Mia	$\dfrac{2}{5}$

Which student spent the **greatest** amount of time on the test?

Show how you figured out your answer.

C. **Solution**: Keith.

D. **Problem Intent**: This problem helps to begin a conversation around the need for students to have experiences in defining the unit based upon a problem context. Teachers should discuss what decisions are involved in the process of defining the unit. How do these decisions affect what results as a solution to the problem?

E. **Discuss** the strategies used to solve this problem. Pay particular attention to different types of fraction representations, especially those that allow for accurate comparison of quantities. Representations that you may see include: percents, using proportions to find common denominators or common numerators, decimals, clocks, and fractions of a region.

F. **Discuss** the following mathematical idea:
   a. When **cross products** are used as a means to determine which fractional quantity is larger, discuss how/why this method works. Try and identify what cross products represent. (A cross product is the numerator of each fraction given a common denominator). This method may be a quick method to compare two quantities, but it does not help students to develop fraction sense, especially if they are already shaky in their current understanding. Cross products mask the relative nature of fractions by removing the quantity in the denominator. Likewise, the

method of cross-multiplication used to solve proportions can also make it difficult for students to fully develop fraction sense.

Part III: Looking at Student Work (LASW), continued

G. **Misconceptions/Questions that May Arise:**
   a. Do students understand the concept of fractions or the skill of finding a common denominator, or both? How do you know?
   b. If the solution to this problem serves as baseline information about this student's understanding of fractions, how would you build on it?

Options for Customization

H. **Guiding Questions:** Two Guiding Questions have been provided in the Student Work Analysis Grid for this problem. As a way to customize the LASW process, you (or your group) will need to decide on the remaining two questions. You can use the two questions listed below that are specific to this problem or refer to the list of generic questions found on Page 5 in Session 7.
   a. What strategy is used to compare the different amounts of time spent on the test?
   b. Can the student's strategy used be generalized to use with other fractional amounts? If so, how? If not, why?

I. **Using A Group Member's Student Work:**
   a. *Prior to offering this session to your MLC, you need to collect student work samples for the task:* **Time Spent on Test.**
   b. See Page 5 in Session 7 for more details on collecting student work samples. For this task, select 4 samples to discuss during the MLC session. Prior to photocopying samples, mark them as **A, B, C,** and **D.**

Part IV: Our Learning
Approximate Time: 20 Minutes
Grouping: Whole Group

A. **Discussion**: After evidence of student understanding has been discussed as a whole group, you want to facilitate discussion around how the LASW process will impact what teachers do within their classrooms. Some questions to help guide discussion include:
   a. What do we take away after LASW?
   b. What did we learn? About student thinking? About our own knowledge?
      1. Refer back to chart made at the beginning of the session
   c. How does today's session relate to important mathematical content and pedagogy?
   d. How does it impact **your** practice at **your** grade level? *(Note: In order to help teachers connect this session to the mathematics within their own grade level, refer to the information below).*

> **Making Connections** Across the Grade Levels
>
> **K – 2**: This session highlights the complexity of fraction notation and representation. The development of fraction sense arises at this stage in school as students realize that in some contexts it is impossible to fair share – how can toys or food be split evenly amongst a group of children when there are not enough whole pieces to go around? Representing common fractions using various models begins here as well. (1.G.3, 2.G.3)
>
> **3 – 5**: The LASW problem is appropriate for Grades 4 & 5 to explore and specifically addresses learning standard 4.NF.2. In addition, the Math Metacognition problem is a Grade 5 problem. During these years in upper elementary school, it is important for students to really develop their fraction sense by having lot of experiences with different models of fractions and by spending a good deal of time thinking about fractions as quantities **prior** to performing operations on them. (3.NF.1, 3.NF.2 a-b, 3.NF.3 a-d, 3.G.2, 4.NF.1, 5.NF.2, 5.NF.3)
>
> **6 – 8**: Working with fractions, decimals and percents is considered to be one of the "hallmarks" of number sense in middle school (with the other being integers). It is imperative that students have a solid foundation of fraction sense developed during grades K – 5 so that successful computation with fractions can occur in Grades 6 – 8. Rational numbers are extremely complex and worthy of a great deal of study and practice. (6.RP.3, 6.NS.6 a-c, 6.NS.7, 7.NS.1 a-d, 7.EE.3)

B. **Writing a Problem or Task**: As a way to synthesize learning from today's session, ask MLC members to come up with a math problem or task that would embody the ideas discussed today. The problem should be appropriate to use at their grade level. Writing these problems will help both you as the facilitator and the other group members to develop a stronger sense of how these mathematical ideas show up in classrooms from grades K – 8. (*Note: See Part IV in Session 1 for more details*).

Part V: Feedback & Wrap-up

Approximate Time: 5 Minutes
Grouping: Individual

A. **Closing:** Close the session with a message such as: "Hope you leave here with more questions – about student thinking, about your teaching, and ways that we as a group can help support one another." Have group members keep in mind the following: Dialogue, Reflection, and Inquiry are the keys to successful learning.

B. **Exit Cards:** Pass out exit cards for group members and ask them to provide some feedback to you as the facilitator. Select one or two questions from the list below to help them summarize their thinking about the mathematics from today's session. Collect exit cards so that a summary can be shared during the next session.

Feedback / Exit Card Questions

- How does the mathematics that we explored connect to your own teaching?

- How do I see what we've done today relate to key mathematical ideas or pedagogical content knowledge?

- What idea or discussion topic did you find most interesting from today's session. Why?

- How was this session for you as a learner?

- What ideas were highlighted for you in today's session that you had not previously considered?

- What are you taking away from today's session?

Related Student Discourse Video Clip – LASW Problem

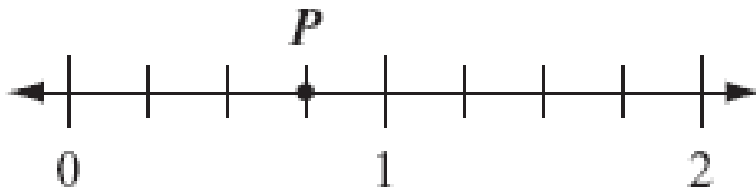*Content-Focused Coaching*
Disc 2: Grade 4
Problem: Representing Different Fractions on a Geoboard

Session Reference

- *Content-Focused Coaching: Transforming Mathematics Lessons*, by L. West and F. Staub, Heinemann/University of Pittsburgh, 2003.

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Problem Source: MA DESE Released Test Item, 2006 Grade 5 Mathematics MCAS

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Student Work Analysis for: Time Spent on Test

Student	What method(s) are used to find the solution to the problem?	How does the student's work show evidence of the understanding of fractions?		
A				
B				
C				
D				